Presented at the 17th Annual International Symposium on Forecasting 97, June 20 1997 in Barbados.

Forecasting Exchange Rate Dynamics Using GMM, Estimating Function and Numerical Conditional Variance Methods.

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A stochastic differential equation is at the heart of short term interest rate dynamics. Various parameter restrictions on the unrestricted model lead to eight special cases which are: Brennan and Schwartz, Cox-Ingersoll and Ross (CIR) square root process, Vasicek, CIR variable rate process, Dothan, constant elasticity of variance (CEV), geometric Brownian motion, and Merton. We apply these models to exchange rate dynamics and evaluate which model best fits the data from the viewpoint of out-of-sample forecasting. The estimation by generalized method of moments (GMM) methods is supplemented by newer small sigma asymptotics estimating function (SSA-EF) methods suggested in Vinod (1996) and a numerical conditional variance (NCV) method proposed here. Using daily, weekly, and monthly rates for the British pound and the Deutsche mark for the period 1975 to 1991, we find that the SSA-EFs and NCV methods outperform the traditional GMM methods by orders of magnitude of out-of-sample forecast errors.

1. Introduction

The models used to characterize short term <u>interest</u> rate movements capture the dynamics by a familiar stochastic differential equation (SDE). We claim that the same equation is applicable to short term <u>exchange</u> rate movements and verify this from an empirical standpoint. If true, this has implications for forecasting exchange rate changes, as well as for pricing currency derivatives and for hedging currency risk. In a recent interview of Clive Granger in *Econometric Theory* by Peter Phillips (1997, p. 272), Granger was asked "What would you like to see people (theoretical econometricians) doing?". His answer was to use a "predictability test, not a test of fit, so the fact that your model fits in-sample does not mean that it is going to forecast out of sample. The test that I push is that you actually build in-sample models ... then you ask which model actually forecasts the better out of sample, using a comparison of forecasts test."

The approach of this paper is straightforward. To test the claim mentioned above, we first use the Generalized Method of Moments (GMM) technique of Hansen (1982) outlined in Chan et al (1992) for evaluating alternative short term interest rate models. We find that in using these interest rate models to model exchange rates, the out of sample forecasts by

GMM yield intuitively large[1] sum of squared errors (SSE). Hence this paper proposes two new alternative estimation methods, SSA-EF and NCV to determine whether their out of sample forecasts have a reduced SSE . Before explaining the new estimation methods, let us review the traditional SDE used to model short term interest rate dynamics.

There are a number of specifications that have been proposed to capture the dynamic behavior of the short term interest rate[2]. We use a general SDE to characterize the short term behavior of the exchange rate x. This differential equation specification is similar to that used to describe the short term interest rate processes and takes the following form:

(1) $$dx=(\alpha+\beta x)dt+\sigma x^{\gamma}dW$$

<u>Remark 1 (Drift part)</u>: In our context, dx is the change in spot exchange rate, $(\alpha+\beta x)dt$ is the drift part. Now $(\alpha+\beta x)=\kappa(\mu-x)$ is the conditional mean of dx (in a long-run equilibrium) which depends on x through β, where the notation $\kappa\mu=\alpha$ and $\kappa=-\beta$ clarifies its relation to mean reversion toward μ at the speed of adjustment κ. This then represents mean reverting dynamics if and only if $\kappa>0$. Usually, statistically significant negative β values prevent explosive growth of the drift part.

<u>Remark 2 (Diffusion part)</u>: Since the conditional variance of dx is $(\sigma x^{\gamma})^2$, the volatility (measured by the standard deviation σx^{γ}) obviously depends on the level of the exchange rate x through γ. When $\gamma>1$, volatility is highly sensitive to the level. The dW in (1) represents a standard Wiener process or Brownian motion, Campbell et al (1997, p. 344). The increments dW are normal random variables with $E(dW)=0$, and variance $E(dW)^2=dt$.

Depending on the restrictions imposed on the parameters α, β, and γ in equation (1) one obtains several nested models. We retain the names of these alternative models in accordance with their corresponding interest rate counterparts. Table 1 below shows the nine models (including the unrestricted) considered here and explicitly indicates parameter restrictions for each model. We estimate a discrete time specification of equation (1) using daily, weekly, and monthly values for the British pound and the German mark expressed in terms of one US dollar.

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[1] The intuition is confirmed by our tables 4 through 9. For daily british pound the SSE of 47.44 is large.

[2] For a listing of some of these models the reader may refer to Chan et. al (1992) and Fabozzi and Fabozzi(1995).

Table 1

Nine Models Considered for Short-Term Exchange Rate Dynamics With Explicit Listing of Parameter Restrictions on the Unrestricted Model $dr = (\alpha + \beta r)dt + \sigma r^{\gamma} dZ$					
Model Name	α	β	γ	σ	**Stochastic Differential Equation**
1. Brennan-Schwartz			1.0		$dr = (\alpha + \beta r)dt + \sigma r dZ$
2. CIR SR			0.5		$dr = (\alpha + \beta r)dt + \sigma r^{1/2} dZ$
3. Vasicek			0		$dr = (\alpha + \beta r)dt + \sigma dZ$
4. CIR VR	0	0	1.5		$dr = \sigma r^{3/2} dZ$
5. Dothan	0	0	1.0		$dr = \sigma r dZ$
6. CEV	0				$dr = \beta r dt + \sigma r^{\gamma} dZ$
7. GBM	0		1.0		$dr = \beta r dt + \sigma r dZ$
8. Merton		0	0		$dr = \alpha dt + \sigma dZ$

The first three models impose no restrictions on either α or β. Models 4 and 5, set both α and β equal to zero, while Models 6, 7 and 8 set either α or β equal to zero. Model 1 used by Brennan and Schwartz (1980) implies that the conditional volatility of changes in the exchange rate is proportional to the level of the exchange rate r. Model 2 is the well known square root model of Cox Ingersoll and Ross (CIR) (1985) which implies that the conditional volatility of changes in the exchange rate is proportional to the square root of the level of the exchange rate r. Model 3 is the Ornstein-Uhlenbeck diffusion process first used by Vasicek (1977). The implication of this specification is that the conditional volatility of changes in the exchange rate r is constant. Model 4 was used by CIR (1980) and by Constantinides and Ingersoll (1984) indicates that the conditional volatility of changes in the exchange rate is highly sensitive to the level of the exchange rate. Model 5 was used by Dothan (1978), Model 6 is the constant elasticity of variance (CEV) process proposed by Cox (1975) and Cox and Ross (1976). Model 7 is the famous geometric Brownian motion (GBM) process first used by Black and Scholes (1973). Finally, Model 8 is used by Merton (1973) to represent Brownian motion with drift.

