UNDERSTANDING MATTERS FROM A LOGICAL ANGLE[1]

First appeared in: *Semiotische Bericthe-Doxa* (joint issue), 3-4/11(1987), pp. 101-125; reprinted in: G. Klima: *ARS ARTIUM: Essays in Philosophical Semantics, Medieval and Modern*, Budapest: Institute of Philosophy of the Hungarian Academy of Sciences 1988; and *Annales Universitatis Budapestinensis*, Sectio Philosophica et Sociologica, 22-23(1990), pp. 37-62.

Logical Aspects of Understanding

As the recently almost unlimited multiplication of several kinds of logical systems tends to turn the term "logic" somewhat ambiguous, perhaps the safest way to speak about the logical aspects of anything is by presenting a logical system which one considers the most suitable in treating the subject. At any rate, this is what I am going to do in this essay. But first, some preliminary remarks.

1. UNUNDERSTANDING

Understanding, it seems, may best be understood **ex opposito**, namely from cases of ununderstanding. Now, even if we restrict our attention to linguistic understanding (setting aside e.g. metacommunication, on the one hand, or understanding a thing's nature, or one's feelings or motives, on the other) what these cases reveal above all is that understanding is a highly precarious business, depending on many factors for its success.

The most trivial case of linguistic ununderstanding is when the listener simply does not know the language of the speaker. From this case of absolute ununderstanding there is a more or less continuous transition, through several degrees of understanding achieved between occasional users of the same language (depending on the linguistic competence of both parties), up to the degree of understanding achieved between native speakers of the same language.

But, as we all know well, even mastery of the same idiom by both parties is all too often insufficient to avoid misunderstandings. Languages abound with ambiguities, which leave open the possibility of different interpretations of the same expression by different users of the language. These differences of interpretation may be the source of either occasional, easily corrigible, or systematic, sometimes incorrigible misunderstandings.

Now what seems to be the common source of all these kinds of ununderstanding is either the lack (in the case of absolute ununderstanding), or the difference (in cases of misunderstanding) of interpretation of the same linguistic sign. This difference of interpretation may, again, stem from different sources.

First, it may be due to some objective ambiguity, inherent in common language usage: the same expression may have different (socially) objective meanings.

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[1] I hope Professor Quine and Professor Geach will pardon me for the trifling plagiarisms committed in the main title of this paper - if not for the paper itself.

Second, it may be due to some sort of ignorance, or (semantic) incompetence of the interpreter (who may be the speaker as well as the listener), who, in consequence, assigns a subjective interpretation to the expression in question which is different from its (socially) objective meaning.

Third, it also may be the case that the objective meaning of an expression is vague, underdetermined, and so (perhaps, depending on context), users of the language are quite free to assign to it different subjective interpretations without being ignorant or incompetent.

Now what all this boils down to seems to be that a logic which is to be able to give an account of understanding (and, by the same token, of ununderstanding) should incorporate some notion of objective, socially established meaning, on the one hand, and of subjective, individual interpretations of intersubjective linguistic signs, on the other.

The task of the subsequent discussion is to draw the main outlines of such a logic.

2. FROM INTENSIONS TO SIGNIFICATIONS

In our days the most powerful tool for the exact study of meaning is model theoretic semantics, in particular, its branch called intensional logic.

Nevertheless, by now it seems to be quite clear that the classical notion of intension does not capture the intuitive notion of meaning. Intensions, conceived as functions from possible worlds, or indices (ordered pairs of possible worlds and time-points) to extensions or factual values (cf. Montague, 1973), are too "coarse-grained" in that they fail to distinguish between non-synonymous, merely necessarily coextensive expressions. (cf. Lewis, 1976)

Recently several attempts have been made to produce semantics that would provide more "fine-grained" intensions. However, as these attempts are either still under formation (see e.g. Perry, 1986), or, at least, under discussion (see e.g. Anderson, 1987), let me bluntly proceed on my own way and sketch my alternative approach to offer it for similar discussions.

The approach I am about to present has the following distinguishing features on account of which, I think, it deserves to be called "alternative":

1. It is based on the so-called "inherence theory" of predication, (see e.g. Moody, 1957)
2. Accordingly, it incorporates the Aristotelian notion of truth, defined in terms of actual existence. (cf. Matthen, 1983; Hintikka, 1986; Jacobi, 1986; Weidemann, 1986)
3. It provides an intensional logic without any reference to possible worlds.
4. It provides suitably "fine-grained" intensions and an adequate synonymy relation with no reference to "unique, non-circular definitions". (cf. Bealer, 1982; Anderson, 1987)
5. It provides room for objective meanings as well as for subjective concepts - thereby being able to solve the intentional paradoxes and to give an account of the abovementioned types of ununderstanding. (cf. Kamp, 1981; Landman, 1986)

For the sake of simplicity, let me start with (the very primitive) language of first-order uniform quantification theory, containing only the usual logical connectives, the two quantifiers, variables, one-place predicates and formulas built up from these according to the usual construction rules.

According to the usual semantics for this language, the semantic value of a predicate P is a subset of the universe of discourse W of a model M, and an atomic formula of the form ‘Px’ is true according to (or is satisfied by) an assignment of values f, if and only if the element of W assigned by f to ‘x’ is an element of this subset of W. So far, so good.

Nevertheless, this is not the only possible way to specify the truth conditions of such a simple predication. Intuitively, what the above formal description says is that the semantic function of predicates is to denote sets, and the truth of a predication depends on membership in the set denoted by the predicate. For example, on the above account the function of the predicate ‘is red’ is to denote the set of red things, and the truth of the predication ‘x is red’ depends on whether the thing assigned to ‘x’, say u, is a member of this set.

However, we may also say, and perhaps even more intuitively, that the function of this predicate is to signify the property of redness, and the truth of this predication depends on whether u has redness, i.e., whether u’s redness exists. (Disregarding for the moment Quinean and Geachean qualms about properties.)

So what the predicate of this predication, according to this description, signifies is u’s particular redness at time t of the predication (an “individualized form” as Prof. Geach would call it. Cf. Geach, 1969) Let me call this particular redness **the significate of** the predicate “is red” **in** u **at** time t, or, in symbols: Sgt(R)(u)(t). It is the actual existence of this significate that accounts for the truth of this predication. But what about the case when this significate does not exist?

Well, then it either **can** exist or not. So, if this significate is not actual, then it is either potential or impossible, and, accordingly, the predication is either possible or impossible at the time of the predication. (Concerning reference to and quantification over non-actual individuals see Essay III.)

But it may also be the case that the given predicate is simply not applicable to a given thing; say, the predicate ‘is red’ to the number 2. In this case one would rather be inclined to say that the predicate has no significate in this thing at all, neither actual, nor non-actual, for the predicate is simply undefined for this thing. We may represent this case by introducing a zero-entity, 0, which falls outside the whole universe of discourse W: that a predicate’s significate in a thing at time t is 0 means that the concept of the predicate is undefined for that thing.

So, to put this in formulas:

$$\mathrm{Sgt}(R)(u)(t) \in A(t) \cup P(t) \cup I(t) \cup \{0\},$$

where A(t), P(t), and I(t) are disjoint subsets of the universe of discourse W (representing the sets of signifiable things which are actual, potential or impossible at time t, respectively) and 0 is not an element of W.