Estimating the parameters of many of these models by the maximum likelihood (ML) method requires closed forms of their likelihood functionswhich are often either unavailable or too difficult to implement. For example, the CIR-SR implies a noncentral χ^2 and the GBM implies a lognormal density, for which closed forms are available. However, the closed form for the CEV
involves the Bessel functions and is difficult to implement and for some no closed form is available. The appeal of the GMM for estiamating the SDEs is that it does not require such closed forms.

Section 2 describes the three estimation methodologies starting with the GMM method. Sections 2.1 and 2.2 describe the newer SSA-EF and NCV methods. Section 3 describes the data and its source. Section 4 presents the estimation and forecasting results. Sections 5 summarizes and concludes the paper.

2. The GMM Estimation Methodology

In this section we briefly review the traditional GMM methodology[3] and explain how one uses it to estimate the parameters of the nine SDEs. As several previous studies have done we estimate the parameters in equation (1) using a discrete-time specification[4]. The discrete-time specification of equation (1) can be written as

(2) $$r_{t+1} - r_t = \alpha + \beta r_t + \varepsilon_{t+1}$$

(3) $$E[\varepsilon_{t+1}] = 0$$

(4) $$E[\varepsilon_{t+1}^2] = \sigma^2 r_t^{2\gamma}$$

Let θ be the vector of parameters α, β, σ^2 and γ. The GMM technique requires the estimation of the parameters in θ using a set of instruments z_t that satisfy the orthogonality condition $E[f_t(\theta)] = 0$, where $f_t(\theta) = \varepsilon_t \otimes z_t$. That is, f_t is a vector of cross-products of each instrument in z_t with each element of the residual vector ε_t. We define the vector $f_t(\theta)$ as

(5) $$f_t(\theta) = \begin{bmatrix} \varepsilon_{t+1} \\ \varepsilon_{t+1} r_t \\ \varepsilon_{t+1}^2 - \sigma^2 r_t^{2\gamma} \\ (\varepsilon_{t+1}^2 - \sigma^2 r_t^{2\gamma}) r_t \end{bmatrix}$$

Let $g_T(\theta)$ contain the sample averages corresponding to the elements in $f_t(\theta)$, that is

(6) $$g_T(\theta) = \frac{1}{T} \sum_{t=1}^{T} f_t(\theta)$$

GMM minimizes with respect to the parameter vector θ the quadratic form:

(7) $$J_T(\theta) = g_T(\theta)' W_T \, g_T(\theta)$$

where W_T is a positive-definite weighting matrix. The first-order condition is

(8) $$D_T(\hat{\theta})' W_T \, g_T(\hat{\theta}) = 0$$

where $D_T(\hat{\theta})$ is a matrix of partial derivatives defined by $\partial g_T(\theta)/\partial\theta$. For the unrestricted model the parameters are just identified. For the nested models the system is overidentified and the GMM estimates depend on the choice of the weighting matrix W_T. If we let $W_T = S^{-1}(\theta)$, where

(9) $$S(\theta) = \lim_{T\to\infty} \mathrm{Var}[T^{1/2} g_T(\theta)]$$

[3]The reader interested in a rigorous treatment of the GMM technique can refer to Hansen (1982), Hansen and Singleton (1988), Chamberlain (1987), Davidson and MacKinnon (1993), Hamilton (1994), Newey (1985), Newey and West (1987) and Pagan and Wickens (1989).

[4] The discretized process is an approximation to the continuous process and we acknowledge the temporal aggregation issue associated with this.

then the GMM estimator of) has the “smallest" asymptotic covariance matrix. Also when the weighting matrix W•"Ð)Ñ is used, X times the minimized value of the quadratic form in (7) is distributed as a ;# with degrees of freedom equal to the number of orthogonality conditions minus the number of parameters to be estimated.

Having obtained the GMM estimates of !, ", # and 5 we need to compute the out of sample forecasts. It is convenient to discuss this in the following subsection where we explain the rationale used for the forecasting equation (16).

2.1. Small Sigma Asymptotics - Estimating Functions (SSA-EF) Methods

Vinod (1996) proposes an alternative to the GMM using Godambe (1960) and Durbin's (1960) estimating functions (EFs). The EFs are defined as functions of data and parameters, g(y,))=0. The EF estimators are the roots of g=0, and need not be different from the familiar maximum likelihood, or instrumental variables estimators. Surveys by Godambe and Kale (1991), Dunlop (1994), Liang and Zeger (1995) explain that EF estimators can be used when the ML fails. For SDE estimation the EFs have the same appeal as the GMM. Furthermore, EFs satisfy an <u>appealing</u> <u>small-sample</u> <u>property</u> of Gauss consistency from early 19th century, which requires that the method of estimation should yield the correct estimates when all model equation errors are identically zero. Kadane's (1970) small-sigma asymptotics (SSA) explained in Vinod and Ullah (1981, p. 162) can be viewed as an implementation of Gauss's notion oc consistency. Kadane writes the error as 5u, by injecting an additional constant 5 and finds limits as that constant 5 Ä 0. This constant 5 should not be confused with the 5 of the stochastic differential equation (1).