Now from such a significate of R, a particular property of a thing at a certain time, we get what I call **the signification of** R, the universal property of R-ness (for a parallel distinction between “concrete” and “abstract” properties see Küng, 1967), by abstracting from its individualizing conditions, namely time and subject, i.e., in formal terms, this significate being a value of a function, by iterated applications of functional abstraction. (For the close parallelism between functional abstraction and the traditional, Aristotelian conception of abstraction see Geach, 1969 and Klima, 1984.) Functional abstraction in general is effected by the application of Alonzo Church’s lambda-operation. (Curch, 1956) (To be sure, since significates are denoted here as values of functions— which, again, are values of functions, and so on—, as the pairs of parentheses, the signs of function-application show, we could denote these functions directly by simply omitting the pairs of parentheses following the functional signs. However, with this notation I wish to make even more explicit the derivation of significations as “unsaturated” functions, by abstracting from their arguments. For the exact description of the working of this operation see Appendix.) So we can write:

$$Sg(R)(u) = \lambda t(Sgt(R)(u)(t)),$$

whence, further,

$$Sg(R) = \lambda u(Sg(R)(u)) = \lambda u(\lambda t(Sgt(R)(u)(t))).$$

Now predicate-signification so-conceived is sufficiently fine-grained to distinguish between non-synonymous, merely necessarily coextensive predicates. (cf. Sober, 1982)

For example, even if necessarily, for every x,

x is trilateral iff x is triangular,

that is, for every u and t,

$$Sgt(trilateral)(u)(t) \in A(t) \text{ iff } Sgt(triangular)(u)(t) \in A(t)$$

since for any u and t, it is **not** the case that

$$Sgt(trilateral)(u)(t) = Sgt(triangular)(u)(t),$$

therefore, it is **not** the case that

$$Sg(trilateral) = Sg(triangular).$$

But what about sentence-meanings? Well, having predicate-significations at our disposal, we can easily construct the significate of a simple predication at a time t according to a given assignment as follows:

$$Sgt(Px)(t)(f) = Sgt(P)(f(x))(t).$$

But how can we proceed to more complex formulae? For in order to have a complete truth-definition (let alone a complete synonymy relation) that covers all possible formulae we must be able to construct somehow the significates even of complex formulae. (Of course, in accordance with the principle of compositionality, as functions of the semantic values of their constituents.) To this end we must take into consideration even the significates of logical constants in respect of the significates of their argument-expressions.

So first, the significate of negation:

$$Sgt(\sim)(s)\in A(t) \text{ iff } \textbf{not}: Sgt(\sim)(s)\in A(t),$$

where s is a sentence-significate, i.e., the significate of some sentence (formula) p at time t, according to an assignment f.

Whence, the significate of a negated sentence, ~p, is constructed as follows:

$$Sgt(\sim p)(t)(f) = Sgt(\sim)(Sgt(p)(t)(f))$$

Similarly with conjunction:

$Sgt(\&)(s_1)(s_2)$ is an element of

/i/ A(t), if s_1 and s_2 are elements of A(t)

/ii/ I(t), if s_1 or s_2 is an element of I(t) and the other is not 0, or if s_1 is an element of A(t) then s_2 is not an element of A(t)

/iii/ {0}, if s_1 or s_2 is 0,

/iv/ P(t) otherwise[2]

whence

$$Sgt(p \,\&\, q)(t)(f) = Sgt(\&)(Sgt(p)(t)(f))(Sgt(q)(t)(f))$$

Now with the quantifiers the situation is somewhat more complicated. But let us just consider: what does a formula of the form (x)(Fx) say? Well, it says that for some value of x, the function associated with F is satisfied, i.e., in our present framework, that for some value of x the value of the function associated with F (namely, the signification of F) is actual. But what about more complex formulae that can stand behind the quantifier? Well, if we look at the signification of F as being derivable from that of Fx by λ-abstraction, then the question is not so difficult to answer.[3] As we know the significates of Fx for any value of x, we may construct the signification of λx(Fx) from these, again, by λ-abstraction. Let

(1) Sgt(λx(Fx))(t)(f)(u)

be the significate of λx(Fx) at time t according to f in u∈W, and let this be identical with

(2) Sgt(Fx)(t)(f[x:u]),

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[2] As can be seen, the rationale for this rule is the following: a conjunction is /i/ true (its significate is actual) if both of its conjuncts are true, /ii/ impossible, if 1. one of its conjuncts is impossible , or 2. if the truth of one conjunct implies the falsity of the other, i.e., if they are incompossible, /iii/ undefined, if one of its conjuncts is undefined, /iv/ otherwise it is possible. We could have given a similarly complicated rule also for negation, specifying e.g. that the negation is undefined if the negated sentence is undefined, but I wanted to give only the minimal specification of what may count as negation. Surely further complications would be needed e.g. to distinguish external and internal negation, but this is not of our present concern.

[3] In what follows I will use λ also as a syntactic operator: a metasign prefixed by λ and a metavariable stands for the object-language string that results from the object-language expression denoted by the metasign by omitting the free occurrences of the variable denoted by the metavariable. So e.g., if the metavariable v stands for the object-language variable y, and the metasign A stands for the object-language formula 'F(y) & (∃y)(Gy))', then λv(A) stands for the string: 'F( ) & (∃y)(Gy))'. Henceforth I will call this the "verb-phrase" of A. In the example in the main text I use x, F and Fx autonomously in the metalanguage.

where f[x:u] is the same as f except that it assigns u to x.[4] Now as we know (2), we also know (1). But from this we can get the signification of $\lambda x(Fx)$ by abstracting from u:

$$Sg(\lambda x(Fx))(t)(f) = \lambda u(Sgt(\lambda x(Fx))(t)(f)(u))$$

But then, in a similar vein, we can construct the signification of any λ-abstract of any complexity, thereby providing suitable associated functions for quantified formulae.

So, to sum up,

$$Sg(\lambda v(A))(t)(f) = \lambda u(Sgt(\lambda v(A))(t)(f)(u)),$$

where

$$Sgt(\lambda v(A))(t)(f)(u) = Sgt(A)(t)(f[v:u]).$$

And so the signification of the existential quantifier can be defined as follows:

$Sgt(\exists)(V)$ is an element of

/i/ $A(t)$, if for some $u \in W$, $V(u) \in A(t)$,

/ii/ $I(t)$, if for every $u \in W$, $V(u) \in I(t)$,

/iii/ $\{0\}$, if for every $u \in W$, $V(u)=0$

/iv/ $P(t)$, otherwise,

where $V(u) \in A(t) \cup P(t) \cup I(t) \cup \{0\}$ (intuitively, V is the place-holder of the signification function of the “verb-phrase” of the formula); whence the significate of an existentially quantified formula:

$$Sgt(\exists v(A))(t)(f) = Sgt(\exists)(Sg(\lambda v(A))(t)(f)).$$

I do not want to bore the reader with the rest of the logical connectives, I think it should be quite clear by now that by the same recursive method the significates of any complex formula can be determined, wherefrom, by λ-abstraction, its signification can be constructed.

In general, if $Sgt(exp)(e_1)...(e_n)$ is the significate of any expression in respect of any entities whatever, then its signification—Sg(exp)—is gotten from this by “chopping off” $e_1...e_n$ by iterated applications of λ-abstraction.

Now this general definition of signification provides us with a very natural general notion of synonymy: two expressions are synonymous if and only if their significations are identical, i.e., if they have the same meaning.