To illustrate SSA-EF, Vinod (1996) considers a representative agent model leading to consumption-based capital asset pricing (C-CAPM) model, see Tauchen (1986) and Ogaki (1993). C-CAPM is traditionally estimated by the GMM methods. Vinod (1996) rewrites the expression from economic theory as E[g̃(y,))]=1, where E is the expectation operator and g̃ represents a nonlinear expression. The EF estimates of discounting parameter " (<1) and risk aversion parameter # (<2) of the C-CAPM model are shown to be easier to estimate and economically more meaningful than the GMM estimates. This is not surprising, since optimal EFs attain the Cramer-Rao lower bound on variance. Hence we proceed to derive a similar model for exchange rate dynamics.

First, let us we rewrite (4) as

(10) $$E\,(5^{-2}\ B_{>}^{-2\#}\ '^{2}_{t+1}) = 1$$

where we can replace the $'_{t+1}$ by [$B_{>+1}$ • $B_{>}$ • ! • " $B_{>}$] from (2). When we remove the expectation operator from (10) we are considering individual observations, not their overall average. Hence we must introduce an error term (using the small-sigma notation) 5 u_{t+1}, which should be zero when the expectation is evaluated E(5u_{t+1})=0. Hence (10) becomes:

(11) $$(5^{-2}\ B_{>}^{-2\#}\ '^{2}_{t+1}) = 1 + 5u_{t+1},$$

Now take logs of both sides to yield:

(12) • 2 ln(5) • 2# ln(B>)+2 ln '$_{t+1}$ =ln(1+5u$_{t+1}$)

where the right hand side can be written as 5u$_{t+1}$– 5^2 u$^2_{t+1}$/2+ 5$^\$$ u$^\$_{t+1}$/\$ • â . The small-sigma asymptotics means 5 Ä 0, and a linear approximation means that we omit all terms with 5^j for j 2. Then the right hand side of (12) becomes w$_{t+1}$= 5u$_{t+1}$, which is viewed as just another true unknown regression error w$_{t+1}$. Substituting the earlier expression for '$_{t+1}$, we rewrite (12) as:

(13) • ln(5^2) • # ln(B>)^2 + ln[B>$_{+1}$ • B> • ! • " B>]^2 = w$_{t+1}$.

This may be called a nonlinear SSA-EF regression equation for the four unknowns !, ", # and 5. A convenient iterative estimation is possible with two SSA-EF regressions. First regression uses (2) to estimate ! and ". For brevity, denote D>€"=[B>$_{+1}$ • B> • $ • S B>]. To avoid the log of a zero in computing ln(D>€")^2 in (13) we “winsorize" with a tolerance constant[5] 7 (=0.0001, say) as follows. First initialize winsorized D>€" denoted by wins(D>€") =D>€" for all relevant values of t. Only if |D>€"|<7, it is too close to zero requiring the following correction: (i) if D>€"<7, force D>€"= • 7, and (ii) if D>€" 7, make it D>€"=7. Our second SSA-EF regression involves rearranging (13) as

(14) ln(wins(B>$_{+1}$ • B> • $ • S B>))$^\#$ œ \$ € # lnÐB>)$^\#$ € A>€"

where \$=ln(5^2). Thus 5=exp(0.5 \$) and # is estimated by the slope coefficient. The winsorization has the added bonus that it prevents economically meaningless behavior near the boundary. In the familiar interest rate models the boundary is near zero interest rates.

<u>Discrete Approximation to SDE</u>: For the iterations we need to use these estimates of 5 and # in conjunction with those of ! and ". Let us reconsider (10) without the E operator as 5^2 œ B>$^{-2\#}$ '$^2_{t+1}$. Since 5$^\#$>0, only the positive square root of both sides is of interest. Hence we can substitute '$_{t+1}$=5 B>$^\#$ in (2) to read

(15) B>€" • B> • 5ÐB>Ñ$^\#$ œ ! € "B>

Upon substituting the estimates of parameters we have a forecasting equation

(16) B>€" œ B> € 5ÐB>Ñ$^\#$ € $ € SB> € forecast error

This derivation represents a new way of implementing the diffusion process. It provides a convenient method of approximately discretizing the continuous SDE of (1). See Remark 2, above for relevant interpretations.

Substituting the estimates of 5 and # on the left side of (15) yields a feasible regression for each iteration. Denote the new estimates by $(i) and S(i) for i-th iteration. Now let i=1, and compute the corresponding iterated estimates of 5 and # from a regression similar to (14):

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[5] Winsorization is a terminology familiar in statistical literature dealing with robust estiamtion. It is an alternative to trimming where instead of omitting extreme values they are kept at the farthest extreme value. We used a different tolerance constant 7=0.00001 for calibration and found that our results are not sensitive to its choice.

(17) ln(wins(B>+1 • B> • $(1) • S(1) B>))# œ $ € # lnÐB>)# € A>€"

Hence we have §(1)=exp(• 0.5 $) and #(1) from the intercept and the slope. Clearly for any i>1, one can compute the two simple linear regressions (15) and (17); thereby avoiding nonlinear estimation which is often sensitive to initial values of $, S, § and #. The iterations should end when the absolute value of the change in parameter estimates is less than a tolerance constant 7' (0.001, say).

Since this paper emphasizes forecasting, we reserve a reasonable number (=N'=(1/17)N, one year out of 17) of the original N observations for out-of-sample forecasting. Make all estimates by using only the initial (N • N') observations. Since the true values of the last B> are known, the criterion for ending the iterations is based on the N' , 1 vector of forecast errors $(i). For the i-th iterate the typical forecast error is defined by:

(18) $>€"(i) œ B>€" • Ò$(i) € Ð" € S(i) ÑB> € §(i) (B>)#(i)Ó

The sum of squares of forecast errors for the i-th iterate is denoted by SSE =[$(i)'$(i)]. At each iteration, whether to revise the current i-th estimates of (!, ", 5 and #) depends on whether the SSE is reduced. If SSE is increased by the (i+1)th iteration, one should reject the (i+1)-th estimates and try the (i+2)-th iteration, and so on. In our sample data sets, the termination of iterations by considering closeness of coefficients (using 7') or by using forecast errors yields the same results. In other words, one ends up choosing the same iteration number, whether one uses the 7' or SSE.