Well, so far, so good, as far as the formal theory is concerned—one might say—but how do we, as users of a language, come to know, if ever, this all-embracing relation? How is it possible

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[4] That is, the significate of a “verb-phrase”, i.e., what results from a sentence by leaving off a denoting expression, in respect of a thing is the same as the significate of the sentence when the denoting phrase in question is taken to refer to the thing in question.

that despite all our everyday semantic competence we have so much trouble in identifying meanings, if things are so simple as they seem from the above description?

Well, things are not so simple as that at least for two reasons.

1. In actual language usage, when we identify the meanings of expressions, we cannot appeal to well-defined functions in a well-defined model: all we have are other expressions, and we try to specify the meaning of one expression by trying to construct other expressions which we judge to have the same meaning as our explanandum. So we can never give the meaning of an expression absolutely, by constructing it as an abstract object, as it were; all we can do is to specify it relatively, by trying to supply synonymous expressions. The situation is somewhat similar to that when we have to judge the length of things without an étalon. In this case all we can do is to say which are and which are not of the same length, relying on our fallible sight. Nevertheless, this does not mean that they do not have a length of their own. The setting up of a model of the abovementioned type is like establishing an étalon. If we "calibrate" the significations of the primitive expressions relative to a model, then, on the basis of these "absolute meanings" we can make judgement about the synonymy relations holding between them and between expressions built up from them, without having to rely on our vague intuitions. So in the formal theory what comes first is absolute meaning, which defines synonymy; in actual usage, however, what comes first is some fragment of the synonymy relation, which specifies meanings. But things are not even so simple as that.

2. For when we acquire a language we do not get even these splinters, as it were, of the synonymy relation. As Peter Bosch most aptly wrote: "... in the acquisition of our native language we get started with partial explanations of the use of words; in the extreme case, these partial explanations are ostensive definitions. We are shown positive and negative instances and we are told: 'This is a rose, and also that and that; but this here is not a rose'. But in the majority of cases we are not given any explicit definition at all, not even an ostensive one. Rather we have to pick up the use of an expression from the various concrete applications we happen to come across. Now, these are always applications to a limited number of things, always in different contexts. ... Bit by bit, we might say, the child who is exposed to those partial explanations witnesses the growth of something more appropriately called a jungle of definitions: intersecting each other, complementing each other, pushing each other aside, and still leaving whole areas of untouched desert land in between and wide unexplored oceans around the buzzing jungle. Still, as long as we move in our jungle, somehow we seem to be doing alright, communication flows uninhibited. Only when we find ourselves unexpectedly in one of the deserts, or are driven into the ocean, we stand speechless: for those regions our predicates are not defined". (Bosch, 1983:191)

So the objective, but "intangible" synonymy relation is not only elusive in that it can be captured by us only in its instances, but, owing to the finitude of our mind and experiences, we can know it only partially and in a rather unsystematic manner, despite all our everyday linguistic competence.

So if our logic is to give an account of our troubles in identifying meanings, which lies at the heart of our misunderstandings, then it is bound to provide us also with subjective concepts, which are somehow partial relative to objective meanings.

3. FROM SIGNIFICATIONS TO CONCEPTS

In the present theory, the significate of a predicate P in a thing u at time t is a particular, individualized property of the thing. We get from this the universal property of P-ness by abstracting from these two individualizing factors, namely from time and subject. On the other hand, by the converse operation, concretion, as it may be called, we can arrive again at the particular individualized property of an individual signified by the predicate P at a certain time.

Now, what if we fill in the empty argument-place of this universal (the signification of P) with a thing that is apt not only to **bear** such an individualized property, but which is apt also to **represent** such properties, namely a human mind?

Well, then what we get is the significate of the predicate P in this mind, say m, at time t: Sgt(P)(m)(t), i.e., the mental significate of P at t: $Sgt_m(P)(t)$. This is a particular, individual **concept**, belonging precisely to **this** mind. However, this does not mean that this concept is not universal in another respect, namely in regard to its objects. For even if this concept is particular in that it belongs to this mind, it is universal in that it is a common representation of many individuals. (cf. Aquinas, 1933:29.)

But how is this one-to-many relation established? Well, by abstraction, of course. The mind generates this universal concept by abstraction from particular representations of individuals, from **phantasms**, to use Aristotle's happy term. Now for the sake of brevity somewhat simplifying matters let me give a sketch of how I think from these mental representations (concepts and phantasms) we can construct a **mental language**, which may prove useful in our attempts at understanding understanding.

As I have said, the significate of a predicate P in a mind m at time t is a subjective concept, m's concept of P at time t:

$$Sgt(P)(m)(t) = Sgt_m(P)(t) = Con(m,t)(P)$$

But this concept is universal in that it is got by abstraction from phantasms:

$$Con(m,t)(P) = \lambda\varphi(Con(m,t)(P)(\varphi))$$

Now since phantasms are representations of individuals, we can speak of (here comes the simplification, but see the next section) the phantasm of an individual u in a mind m: $Phant_m(u)$. But then, we can suppose that the variables of our language, just as they range over individuals **ad extra**, so they range over corresponding phantasms representing these individuals **apud mentem**. That is, we may suppose that for every assignment of variables f, which assigns to a variable a thing **ad extra**, there is another assignment **apud mentem**, f_m, assigning to the variable in question the phantasm representing the thing:

$$f_m(x) = Phant_m(f(x))$$

But then we can construct the significate of an atomic sentence Fx in a mind m at a given time t according to an assignment **apud mentem** f_m (a "mental sentence", a thought) as follows:

$$Sgt_m(Fx)(t)(f_m) = Con(m,t)(F)(f_m(x))$$

Now, to round out the picture, again, just as we supposed a frame of reference **ad extra** consisting of sets of actual and non-actual individuals and their properties, so we have to suppose in the mind a representational frame containing phantasms with their properties assorted in classes according as the mind **takes them** to be actual or non-actual.

So, accordingly,

$$Con(m,t)(P)(\varphi) \in A_m(t) \cup P_m(t) \cup I_m(t) \cup \{0\},$$

where Am(t), Pm(t) and Im(t) are disjoint subsets of the "discourse-universe" of the mental representational frame, W(m,t), (note that W(m,t) may change in the course of time) which is a finite proper subset of W and φ is some phantasm: $Phant_m(u) \in W(m,t) \cup \{0\}$. (The case $Phant_m(u)=0$ represents the situation when m has no phantasm of u.)

Now, on this basis, by a method exactly parallel to the one applied above in connection with sentence-significates **ad extra**, namely by defining the mental significates of the logical connectives, we can determine the mental significates of any complex formula.

For example,

$$Sgt_m(\sim)(s_m) \in A_m t \text{ iff } \textbf{not}: s_m \in A_m(t),$$

and so on.

So in this way for any given formula A we can construct two corresponding significates (at a given time, according to a given assignment): one **ad extra**, the actuality of which means that A is true (at the given time, according to the given assignment), and another **apud mentem**, which being in the actual domain of the representational frame of a subject means that the subject assents to A (at t according to the corresponding assignment **apud mentem**). So whereas actuality **ad extra** founds the truth of, actuality **apud mentem** founds belief in a proposition.

So we can see that the same syntactic structure allows us to construct to it two different, but strictly corresponding significates. (cf. Bealer, 1982) And thus, just as the network of significates **ad extra** defines for us a synonymy relation between significations, i.e., (socially) objective meanings, so the network of significates **apud mentem** defines a synonymy relation **apud mentem**, between subjective concepts.

But we have this pretty rich model for a pretty poor language. Indeed, this language is so poor that despite the richness of its semantics it could only very poorly reflect the problems involved in understanding or misunderstanding each other in a human language. As a matter of fact, it could only serve to illustrate the semantic method by which we can handle even much richer formal languages approximating better the complexity of natural languages.

In the next section, therefore, I sketch some extensions of our formal language at hand, in order to show the ways in which I think we can get closer to the actual syntax and semantics of natural languages.

4. APPROACHING NATURAL LANGUAGE

The first thing to do by way of extension is to introduce many-place predicates. Now, as already one-place predicates are treated as if they had an extra-argument for time, this sort of extension comes very naturally as follows:

$$Sgt(P^n)(u_1)...(u_n)(t) \in A(t) \cup P(t) \cup I(t) \cup \{0\},$$

(of course, if u_1 or ... u_n=0, then it is not the case that $Sgt(P^n)(u_1)...(u_n)(t) \in A(t)$), whence the significate of an atomic sentence with an n-place predicate is constructed as follows:

$$Sgt(P^n(tm_1)...(tm_n))(t)(Sp)=Sgt(F)(Sp(tm_1)(t))...(Sp(tm_n)(t))$$

where $tm_1...tm_n$ are terms and $Sp(tm_1)(t)...Sp(tm_n)(t)$ are their **supposita** at time t.

Supposition, as it was conceived by medieval logicians, is the referring function of general, as well as of singular terms. As I have argued elsewhere (See Essay II. of this volume), supposition therefore should be regarded as an extended assignment function, extended in that it assigns referents not only to simple variables, but also to general terms, which are thus to be regarded as restricted variables.

The introduction of restricted variables into the apparatus of standard quantification theory greatly increases the expressive power and faithfulness of quantification theory in representing the workings of quantificational and cross-referential devices (determiners, pronouns) of natural languages. (See Essay III. of this volume.)

The introduction of restricted variables into the present framework is served by the following clauses:

/i/ If v is a variable and A is a formula, then v.A is a restricted variable

(Notice that this clause is applicable recursively, so that it also allows of "nested" occurrences of restricted variables. (Cf. Prullage, 1976))

/ii/ If x is a simple variable, then $Sp(x)(t) \in W$

/iii/ If v.B is a restricted variable, then $Sp(v.B)(t)=Sp(v)(t)$, if $Sgt(B)(t)(Sp) \in A(t)$,

otherwise $Sp(v.B)(t)=0$

Of course, the introduction of restricted variables has to affect also the clauses determining the significates of quantifiers. On the other hand, since with restricted variables we have to regard all quantifiers as two-place functors (at least, see again Essay II. of this volume), here we are able to formulate these clauses so generally that they cover not only the standard quantifiers ("every" and "some"), but also the astounding multitude of natural language determiners such as "most", "the", "less than half of the", "three", etc. (See Keenan & Stavi, 1986)

The relevant clauses are the following:

If Q is a determiner, then Sgt(Q)(N*)(V) is an element of

/i/ A(t), if for Q’$u \in N^*$, $V(u) \in A(t)$,

/ii/ I(t), if it is not the case that for Q’$u \in N^*$, V(u) is not an element of I(t)

/iii/ 0, if it is not the case that for Q’$u \in N^*$, V(u) is not identical with 0,

/iv/ P(t) otherwise,[5]

where Q’ is the natural language determiner represented by Q, N* is a subset of $W \cup \{0\}$, and V is a function, such that for every $u \in W \cup \{0\}$, $V(u) \in W \cup \{0\}$. (Intuitively, N* is the place-holder of the range of the noun-phrase determined by Q, while V is the place-holder of the signification function of the verb phrase of the sentence in which Q occurs.) Hence,

$$Sgt((Qv.B)(C))(t)(Sp)=Sgt(Q)(Rg(v.B)(t)(Sp))(Sg(\lambda v.B(C))(t)(Sp))$$

where Rg(v.B)(t)(Sp), the **range** of v.B, is defined as follows:

> $Rg(v.B)(t)(Sp)=\{u \in W: Sgt(B)(t)(Sp[v:u]) \in A(t)\}$, if this set is not empty, otherwise $Rg(v.B)(t)(Sp)=\{0\}$. (Recall also the definition of the signification of λ-abstracts above, substituting Sp for f in it.)

Now, our new terms, the restricted variables, have not only supposition, but also signification: a restricted variable, say, x.Fx, has not only the function to refer to (to denote) a thing which is F, but also to signify (to connote) the F-ness of this thing. But this connoted property need not always be identical with the significate of the matrix of such a variable: for example, Sgt(x.Fxy)(t)(Sp) need not be identical with Sgt(Fxy)(t)(Sp), for in that case it should also be identical with Sgt(y.Fxy)(t)(Sp); but, say, Plato’s love towards Socrates need not be identical with Socrates’s being loved by Plato.

So we can suppose that the dot, the term-forming operator of restricted variables (the natural language equivalents of which are relative pronouns), does double duty: it not only restricts the range of the operator variable, for it does so only by making the matrix connote a property in the supposita of the operator variable; indeed, the restricted variable stands only for those supposita of the operator variable which actually have this property.

So the clause determining the signification of the dot (of relative pronouns) runs as follows:

$Sgt(.)(u)(s) \in A(t), I(t), \{0\}$, or P(t) according as $s \in A(t), I(t), \{0\}$, or P(t), respectively, whence the significate of a restricted variable at time t according to a given supposition is constructed in the following manner:

$$Sgt(v.A)(t)(Sp)=Sgt(.)(Sp(v)(t))(Sgt(A)(t)(Sp)).$$

Now recognizing this connotative function of general terms should make us rethink what I have said about phantasms, the natural candidates for being the mental supposita of general

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[5] Compare the parallel definition of the significate of the existential quantifier above.

terms in their referring function. Phantasms, as I have said, are mental representations of individuals. But individuals as a rule do not appear to us as **nude** individuals, bereft of all properties. Phantasms always represent individuals as being endowed with general properties, signifiable by general terms. As St. Thomas Aquinas puts it in his **Commentary on the Posterior Analytics**: "... the senses perceive somehow even the universal. For they perceive Callias not only as being Callias, but also as being this man, and similarly Socrates, as being that man. Hence, such a perception of the senses being at hand, the intellect is able to see man in both ..." (Aquinas, 1882, lb.2.lc.20.) So, instead of speaking about the phantasm of a thing absolutely (this was the simplification noted above), we should rather speak of the phantasm of a thing u (in a mind m, at time t), **as bearing a certain property p**: Phant(m,t)(u,p), where $\text{Phant}(m,t)(u,p) \in W(m,t) \cup \{0\}$, whence the mental suppositum of a restricted variable (at time t) is:

$Sp_m(v.A)(t)=\text{Phant}(m,t)(Sp(v.A)(t),Sgt(v.A)(t)(Sp))$.

Now, as we can see, in this way the same thing may be represented by different phantasms, and different things may be represented by the same phantasm. But then even terms which are coreferential **ad extra** may have different supposita **apud mentem**.

But since, as I have said, what founds belief, is the actuality of the mental significate of a sentence in one's mental representational frame, no wonder if one who does not know about the coreferentiality of two terms **ad extra** may assent to a sentence with the one term, while dissent from the like sentence with the other. And so failure of substitutivity of coreferential terms gets a rather smooth explanation in this framework.