Our iterative EF estimation can handle all nine models of Table 1 quite readily, whereas the GMM algorithm failed to estimate any values for models 4 through 8. If both ! and " are preassigned to be zero, (CIR-VR and Dothan models) there is no need to estimate them by the regression (15). Then the left side of (17) simplifies to be ln(wins(B>+1 • B>))#. When # is preassigned as in seven of nine models, (15) simplifies to a version where # is simply replaced by its preassigned value. Obvious care is needed in avoiding taking logs of zeros, and forcing the regression through the origin when the intercept is absent. One should simply use the sample mean of the dependent variable as the regression coefficient when the regressor is a column of ones, instead of running such a regression.

In structural estimation, the usual F tests involve a comparison of the restricted residual sum of squares (RRSS) with the unrestricted (URSS). The choice of the model may be based on a ranking of the estimated F values for each of the eight models in comparison with the unrestricted, subject to the requirement that each parameter estimate be meaningful in terms of economic theory. These comparisons focus on in-sample fits only. Since we have chosen the out-of-sample forecasting as our criterion, we do not implement the traditional F tests. Instead, we use the SSE ranks for comparisons.

2.2. Numerical Conditional Variance (NCV) Method

When one considers the discretization of the SDE (1) by (2) and (4), it is clear that at each date t, (4) involves a conditional variance based on all information till date t-1, without access to any subsequent data. Since numerical estimation is not difficult these days, one can directly compute this, except for a few initial observations. For each t we have a numerical <u>brute force</u> estimate of '$^{\#}_{>}$ obtained from the actual variances of residuals of (2) using the data till date t-1 only. At t=6, we start with five initial observations to compute the regression in (2) to estimate $ and S. There will be five regression residuals, some of which may well be zero. Now we compute their variance as an estimate of '$^{\#}_{6}$ associated with the 6th date. For each additional date, our NCV method involves recomputing the $ and S and new sets of residual variances. This will create a time series of '$^{\#}_{>}$ for t=6 to t=N of length N • 5 representing the conditional variances implicit in the left side of (4). There is a corresponding time series of spot rates B>. Taking logs of (4) we have an estimation equation for # and 5.

Ð19Ñ ln S$^{2}_{t+1}$ œ ln(5^{2}) € # ln(B$^{2}_{>}$) + %>€" (t=5, â N-")

To remain comparable, we estimate ! and " from the regression (2) for the same data sample (t=5, â N-").

Having obtained the NCV estimates of !, ", # and 5 we compute the out of sample forecasts from the forecasting equation (16).

3. The Data

The data set used in this analysis was obtained from Bekaert (1995). It consists of daily observations on the British pound and Deutsche mark for the period January 1, 1975 to July 19, 1991. All values are the averages of the bid and ask rates. For the weekly and monthly analysis the data was sampled from this each Wednesday. Summary statistics for the data are presented in Table 2 and Table 3.

<u>Table 2. Summary Statistics</u>
Means, Standard deviations and signs of autocorrelations of daily, weekly and monthly changes in the value of the British pound from January 1, 1975 to July 19, 1991.

Variable: B>€" • B>	obs.	mean	s.d.	Are 3" to 3$_{20}$ consistently € or • ?
Daily Changes	4144	.000042	.004389	No.
Weekly Changes	828	.000212	.009460	No.
Monthly Changes	206	.000979	.019643	No.

<u>Table 3. Summary Statistics</u>
Means, Standard deviations and signs of autocorrelations of daily, weekly and monthly changes in the value of the Deutsche Mark from January 1, 1975 to July 19, 1991.

Variable: B>€" • B>	obs.	mean	s.d.	Are 3" to 3$_{20}$ consistently € or • ?
Daily Changes	4144	• .000153	.015853	No.
Weekly Changes	828	• .000753	.033394	No.
Monthly Changes	206	• .002785	.071338	No.

4. The Estimation Results

In this section we present the daily, weekly and monthly frequency results of the three estimation methods GMM, SSA-EF and NCV for both currencies. Tables 4 through 9 report the parameter estimates with asymptotic t-statistics, predicted out of sample SSE for the nine models and their SSE ranks. All these tables are divided into section 1 for the GMM, section 2 for the SSA-EF and section 3 for the NCV estimators. In light of remark 1 our one-sided tests for " for the unrestricted model have the negative sign and and absolute t-values are mostly larger than 1.645, (one tail test at 5%) suggesting evidence of mean reversion for the majority of estimates. Only five sections out of a total of eighteen (Tables 6.2, 7.3, 8.3, 9.2 and 9.3) have statistically insignificant s for the unrestricted model, implying that these models do not confirm statistically significant mean reversion. In fact, whenever s's are statistically signifcant, they are always negative, except for the GMM estimate of DM currency in tables 7.1, 8.1 and 9.1, suggesting explosive exchange rate dynamics. We may regard this as a peculiarity of the GMM estimator.

Also, from section 1 of all these tables one does not reject the null hypotheses #=0 in the unrestricted model for the GMM, implying no strong dependence of conditional volatility (of exchange rate changes) on the level of exchange rate. From section 2 of these tables for SSA-EF one would reject #=0, except for monthly data for both currencies.. From section 3 of these tables for NCV, one would reject #=0, except for monthly data for the DM currency. Thus both the new estimators support at least weak dependence of conditional volatility on the level of exchange rate. Large magnitudes (at least 0.96) of \$ for the unrestricted model from the SSA-EF estimation implies that the conditional volatility is highly sensitive to the level of exchange rates. The magnitude of \$ from the NCV estimation is less than 0.5, except for weekly and monthly data on the British Pound. Also, the NCV method produces negative (although small in magnitude) estimates for # from daily and weekly data on the DM. Since these negative estimates are statistically significant this suggests that daily and weekly data on the DM exhibits an inverse association between conditional volatility and the level of exchange rates. In addition, for both currencies the NCV estimates of # are smaller in magnitude than the SSA-EF estimates.