For an adaptation of cases involving proper names we should suppose that even a proper name can have a significate distinct from its suppositum. Well, why not? After all, getting to be named, say, John, confers upon one the property of being named John, i.e., of being the suppositum of the name "John".

So, accordingly, $Sgt(j)(u)(t) \in A(t)$, if Sp(j)=u, otherwise it is 0, where $Sp(j) \in W$, whence

$$Sp_m(j)(t)=\text{Phant}(m,t)(Sp(j),Sgt(j)(Sp(j)(t))).$$

This much having been said, I think I should also add that in the present theory only simple variables have no such kind of connotation: $Sgt(x)(t)=Sp(x)(t) \in W$.

Now what we have here is already a quite rich intensional theory, but with an extensional language, in the sense that we still do not have in our language intensional expressions such as modalities, tense operators or attitude verbs.

However, with this rich theory at hand we do not even need these various kinds of special expressions, the so-called intensional operators. In line with George Bealer's arguments, we may abandon the "multiple-operator approach" in favour of an "intensional abstraction approach" by introducing terms referring to "intensional entities"—to significates of expressions. The introduction of these new terms (**sentential terms** as I will call them, representing sentence-nominalizations) is served by the following clauses:

/i/ If A is a formula, then [A] is a (sentential) term

/ii/ $Sp([A])(t)=Sgt(A)(t)(Sp); Sp_m([A])(t)=Sgt_m(A)(t)(Sp_m)$

/iii/The significates of the above terms are identical with their supposita.

Now with these terms at hand we can offer satisfactory formulations and solutions of the intentional paradoxes. (I think, the representation of modal discourse is trivial in this setting: a possibility predicate, ‘it is possible’, states of a sentential term, ‘that p’, that its suppositum, i.e., the significate of p at t, is possible, i.e., is an element of P(t).) Let me sketch just one example:

(1) Plato is identical with Aristocles

(2) John believes that Plato is wise

(3) John believes that Plato is Aristocles

John believes that Aristocles is wise

(1) and (2) do not imply the conclusion, however, (2) and (3) do.

In a straightforward formalization the argument looks as follows:

(1) p=a

(2) jB([Wp])

(3) jB([p=a])

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jB([Wa])

Here the natural stipulation on the interpretation of B is the following:

$Sgt(jB(trm))(t)(Sp)=Sgt(B)(Sp(j))(Sp_{jm}(trm)(t))(t)\in A(t)$ if and only if $Sp_{jm}(trm)(t)\in A_{jm}(t)$, where trm is a sentential term, and ‘jm’ denotes John’s mind.

“Plato” and “Aristocles” being different names, though they stand for the same individual **ad extra**, they may have different phantasms in John’s mind as their mental supposita. But then the mental significates of Wp and Wa, i.e., the mental supposita of [Wp] and [Wa], may also be different. And so also the significates of (2) and the conclusion may be different, despite the truth of (1). So (1) and (2) may both be true, while the conclusion false. On the other hand, if (3) is true, then the names p and a must stand for the same phantasm in John’s mind. But then also [Wp] and [Wa] must stand for the same thought (of John’s mind), and so the significate of (2) and that of the conclusion are the same. Hence, if (2) and (3) are true, then so is the conclusion.

Now I think it is easily imaginable how this kind of treatment can be extended to cover also other intentional paradoxes, such as e.g. Kripke’s puzzle about belief. (Cf. Laurier, 1986:47) But then with a theory at hand that is able to offer a satisfactory treatment of the intentional paradoxes we may be more confident about what it brings out concerning the problems of understanding.

5. TOWARDS UNDERSTANDING

I hope that even from the above rather sketchy presentation a quite rich picture of a semantics emerged: we have a quite complex language with a twofold interpretation, one part of which is **ad extra** and the other is **apud mentem**. Both define a network of semantic values, significations **ad extra** and concepts **apud mentem**, characterizable by their respective synonymy relations. Concepts are partial relative to significations in that they are defined only for finite real subsets of the domain of significations. (At least, in their relation to one's subjective representational system. For more on this see Appendix and Essay VII.) Still, there is some similarity between the network of significations and that of concepts. Indeed, a suitably defined **homeomorphism** (i.e. partial isomorphism) of their respective synonymy relations may express their conformity. And what this conformity expresses should be nothing but the degree of the subject's semantic competence who possesses this network of concepts. For apart from John's case above, who simply does not know that Plato was originally called Aristocles, it may also be the case that, due to our finitude of mind, we do not know the whole of the synonymy relation of our language. This is why we have so many troubles with providing correct definitions or unexceptionable taxonomies. And this is why Theaetetus, in his quest for knowledge, might come to know without any kind of external experience what he, despite his everyday semantic competence, formerly had not known: by Socrates's questions he was led to adjust the synonymy relation of his subjective representational system (remember, it changes in time!) to the objective synonymy relation existing in his language. (Cf. Bealer's treatment of the paradox of analysis.)

On the other hand, as I have already indicated, the objectivity of this synonymy relation is **social** objectivity. This is the kind of objectivity possessed by e.g. prices. This kind of objectivity derives from the behaviour of groups of people. (Whether or not "behind" this kind of objectivity there is some more basic, unchanging objectivity, limiting the possible variety of human concepts and languages, inherent in human nature or the nature of things is a further question. Notice that the theory presented is, as a good logical theory should be (cf. Henry on "logical aloofness", Henry, l984), able to give room for a great variety of ontologies ranging from extreme realism to utmost nominalism and conventionalism. For more on this see Appendix.) But then, since people are free agents, if their behaviour changes, this changes also the structure of the kind of objectivity their behaviour constituted. And this is why meanings can change through the activity of users of the language. If a rule is strong (many people keep to it), then acting against it is just to make a mistake, if, on the other hand, the rule is weak, or there is no rule at all, then users of the language are quite free to create new rules thereby changing the whole network of significations. (Just consider the change caused by the change of the concept of motion. When "moves" (movetur) ceased to mean "is being moved /by something/" (movetur /ab aliquo/), then arguments for a prime mover became pointless.)

Now suppose there is a population of people who speak the same language. In our present framework this means that they use the same expressions in a way which constitutes an objective synonymy relation between these expressions. We may say that these people generally understand each other if and only if in keeping with the rules according to which the significates of expressions are to be constructed, possessing homeomorphic synonymy relations, they are able to construct in their mental representational systems those mental significates of these expressions which correspond to the same significate of these expressions **ad extra**.

But this kind of understanding may be hampered by several kinds of impediments:

1. SYNTACTIC AMBIGUITIES

The construction of the significates of a complex expression, as we have seen, depends on its syntactic structure. So if a complex expression has different syntactic analyses (e.g. "Every boy loves some girl)", "A white thing can be black"), then surely different significates may pertain to it as it is analysed in one way or another.

2. SEMANTIC AMBIGUITIES

Languages abound with equivocal terms. (pot_1=earthenware vessel; pot_2=marijuana) Equivocal terms are those which have different significations. But then the same word construed in one way or another may contribute to the construction of non-corresponding significates of the same expression in the minds of different language users.

3. PRAGMATIC AMBIGUITIES

Some expressions are unspecific on purpose. They contain some hidden variable element the value of which is specified by context. For example, as Dag Westerstahl has convincingly argued, the definite article is to be regarded as a context set indicator. (Westerstahl, l985) It requires uniqueness of reference of the noun phrase following it only in respect of a limited set of things: when I say "I was in the kitchen", I do not want to imply that I was in the only kitchen in the world. (For the introduction of context sets into the theory of restricted variables see Essay III.) Now failure to identify context sets properly is a very frequent source of misunderstandings.