Recalling Granger's quote in our introduction, a comparison between estimators may properly rely on the out of sample predicted SSE. According to this criterion, the performance of GMM is extremely poor relative to the newer methods. For example, in Table 4 the SSE for the GMM is at least seven thousand times that of the SSA-EF and at least five thousand times that of NCV. Note that the unrestricted model, which has lowest within sample error sum of squares, does not give the best out of sample forecasts.

For both SSA-EF and NCV the performance of all nine models is good, and significantly better than GMM. Our tables report rankings as Rank=1 for the model with the lowest

SSE and the Rank=9 for the model with the largest SSE. All estimators generally give the worst rank to the Vasicek model which preassigns #=0. However, the differences among the SSE values are not large enough to conclusively suggest the preference for one over the others. If we must prefer among them, an average rank across all models and all data sets may be considered. We are somewhat surprised to note that the unrestricted model is not among the top three. The average rank for the CIR-VR is 1.5, for the GBM it is 3, and for the Dothan model it is 3.5.

V. Conclusion

In this paper we show that the models used to characterize short term interest rate movements can also capture the dynamics of short term exchange rates. We consider eight alternative models that are nested within a general dynamic specification. The interpretation of the coefficients of the general model is given in remarks 1 and 2. We estimate the coefficients of these models using daily, weekly and monthly data on the British pound and the Deutsche mark.

The results of the tests for daily, weekly and monthly exchange rates indicate significant mean reversion. There is statistical evidence that the conditional volatility of changes in the short term exchange rate is sensitive to the level of exchange rates. However, we cannot conclusively determine whether that sensitivity is high (#   1) or low (# Ÿ 0.5). Interestingly, the more commonly used interest rate models such as Vasicek (1977), CIR SR (1985) and Brennan-Schwartz (1980) did not do well in capturing the short term dynamics of exchange rates. The top two models in terms of the average rank (in out of sample predicted SSEs) across all data sets and estimators are the CIR-VR and the GBM with preassigned #=1.5 and #=1, respectively. Note that the GBM model is widely used for the pricing of currency options. In any case, the results of this empirical evaluation have important implications for the pricing and for hedging of currency dependent contingent claims.

The main contribution of this paper is to suggest two new methods as alternatives to the usual GMM method for estimating the fundamental stochastic differential equation. The new methods are flexible enough to estimate all nine models, whereas the GMM fails to converge for five out of nine models. Since a comparison between estimators may rely on the out of sample predicted SSE, the performance of GMM is extremely poor relative to the newer methods. Thus for exchange rate data, any use of GMM requires caution.

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Table: 4 Parameter values and out of sample predicted SSE from GMM, EF and NCV estimation methods for nine models using DAILY dollar values on the British Pound.

Model Name	!	"	#	5	**SSE**
4.1: GMM					
Unristricted	1.2270	-1.4550	3.7929	0.047829	47.440
t-values	3.6657	-4.5191	0.019115	0.023187	Rank=1
Brenn-Sch.	1.2407	-1.4623	1.5000	0.029855	51.155
t-values	20.924	-15.917		1.7512	Rank=2
CIR SR	1.2599	-1.4857	1.0000	0.025816	53.092
t-values	18.308	-14.349		1.6226	Rank=3
Vasicek	1.3084	-1.5461	0.0000	0.018035	57.659
t-values	11.532	-9.8580		1.2719	Rank=4
4.2: EF					
Unristricted	0.0007120	-0.001166	1.32887	0.00460289	0.00667
t-values	3.68896	-9.41576	19.5728	25.3440	Rank=6
Brennan-Sch.	0.0007120	-0.001166	1.00000	0.00383589	0.00669
t-values	1.78059	-7.41048		83.9344	Rank=7
CIR SR	0.0007120	-0.001166	0.50000	0.00290742	0.00673
t-values	-1.01203	-4.50858		82.6085	Rank=8
Vasicek	0.0007120	-0.001166	0.0000	0.00220368	0.00677
t-values	-3.73006	-1.72711		80.3279	Rank=9
CIR VR	0.00000	0.00000	1.50000	0.00505881	0.00659
t-values	0.00000	0.00000		0.00000	Rank=2
Dothan	0.00000	0.00000	1.00000	0.00383434	0.00663
t-values	0.00000	0.00000		0.00000	Rank=4
CEV	0.0000	1.76341e-05	1.33973	0.00462933	0.00662
t-values	0.0000	-33.0827	19.7841	25.3610	Rank=3
GBM	0.0000	1.76341e-05	1.00000	0.00383477	0.00664
t-values	0.0000	-32.3805		83.9634	Rank=5
Merton	-0.00217141	0.00000	0.00000	0.00220341	0.00574
t-values	-31.0894	0.00000		80.3256	Rank=1
4.3: NCV					
Unristricted	0.00072041	-0.0011785	0.40313	0.0044649	0.0085348
t-values	1.7980	-1.7432	33.394	142.60	Rank=7
Brenn-Sch.	0.00072041	-0.0011785	1.0000	0.0062146	0.0083991
t-values	1.7980	-1.7432		371.41	Rank=4
CIR SR	0.00072041	-0.0011785	0.50000	0.0047111	0.0085110
t-values	1.7980	-1.7432		470.11	Rank=6
Vasicek	0.00072041	-0.0011785	0.0000	0.0035713	0.0086415
t-values	1.7980	-1.7432		417.85	Rank=9
CIR VR	0.0000	0.0000	1.5000	0.0081993	0.0081996
t-values	0.0000	0.0000		268.23	Rank=1
Dothan	0.0000	0.0000	1.0000	0.0062156	0.0082894
t-values	0.0000	0.0000		371.35	Rank=2
CEV	0.0000	1.8441e-05	0.40353	0.0044667	0.0084355
t-values	0.0000	0.15632	33.398	142.48	Rank=5
GBM	0.0000	1.8441e-05	1.0000	0.0062157	0.0083057
t-values	0.0000	0.15632		371.31	Rank=3
Merton	3.2686e-05	0.0000	0.0000	0.0035719	0.0085759
t-values	0.46751	0.0000		417.51	Rank=8

Table: 5 Parameter values and out of sample predicted SSE from GMM, EF and NCV estimation methods for nine models using WEEKLY dollar values on the British Pound.

Model Name	!	"	#	5	**SSE**
5.1: GMM					
Unristricted	0.85487	-1.4344	-2.5363	0.00065407	0.37964
t-values	1.7539	-1.6455	-0.029130	0.0066966	Rank=1
Brenn-Sch.	0.74455	-1.1911	1.5000	0.065840	0.75729
t-values	1.8434	-2.0393		0.70844	Rank=3
CIR SR	0.78762	-1.2897	1.0000	0.045899	0.63888
t-values	1.8654	-2.0119		0.59693	Rank=2
Vasicek	1.0342	-1.6821	0.0000	0.036524	1.2511
t-values	2.2969	-2.7725		0.67909	Rank=4
5.2: EF					
Unristricted	0.003363	-0.00549145	1.60132	0.0108029	0.005943
t-values	3.23937	-5.64967	8.47959	9.15821	Rank=6
Bren-Sch.	0.003363	-0.00549145	1.00000	0.0077399	0.005963
t-values	1.75056	-4.07996		30.2934	Rank=7
CIR SR	0.003363	-0.0054914	0.50000	0.0058659	0.005986
t-values	0.577904	-2.86171		29.8464	Rank=8
Vasicek	0.003363	-0.00549145	0.00000	0.00444563	0.006013
t-values	-0.563511	-1.69331		29.1551	Rank=9
CIR VR	0.00000	0.00000	1.50000	0.0102328	0.005862
t-values	0.00000	0.00000		0.00000	Rank=2
Dothan	0.00000	0.00000	1.00000	0.00775517	0.005874
t-values	0.00000	0.00000		0.00000	Rank=3
CEV	0.00000	9.83822e-05	1.53715	0.0104467	0.005878
t-values	0.00000	-13.9658	7.93574	9.17154	Rank=4
GBM	0.00000	9.83822e-05	1.00000	0.0077559	0.005890
t-values	0.00000	-13.5004		30.3505	Rank=5
Merton	-0.004295	0.00000	0.00000	0.00445515	0.005458
t-values	-12.7847	0.00000		29.2902	Rank=1
5.3: NCV					
Unristricted	0.0035078	-0.0057138	0.87328	0.011517	0.0071555
t-values	1.8086	-1.7473	20.677	40.846	Rank=6
Brenn-Sch.	0.0035078	-0.0057138	1.0000	0.012352	0.0071376
t-values	1.8086	-1.7473		135.27	Rank=5
CIR SR	0.0035078	-0.0057138	0.50000	0.0093689	0.0072136
t-values	1.8086	-1.7473		129.66	Rank=8
Vasicek	0.0035078	-0.0057138	0.0000	0.0071060	0.0073044
t-values	1.8086	-1.7473		109.17	Rank=9
CIR VR	0.0000	0.0000	1.5000	0.016349	0.0069100
t-values	0.0000	0.0000		120.81	Rank=1
Dothan	0.0000	0.0000	1.0000	0.012400	0.0069630
t-values	0.0000	0.0000		137.19	Rank=2
CEV	0.0000	0.00011030	0.85748	0.011462	0.0070149
t-values	0.0000	0.19376	20.604	41.450	Rank=4
GBM	0.0000	0.00011030	1.0000	0.012402	0.0069972
t-values	0.0000	0.19376		137.03	Rank=3
Merton	0.00017043	0.0000	0.0000	0.0071335	0.0072097
t-values	0.50481	0.0000		111.02	Rank=7

Table: 6 Parameter values and out of sample predicted SSE from GMM, EF and NCV estimation methods for nine models using MONTHLY dollar values on the British Pound.

Model Name	!	"	#	5	SSE
6.1: GMM					
Unristricted	0.95883	-1.5775	-1.8696	0.0024608	0.14181
t-values	1.9731	-1.9427	-0.087255	0.028178	Rank=1
Brenn-Sch.	0.79143	-1.2418	1.5000	0.046411	0.19395
t-values	1.8267	-2.1329		0.67533	Rank=3
CIR SR	0.83479	-1.3358	1.0000	0.038091	0.18149
t-values	1.8595	-2.0956		0.70193	Rank=2
Vasicek	1.1140	-1.7532	0.0000	0.048780	0.47807
t-values	2.0861	-2.4720		0.54769	Rank=4
6.2: EF					
Unristricted	0.0139313	-0.0229156	0.957442	0.0143662	0.004294
t-values	0.00000	0.00000	0.00000	0.00000	Rank=7
Brenn-Sch.	0.0139313	-0.0229156	1.00000	0.0147099	0.004293
t-values	0.00000	0.00000		0.00000	Rank=6
CIR SR	0.0139313	-0.0229156	0.500000	0.0111421	0.004306
t-values	0.00000	0.00000		0.00000	Rank=8
Vasicek	0.0139313	-0.0229156	0.00000	0.00843968	0.004321
t-values	0.00000	0.00000		0.00000	Rank=9
CIR VR	0.00000	0.00000	1.50000	0.0200379	0.004263
t-values	0.00000	0.00000		13.2350	Rank=1
Dothan	0.00000	0.00000	1.00000	0.0151778	0.004265
t-values	0.00000	0.00000		13.3725	Rank=3
CEV	0.00000	0.00026	0.693840	0.0127955	0.004267
t-values	0.00000	0.00000	0.00000	0.00000	Rank=4
GBM	0.00000	0.00026	1.0000	0.015168	0.004264
t-values	0.00000	0.00000		0.00000	Rank=2
Merton	0.000578	0.00000	0.00000	0.00870272	0.004282
t-values	0.00000	0.00000		0.00000	Rank=5
6.3: NCV					
Unristricted	0.014977	-0.024521	0.80807	0.022682	0.0048295
t-values	1.8121	-1.7653	9.0836	19.543	Rank=7
Brenn-Sch.	0.014977	-0.024521	1.0000	0.025203	0.0048020
t-values	1.8121	-1.7653		64.892	Rank=5
CIR SR	0.014977	-0.024521	0.50000	0.019153	0.0048769
t-values	1.8121	-1.7653		63.697	Rank=8
Vasicek	0.014977	-0.024521	0.0000	0.014555	0.0049627
t-values	1.8121	-1.7653		54.804	Rank=9
CIR VR	0.0000	0.0000	1.5000	0.03361	0.0045461
t-values	0.0000	0.0000		57.889	Rank=1
Dothan	0.0000	0.0000	1.0000	0.025539	0.0045929
t-values	0.0000	0.0000		66.819	Rank=2
CEV	0.0000	0.000277	0.7696	0.02252	0.00464
t-values	0.0000	0.11556	8.9357	20.185	Rank=4
GBM	0.0000	0.000277	1.0000	0.025554	0.0046123
t-values	0.0000	0.11556		66.596	Rank=3
Merton	0.00060	0.0000	0.0000	0.01475	0.0048034
t-values	0.42329	0.0000		57.023	Rank=6