4. IGNORANCE

In the case of John above, his lack of acquaintance with Diogenes Laertius's piece of information might throw him into serious trouble at a history of philosophy examination. He might /mis/understand the malicious examiner's questions concerning Aristocles as relating to a person whom he should know but (for all his knowledge about Plato) does not.

5. INCOMPETENCE

Incompetence (I mean, semantic incompetence) is some partial anisomorphism between one's subjective synonymy relation and the objective synonymy relation. For example, one who does not know that pot_2=marijuana may badly misunderstand a pot_2 dealer. Now since this kind of impediment of understanding depends on partial anisomorphisms, it may well be the case that two persons who can quite fluently chat about certain things are shocked at hearing the "nonsenses" the other says when it comes to others. I think that here is a quite smooth transition from cases of incompetence, when one clearly breaks a definite rule, to cases of

6. CONCEPTUAL DIFFERENCES

where there is no definite rule to break, so users of the same language are quite free to create their own concepts. (This is why it is so difficult sometimes to tell the mad from the genius.)

These conceptual differences sometimes may prove to be so grave that in some cases two speakers, even if they use the same words, can hardly be said to speak the same language. (Compare e.g. the language of Heidegger and Carnap.)

7. DIFFERENT CONCEPTUAL SCHEMES

Now if we finally take into consideration the fact that when we learn our native tongue we do not learn words by definitions, but **we are trained to recognize things as being such and such**, as falling under this or that concept (remember how, according to the present theory, phantasms represent things **as having some property**), then it is just to be expected that different linguistic communities may develop different conceptual schemes. (In the sense of Walton, l973)

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Having a theory at our disposal which is rich and flexible enough to give an exact reconstruction of the impediments of understanding we may have hopes to give at least correct diagnoses (even if not remedies) of the several kinds of ununderstanding that plague human communication. However, one must not be stupidly optimistic. At the end of this paper I am not quite sure that I have succeeded in making myself understood.

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Appendix

In this Appendix I wish to give an exact description of the formal theory sketched in the body of the paper lest any technically obscure point should remain.

The language of the theory, L, is defined as follows: L:=<C,Pr,V,Trm,F>, where C:={~,&,Q,.,(,),[,],=}, Pr:=Pind∪Ppred, where Pind is the set of individual parameters, or names, Ppred is the set of n-place predicate parameters and V is the set of simple, or proper variables. These sets contain the **primitive symbols** of L. The **complex expressions** of L are contained in Trm, the set of terms, and F, the set of formulae of L. These sets are defined by the following simultaneous recursive definition:

1. If a∈Pind, then a∈Trm.
2. If x∈V, then x∈Trm.
3. If tm∈Trm and B∈F, then tm.B∈Trm
4. If $tm_1,...,tm_n$∈Trm and P^n∈F, then $P^n(tm_1)...(tm_n)$∈F and $(tm_i=tm_j)$∈F
5. If B, C∈F, then ~(B)∈F, (B & C)∈F
6. If v∈Trm and B∈F, then (Qv)(B)∈F
7. If B∈F, then [B]∈Trm

A **semantic system**, SYST, for L is defined as follows:

SYST:=<RF,Sg>,

where RF, the **referential frame** of SYST is defined in the following way:

RF:=<W,A,P,I,T,Ra,Re,Abs,<,0>,

where W is a nonempty set, A(t), P(t), I(t) are disjoint subsets of W for any t∈T, T is a nonempty subset of W ordered by <, Ra is a subset of W, Re is a subset of Ra, Abs is a subset of W, and 0 falls outside W, i.e., 0 is not an element of W. Intuitively, W is the universe of discourse, the set of everything that can be referred to or signified by any means in L. A(t), P(t) and I(t) are the sets of signifiable things, which are actual, potential, or impossible at time t, respectively. T is a set of time points (or intervals), ordered by <, the relation "earlier than". Ra is the set of rationate beings and Re is the set of real beings. (For the intuitive distinction see Schmidt, 1966) Abs is the set of abstract entities, such as sets and functions. 0 is the zero-entity, the semantic value of empty terms, and of predicates in respect of things for which they are undefined.

Sg, the **signification function** of SYST, is defined as follows:

Sg(exp)=λe1(...λen(Sgt(exp)(e1)...(en))...),

where exp is any expression of L (including primitive symbols, except for brackets and parentheses), e1,...,en are any entities whatever (i.e., elements of W∪{0}, elements of L and functions defined thereon) and Sg(exp)∈Abs.

NOTE that I use the λ-operator according to the following equivalences:

If f and g are functions, then

$$f(x)(y)=g(y) \text{ iff } f(x)=g \text{ iff } \lambda y(f(x)(y))=g$$

NOTE also that the distinction between signification (Sg) and significate (Sgt) is intended to reflect the difference between unsaturated and saturated entities, respectively. A significate of an expression in respect of this and that is always a saturated, complete entity. (But note here the special status of concepts. Cf. Essay VII.) The signification of an expression (perhaps, in respect of this or that) is always an unsaturated, abstract entity.

Now the **significates of expressions** of L in an RF are determined by the following clauses:

(1) If $P^n \in Ppred$, $u_1,...,u_n \in W^*$ and $t \in T$, then $Sgt(P^n)(u_1)...(u_n)(t) \in W(t)^*$, where $W^*:=W \cup \{0\}$,

$W(t)^*:=A(t) \cup P(t) \cup I(t) \cup \{0\}$, and if $u_i=0$, then **not**: $Sgt(P^n)(u_1)...(u_n)(t) \in A(t)$, whence

(2) $Sgt(P^n(tm_1)...(tm_n))(t)(Sp)=Sgt(P^n)(Sp(tm_1)(t))$ ... $(Sp(tm_n)(t))(t)$, where $Sp(tm_i)(t)$, is a **suppositum**

of the term tm_i at time t. The **supposition function** of terms is defined by the following clauses:

(i) If $x \in V$, $t \in T$, Then $Sp(x)(t) \in W$

(ii) If $a \in Pind$, then $Sp(a) \in W$, and $Sp(a)(t)=Sp(a)$

(iii) If $tm.B \in Trm$, then $Sp(tm.B)(t)=Sp(tm)(t)$, if $Sgt(B)(t)(Sp) \in A(t)$, otherwise $Sp(tm.B)(t)=0$.

(iv) If $[B] \in Trm$, then $Sp([B])(t)=Sgt(B)(t)(Sp)$

(3) If $s \in W(t)^*$, then $Sgt(\sim)(s) \in A(t)$ iff **not**: $s \in A(t)$, whence

(4) $Sgt(\sim(B))(t)(Sp)=Sgt(\sim)(Sgt(B)(t)(Sp))$.