Table: 7 Parameter values and out of sample predicted SSE from GMM, EF and NCV estimation methods for nine models using DAILY dollar values on the Deutsche Mark.

Model Name	!	"	#	5	**SSE**
7.1: GMM					
Unristricted	-1.0841	0.50577	-0.60515	0.080052	12.184
t-values	-4.6294	5.0289	-0.13851	0.19671	Rank=2
Brenn-Sch.	-0.69166	0.30288	1.5000	0.0012568	10.741
t-values	-1.3715	1.2988		0.38143	Rank=1
CIR SR	-1.3299	0.60493	1.0000	0.013186	30.017
t-values	-8.2814	9.7535		3.0958	Rank=4
Vasicek	-1.1460	0.53645	0.0000	0.042366	15.852
t-values	-3.3940	2.9818		1.4338	Rank=3
7.2: EF					
Unristricted	0.0009986	-0.0005435	1.10746	0.00294457	0.0521507
t-values	1.23379	-6.42478	12.0370	13.5955	Rank=5
Brenn-Sch.	0.0009986	-0.0005435	1.00000	0.00320198	0.0525098
t-values	0.693392	-5.85749		61.1902	Rank=7
CIR SR	0.0009986	-0.0005435	0.500000	0.00472907	0.0545727
t-values	-1.76328	-3.30149		60.8656	Rank=8
Vasicek	0.0009986	-0.0005435	0.00000	0.00698446	0.0574730
t-values	-4.15640	-0.849963		60.0901	Rank=9
CIR VR	0.00000	0.00000	1.50000	0.00216753	0.0508245
t-values	0.00000	0.00000		0.00000	Rank=2
Dothan	0.00000	0.00000	1.00000	0.00320127	0.0522440
t-values	0.00000	0.00000		0.00000	Rank=6
CEV	0.00000	-0.0001072	1.10082	0.00296074	0.0515601
t-values	0.00000	-29.2894	11.8932	13.5961	Rank=3
GBM	0.00000	-0.0001072	1.00000	0.00320295	0.0518742
t-values	0.00000	-29.1128		61.1938	Rank=4
Merton	-0.00719294	0.00000	0.00000	0.00698696	0.0474803
t-values	-28.0877			60.1056	Rank=1
7.3: NCV					
Unristricted	0.00098386	-0.00053304	-0.082929	0.014544	0.092032
t-values	0.68303	-0.83339	-5.1958	78.334	Rank=9
Brenn-Sch.	0.00098386	-0.00053304	1.0000	0.0062508	0.069696
t-values	0.68303	-0.83339		238.41	Rank=4
CIR SR	0.00098386	-0.00053304	0.50000	0.0092314	0.078057
t-values	0.68303	-0.83339		304.07	Rank=5
Vasicek	0.00098386	-0.00053304	0.00000	0.013633	0.08970
t-values	0.68303	-0.83339		351.21	Rank=7
CIR VR	0.0000	0.0000	1.5000	0.0042440	0.063292
t-values	0.0000	0.0000		188.48	Rank=1
Dothan	0.0000	0.0000	1.0000	0.0062677	0.069252
t-values	0.0000	0.0000		240.03	Rank=3
CEV	0.0000	-0.00010314	-0.076169	0.014507	0.090146
t-values	0.0000	-0.90647	-4.8082	78.923	Rank=8
GBM	0.0000	-0.00010314	1.0000	0.0062676	0.068482
t-values	0.0000	-0.90647		240.04	Rank=2
Merton	-0.000197	0.0000	0.0000	0.013670	0.087845
t-values	-0.77046	0.0000		353.98	Rank=6

Table: 8 Parameter values and out of sample predicted SSE from GMM, EF and NCV estimation methods for nine models using WEEKLY dollar values on the Deutsche Mark.

Model Name	!	"	#	5	SSE
8.1: GMM					
Unristricted	-0.96224	0.44260	-0.87654	0.074662	2.2805
t-values	-1.9750	1.7910	-0.10746	0.10832	Rank=2
Brenn-Sch.	-0.68866	0.30007	1.5000	0.0011240	2.2648
t-values	-1.5958	1.5033		0.38420	Rank=1
CIR SR	-1.1764	0.52516	1.0000	0.0078616	5.4713
t-values	-6.8625	7.4333		1.8929	Rank=4
Vasicek	-1.0249	0.47280	0.0000	0.029388	3.0745
t-values	-2.5825	2.2642		1.0524	Rank=3
8.2: EF					
Unristricted	0.0039448	-0.0022442	0.975410	0.00723687	0.0511573
t-values	0.525385	-3.04971	4.41374	5.65455	Rank=7
Brenn-Sch.	0.0039448	-0.0022442	1.00000	0.00709937	0.0510920
t-values	0.582905	-3.10993		25.4909	Rank=6
CIR SR	0.0039448	-0.0022442	0.5000	0.0104863	0.0526982
t-values	-0.576028	-1.90426		25.4179	Rank=8
Vasicek	0.0039448	-0.0022442	0.000	0.015489	0.0550505
t-values	-1.70581	-0.746956		25.1771	Rank=9
CIR VR	0.00000	0.00000	1.50000	0.00479992	0.0498457
t-values	0.00000	0.00000	0.00000	0.00000	Rank=2
Dothan	0.00000	0.00000	1.00000	0.00708984	0.0508687
t-values	0.00000	0.00000		0.00000	Rank=5
CEV	0.00000	-0.0005207	0.964667	0.00726616	0.0503093
t-values	0.00000	-14.2026	4.33673	5.62159	Rank=4
GBM	0.00000	-0.0005207	1.00000	0.00706861	0.0502307
t-values	0.00000	-14.2314		25.2953	Rank=3
Merton	-0.0164349	0.00000	0.00000	0.0154050	0.0487989
t-values	-13.6798	0.00000		24.9502	Rank=1
8.3: NCV					
Unristricted	0.0037947	-0.0021344	-0.11952	0.030955	0.080929
t-values	0.56020	-0.70952	-3.4578	36.176	Rank=9
Brenn-Sch.	0.0037947	-0.0021344	1.0000	0.012931	0.063018
t-values	0.56020	-0.70952		105.93	Rank=4
CIR SR	0.0037947	-0.0021344	0.50000	0.019096	0.069327
t-values	0.56020	-0.70952		136.72	Rank=5
Vasicek	0.0037947	-0.0021344	0.0000	0.028200	0.078282
t-values	0.56020	-0.70952		161.45	Rank=8
CIR VR	0.0000	0.0000	1.5000	0.0088051	0.058269
t-values	0.0000	0.0000		83.935	Rank=1
Dothan	0.0000	0.0000	1.0000	0.013003	0.062648
t-values	0.0000	0.0000		107.62	Rank=3
CEV	0.0000	-0.00047611	-0.10747	0.030832	0.078231
t-values	0.0000	-0.88954	-3.1810	37.013	Rank=7
GBM	0.0000	-0.00047611	1.0000	0.013002	0.061370
t-values	0.0000	-0.88954		107.63	Rank=2
Merton	-0.00093475	0.0000	0.0000	0.028356	0.075556
t-values	-0.77548	0.0000		165.31	Rank=6

Table: 9 Parameter values and out of sample predicted SSE from GMM, EF and NCV estimation methods for nine models using MONTHLY dollar values on the Deutsche Mark.

Model Name	!	"	#	5	**SSE**
9.1: GMM					
Unristricted	-1.0723	0.48787	0.39842	0.019883	0.97324
t-values	-2.0719	1.8779	0.22447	0.35813	Rank=3
Brenn-Sch.	-1.1089	0.47134	1.5000	0.0017671	1.5636
t-values	-3.3632	3.4831		1.0232	Rank=4
CIR SR	-0.96443	0.42964	1.0000	0.0066954	0.95625
t-values	-1.8819	1.7483		0.70481	Rank=2
Vasicek	-1.0654	0.48906	0.0000	0.033410	0.86353
t-values	-2.1832	2.0029		0.92146	Rank=1
9.2: EF					
Unristricted	0.0172444	-0.00974530	1.17541	0.0139102	0.0522332
t-values	0.00000	0.00000	0.00000	0.00000	Rank=4
Bren-Sch.	0.0172444	-0.00974530	1.00000	0.0159538	0.0524855
t-values	0.00000	0.00000		0.00000	Rank=5
CIR SR	0.0172444	-0.00974530	0.500000	0.0235810	0.0535375
t-values	-0.0139639	-1.37021		12.5454	Rank=8
Vasicek	0.0172444	-0.00974530	0.00000	0.0348545	0.0552746
t-values	-0.609495	-0.760751		12.4186	Rank=9
CIR VR	0.00000	0.00000	1.50000	0.0109792	0.0520220
t-values	0.00000	0.00000		12.2801	Rank=3
Dothan	0.00000	0.00000	1.00000	0.0162281	0.0525191
t-values	0.00000	0.00000		11.9713	Rank=6
CEV	0.00000	-0.0022	1.25450	0.01290	0.0515761
t-values	0.00000	0.00000	0.00000	0.00000	Rank=1
GBM	0.00000	-0.0022	1.0000	0.01574	0.0516872
t-values	0.00000	0.00000		0.00000	Rank=2
Merton	-0.00438692	0.00000	0.00000	0.0345012	0.0531179
t-values	0.00000	0.00000		0.00000	Rank=7
9.3: NCV					
Unristricted	0.016467	-0.0093593	0.077539	0.063079	0.079173
t-values	0.56834	-0.72794	1.6123	25.985	Rank=8
Brenn-Sch.	0.016467	-0.0093593	1.0000	0.030719	0.064821
t-values	0.56834	-0.72794		67.901	Rank=4
CIR SR	0.016467	-0.0093593	0.50000	0.045371	0.071347
t-values	0.56834	-0.72794		98.219	Rank=5
Vasicek	0.016467	-0.0093593	0.0000	0.067011	0.080902
t-values	0.56834	-0.72794		115.77	Rank=9
CIR VR	0.0000	0.0000	1.5000	0.021072	0.060231
t-values	0.0000	0.0000		51.281	Rank=1
Dothan	0.0000	0.0000	1.0000	0.031122	0.064560
t-values	0.0000	0.0000		72.825	Rank=3
CEV	0.0000	-0.0021704	0.11320	0.062139	0.074275
t-values	0.0000	-0.94332	2.7056	29.868	Rank=6
GBM	0.0000	-0.0021704	1.0000	0.031115	0.062011
t-values	0.0000	-0.94332		72.907	Rank=2
Merton	-0.0042825	0.0000	0.0000	0.067891	0.075809
t-values	-0.82552	0.0000		131.65	Rank=7