(5) If $s_1,s_2 \in W(t)^*$, then $Sgt(\&)(s_1)(s_2)$ is an element of

/i/ $A(t)$, if $s_1 \in A(t)$ and $s_2 \in A(t)$

/ii/ $I(t)$, if $s_1 \in I(t)$ or $s_2 \in I(t)$ and the other is not 0 or if $s_1 \in A(t)$ then **not**: $s_2 \in A(t)$

/iii/ $\{0\}$, if $s_1=0$ or $s_2=0$

/iv/ $P(t)$ otherwise, whence

(6) $Sgt((B\&C))(t)(Sp)=Sgt(\&)(Sgt(B)(t)(Sp))(Sgt(C)(t)(Sp))$

(7) If u_1, $u_2 \in W(t)^*$, then $Sgt(=)(u_1)(u_2)(t)$ is an element of

/i/ $A(t)$, if $u_1=u_2 \in A(t)$

/ii/ $P(t)$, if $u_1=u_2 \in P(t)$

/iii/ I(t) otherwise, whence

(8) $Sgt(tm_1=tm_2)(t)(Sp)=Sgt(=)(Sp(tm_1)(t))(Sp(tm_2)(t))(t)$

(9) If N* is a part of W*, and V(u)∈W(t)*—intuitively, the place-holders of the range of the “noun phrase” and the signification of the “verb phrase” of the quantified statement, respectively—, where u∈W*, then

Sgt(Q)(N*)(V) is an element of

/i/ A(t), if for Q’u∈N*, V(u)∈A(t),

/ii/ I(t), if it is **not** the case that for Q’u∈N*, **not**: V(u)∈I(t)

/iii/ {0}, if it is **not** the case that for Q’u∈N*,**not**: V(u)=0,

/iv/ P(t) otherwise, where Q’ is a natural language determiner represented by Q. Hence,

(10) Sgt((Qv)(C))(t)(Sp)=Sgt(Q)(Rg(v)(t)(Sp))(Sg(λv(C)(t)(Sp)), where

Rg(v)(Sp)(t)={u∈W: for some Sp’ differing from Sp only in the value assigned to v,[*] Sp’(v)(t)=u}, if

this set is not empty, otherwise Rg(v)(Sp)(t)={0}; Sg(λv(C)(t)(Sp)=λu(Sgt(C)(t)(Sp)(u)),

where for every u∈Rg(v)(t), Sgt(C)(t)(Sp)(u)=Sgt(C)(t)(Sp[v:u]),

where, Sp[v:u](w)(t)=u, if v=w, otherwise Sp[v:u](w)(t)=Sp(w)(t).

And this completes the definition of the signification function for formulas. What remains, then, is to define it for terms. (For terms thus far we have defined only supposition.)

(11) If x∈V, then Sgt(x)(t)=Sp(x)(t)

(12) If a∈Pind, then Sgt(a)(u)(t)∈A(t), if u=Sp(a)(t), otherwise Sgt(a)(u)(t)=0

(13) If tm.B∈Trm, then Sgt(tm.B)(t)(Sp)=Sgt(.)(Sp(tm)(t))(Sgt(B)(t)(Sp)),

where

(14) Sgt(.)(u)(s) is an element of A(t), I(t), P(t) or {0},

according as s is an element of A(t), I(t), P(t) or {0}, respectively.

(15) If [B]∈Trm, then Sgt([B])(t)(Sp)=Sp([B])(t)

Now clauses (1)-(15) /together with clauses (i)-(iv)/ determine the semantic values of L in an RF **ad extra**. What remains to be done, therefore, is to define their semantic values **apud mentem**. To this end first we have to define the **subjective representational frame** of a mind m at time t, SRF(m,t), as follows:

$$SRF(m,t):=\langle W(m,t),A_m,P_m,I_m,T_m,Re_m,Ra_m,Abs_m,<,0\rangle,$$

[*] And, of course, its nested operator variables, if it has any.

where W(m,t) is a finite, proper subset of W, $A_m(t)$, $P_m(t)$ and $I_m(t)$ are disjunct subsets of W(m,t), T_m is a subset of T, Re_m, Ra_m and Abs_m are subsets of W(m,t), < is an ordering on T_m and 0 is the same as above. Intuitively, the elements of SRF correspond to the elements of RF above.

Now a **subjective representational system** of a mind m at time t, SYST(m,t), consists of such a representational frame, containing the mind's **concepts** and **phantasms**:

$$SYST(m,t):=<SRF(m,t),Con(m,t),Phant(m,t)>,$$

where, if $\in_1,...,\in_n$ are elements of W(m,t), then

$$Con(m,t)(P^n)(\in_1)...(\in_n) \in A_m(t)\cup P_m(t)\cup I_m(t)\cup\{0\}$$

and

$$Phant(m,t)(u,p) \in W(m,t)\cup\{0\}$$

where u and p are elements of W-W(m,t), i.e., u is an extramental thing, with p, its extramental property.

The subjective interpretation of the language L, then, is determined by the following clauses:

(1_m) $Sgt_m(P^n)(t)=Sgt(P^n)(m)(t)=Con(m,t)(P^n)$, whence

(2_m) $Sgt_m(P^n(trm_1)...(trm_n))(t)(Sp_m)=Con(m,t)(P^n)(Sp_m(trm_1)(t))...(Sp_m(trm_n)(t))$

where the mental suppositum of a term trm_i is defined:

$Sp_m(trm_i)(t)=$

(i_m) $=Phant(m,t)(Sp(trm_i)(t),Sgt(trm_i)(t))$, if trm_i is a proper variable,

(ii_m) $=Phant(m,t)(Sp(trm_i),Sgt(trm_i)(Sp(trm_i))(t))$, if trm_i is an individual parameter,

(iii_m) $=Phant(m,t)(Sp(trm_i)(t),Sgt(trm_i)(t)(Sp))$ if trm_i is a restricted variable or a sentential term.

The rest of the clauses determining the mental significates of expressions of L run exactly parallel with those determining the extramental significates of the same expressions. (Were this not the case, this would mean that the subject in question has different concepts from those established by common language usage even for the logical connectives. For the representation of this case, however, rather further special syntactic signs would have to be introduced. For example, a sign for Hegelian negation, etc.) Only by way of example:

(3_m) $Sgt_m(\sim)(s_m)(t)\in A_m(t)$ iff **not**: $s_m\in A_m(t)$, whence

(4_m) $Sgt_m(\sim(B))(t)(Sp_m)=Sgt_m(\sim)(Sgt_m(B)(t)(Sp_m))(t)$,

and so on. As no theoretical or practical difficulty is involved in the formulation of the remaining clauses, let me omit their statement here.

However, before completing the formal description of this theory (by giving the definitions of truth and validity) I have to clarify one more, theoretically important point. (I am indebted to István M. Bodnár for having called my attention to this point.) It concerns a highly important distinction of Aristotelian semantics between the immediate and ultimate significates of linguistic expressions. For as we know from Aristotle's *Peri Hermeneias* (lb.I.c.1.), our words signify immediately our concepts, and only by the mediation of these the (properties of) things. The necessity of this distinction also in the present framework can be shown by the following reasoning: in the body of the paper it has been said that it is the actuality of the significate of a predicate in a thing that verifies the predicate of this thing. For example: the predicate "red" is true of this apple if and only if the significate of this predicate in this apple is actual, i.e., iff the redness of this apple exists. On the other hand, we have also said that the significate of a predicate in a mind is a concept. But this would mean that the actual possession of a concept signified by a predicate would verify the predicate of the mind which possesses this concept. For example: on this account by possessing the concept of immateriality a mind would have to be immaterial, and by possessing the concept of materiality the same mind would have to be also material, which is absurd.

However, by drawing the above-mentioned distinction between the immediate and ultimate significate of a word, this absurdity can be avoided. We have said that the significate of a predicate in a mind is a concept. Let us say now that this is the **immediate significate** of this predicate in this mind. But this concept is a universal, i.e., a function, which, when its empty argument-place is filled in with a phantasm, yields a thought within the subjective representational system of the mind. On the other hand, we have said nothing thus far about the application of this concept to the extramental objects themselves. But since according to Aristotle a concept is the natural sign of a (property of) a thing, we can say that a concept, when applied to an extramental thing, i.e., when its argument-place is filled in with a thing, yields an extramental property of the thing as its value, namely the property which this concept is the natural sign of. And this is the **ultimate significate** of the word signifying the concept immediately. And it is the actuality of **this** significate that verifies the predicate of the thing. For example: the word "immaterial" in a mind m signifies immediately the concept of immateriality inherent in this mind. But the inherence of this concept does not verify the predicate of the mind. However, the concept can be applied to a thing, indeed, to the mind itself, when it yields the individualized property inherent in this mind, which is the ultimate significate of the word "immaterial". And it is the actual inherence of this property that verifies the word of this mind. So even if the mind possesses also the concept of materiality, this would not verify of it the word "material", since the ultimate significate of this word is not actual, indeed, as the case may be, it is impossible in this mind. (For more on this cf. Essay VII.)

Now if we denote the ultimate significate of an expression exp in respect of any kind of entities $e_1,\ldots,e_n$, as $Sgt'(exp)(e_1)\ldots(e_n)$, then we can give these ideas a formal expression by the following clauses:

(1') $Sgt'(P^n)(u_1)\ldots(u_n)(t)=Sgt(P^n)(u_1)\ldots(u_n)(t)$, provided $u_1\ldots u_n$ is not a human mind.

(2') $Sgt'(P^n)(u_1)\ldots(u_n)(t)=Con(m,t)(P^n)(u_1)\ldots(u_n)$, provided $u_1\ldots u_n$ is not a phantasm. However, (3’) $Sgt'(P^n)(m)(t)=Con(m,t)(P^n)(m)(t)$, but this is not identical with $Sgt(P^n)(m)(t)=Con(m,t)(P^n)$

(4’) $Sgt'(P^n(tm_1)...(tm_n))(t)(Sp)=Sgt'(P^n)(Sp(tm_1)(t)...(Sp(tm_n)(t))(t)$

(5’) $Sgt'(\sim(B))(t)(Sp)=Sgt(\sim)(Sgt'(B)(t)(Sp))$.

(6’) $Sgt'((B\&C))(t)(Sp)=Sgt(\&)(Sgt'(B)(t)(Sp))(Sgt'(C)(t)(Sp))$

(7’) $Sgt'(tm_1=tm_2)(t)(Sp)=Sgt'(=)(Sp(tm_1)(t))(Sp(tm_2)(t))(t)$,

where identity is to be treated as a distinguished predicate obeying (1’)-(3’).

(8’) $Sgt'((Qv)(C))(t)(Sp)=Sgt(Q)(Rg(v)(t)(Sp))(Sg'(\lambda v(C)(t)(Sp))$, where

$Sg'(\lambda v(C)(t)(Sp)=\lambda u(Sgt'(C)(t)(Sp)(u))$,

where for every $u\in Rg(v)(t)$ $Sgt'(C)(t)(Sp)(u)=Sgt'(C)(t)(Sp[v:u])$.

Now with these formulations at hand we can give as the general definition of **truth** for L in respect of a SYST the following:

(T) If B is a formula, then B is true at time t, if and only if for some Sp, $Sgt'(B)(t)(Sp)\in A(t)$.**

Validity, then, is truth at every time in every SYST, and formal validity of an inference is the validity of the corresponding conditional, as usual.

Synonymy of expressions is identity of their significations. Synonymy of two expressions for a mind is identity of their mental significates or significations (derivable from the mental significates specified above by λ-abstraction). Synonymy of significations or concepts is their identity. Synonymy, of course, may be **partial**, when it is nothing but the partial identity of significations. (That is, identity of significations restricted to a subdomain of their domains.)

A **homeomorphism** of synonymy relations may be defined as follows:

If Syn_1 and Syn_2 are synonymy relations and x and y are elements of subsets of the domain and the range of Syn_1, respectively, then Syn_1 is homeomorphic to Syn_2 iff there is a 1-1 function f such that $<x,y>\in Syn_1$ iff $<f(x),f(y)>\in Syn_2$, where f(x) and f(y) are elements of subsets of the domain and range of Syn_2, respectively.

Finally, I think, some remarks concerning the general character of this system are in order, especially in view of its complexity. As I have noted in the main text, in keeping with D.P.Henry’s aloofness principle, I wanted to construct this system as general as possible, so that it would not impose any artificial restriction either upon the intuitions of users of a language, or upon the possible ontologies available (i.e. formalizable) in it. Now this is one reason for the complicated nature of this system: it is only at the expense of reducing its generality and its expressive power that we could reduce its complexity. For example, by restricting our language only to uniform quantification, or to talk about

** Note how well this definition corresponds to Aristotle’s conception of truth according to which a proposition is true iff what it signifies exists.

extramental entities, excluding even human minds as possible referents of our expressions we could omit several clauses of the above description. But I think this would heavily reduce the philosophical import of this system. On the other hand, with this construction, treated as a general framework, we can give formal expression to a great variety of linguistic forms and related ontological viewpoints. Only by way of illustration let me mention just some of the several possibilities.

(1) By introducing terms referring to significations, and by placing significations in the domain of real beings we get the Platonic world of universal ideas.

(2) By placing significations in the domain of rationate beings and imposing some restrictions upon the discourse about them (cf. Lear, 1982) we get the Aristotelian theory of universals. (For more on this see Essay VII.)

(3) By introducing into the language of the system a copula and an existence predicate, along with appropriate semantic rules, we can offer exact reformulations of a number of traditional metaphysical theses, such as Aquinas's thesis of the real distinction between essence and existence in the creatures, or the thesis of the unity of substantial forms. (See Essay VI.)

(4) By identifying the supposita and significata of (absolute, non-connotative) general terms, and, correspondingly, analyse the copula of the categoricals in terms of identity, we get the metaphysics of 14th century nominalism. (See again Essays VI. and VII.)

Finally, lest anyone should think that this system is able to represent only traditional philosophies let me show in somewhat more detail how I think the ontology of W.V.O. Quine can be accommodated within this system. We have to introduce the following stipulations: W-A!=Abs, where A! is the union of all A(t)'s, i.e., there are no non-actual entities (i.e. merely possible, or impossible individuals, or objects of reference) save the abstract ones. $Sgt(P)(u)(t) \in \{u,0\}$, i.e. predicates signify individuals, not their properties. Whence the extension of a predicate: $Ext(P) := \{u: \text{for some } t, Sgt(P)(u)(t)=u\}$. And so a formula of the form Pa is true iff $Sp(a) \in Ext(P)$. The extension of this approach to many place-predicates is trivial as follows: $Sgt(P^n)(u_1)...(u_n)(t) \in \{<u_1,...,u_n>,0\}$, whence the extension of P^n: $Ext(P^n) := \{<u_1,...,u_n>: \text{for some } t, Sgt(P^n)(u_1)... (u_n)(t)=<u_1,...,u_n>\}$. And so we get back the standard theory of predication.

As I have said, these are only some, rather *ad hoc* chosen examples of the possible applications of this system. I have said nothing in particular concerning how modal, temporal and intentional notions can be introduced, and how the exact reconstruction of these notions within this system can contribute to the systematic study of concept formation, conceptual change, and comparisons of different conceptual schemes in general. I brought up these illustrations only to suggest that this system should not be regarded as the end, but rather as the beginning of an ambitious project.