6 Amortization: saving

The future amortization formula

It's now time to get my daughter's college fund together. I want to have $120,000 available for her education when she turns 18. Our earlier discussions in EXAMPLE (1.2.14) and EXAMPLE (1.4.33) had two morals. The first is that the sooner I start planning for this the better. We'll come back to this point later. The second is that I'll never be able to reach this goal by investing a single lump sum: the amount I'd need is just out of my reach even if I make the investment when she's born. Almost everyone faces a number of similar problems in their life: you know you are going to need a sum of money many times your annual salary all at once. The most universal examples are the money you need to buy a house (or even a car) and the money you will live on when you retire. How do people ever assemble these sums of money? You probably already see the answer: if the sum is too big to put together all at once the only solution is to put it together a bit at a time. People being creatures of habit, the only way most of us can be sure of continuing to put aside the small sums needed is to regularly put aside a fixed sum. This process is called *amortization*. The goals of this section are to understand what the small sums will amount to over time and to draw some conclusions about planning for such major needs. First let's define our terms.

AMORTIZATION (1.6.1) A sum of money which is assembled by making a series of equal deposits at regular intervals into an account which earns a fixed interest rate, or a loan at fixed interest which is repaid by making a series of equal payments at regular intervals is said to be *amortized*.

In this section, we'll consider amortizations which involve saving: we make regular deposits with the goal of having a lump sum of money available *after* the payments are made. Loans, where we receive the lump sum *before* making a series of payments will be discussed in the next section.

We'll continue to use many terms from the preceding section to describe such series of payments. For example, the *period* of the amortization will be the length of time between deposits or payments and we'll again use m to denote the number of periods in a year. The number of years in the *term* of the amortization will again be denoted y and the number of periods (or deposits or payments) will be T: these are still related by the TERM CONVERSION FORMULA (1.1.14) $T = m \cdot y$. The fixed nominal interest rate will be r and the corresponding periodic rate will be $p = \dfrac{r}{100 \cdot m}$ by the INTEREST RATE CONVERSION FORMULA (1.1.11).

SIMPLIFYING ASSUMPTION (1.6.2) In all the problems in this section, we will assume that the compounding frequency of the account into which deposits are made or of the loan against which payments are made is equal to the frequency with which the deposits or payments are made and that deposits or payments are always made at the end of each compounding period.

The assumption that compounding frequency and deposit frequency are equal is definitely not true in many everyday amortizations and you wouldn't even want it to be. For example, you want to have the amount in your retirement account compounded daily (because you get more interest this way) even if you only make deposits into it once a month on payday. What you don't want is to have to learn the very complicated formulas which are needed to compute the amounts in such an account. (Of course, in some professions, these formulas are critical and there are entire courses devoted to them). Our assumption will allow us to work only with very simple formulas. What's more these simple formulas give answers close to, if not quite equal to, those from the more complicated ones so we can draw the important conclusions everyone should know about amortizations from them. In a similar vein, there are lots of amortizations in which the deposits are made at the start rather than the end of each period. The corresponding formulas are only a touch more complicated than the ones we'll use but they are significantly harder to remember and it's easy to confuse the two. We'll leave them to the professionals too.

We will use two new terms for the regular amounts involved:

Deposit (1.6.3) The common amount of each *deposit* will be denoted D although we'll use the term *payment* when a loan is being amortized.

Sum and Balance (1.6.4) The lump sum of money being assembled in a savings account will be denoted S and be called the *sum* of the account or loan. The intermediate sum after the end of the i^{th} period — that is, the amount which has accumulated in the account in the first i deposits will be denoted by S_i. In particular, when a sum is being assembled we will have $S = S_T$, the final intermediate sum after all T periods.

The lump sum of money being repaid in a loan will be denoted B and be called the *balance* of the loan. The intermediate balance after the end of the i^{th} period — that is, the balance outstanding on a loan after the first i payments will be denoted by B_i. In particular, when a loan is being repaid, we will have $B = B_0$ the initial intermediate balance before the start of the first period.

As I hope you'll have guessed from the uses of these letters in earlier sections, we use S for the intermediate sums in a savings account because these are all basically future values and use B for the intermediate balances on a loan because these are all basically present values.

For the rest of this section, we discuss only savings. To start, let's get back to my daughter's college fund. What I'd like to do is open an account which pays a fixed rate of interest — let's say 3.9% — and then make a small deposit into the account at the end of every month from my daughter's birth until she turns 18 at which time

I'd like to have \$120,000 in the account. Since I am making monthly payments $m = 12$ and since I make them for 18 years $y = 18$. Thus the term $T = m \cdot y = 12 \cdot 18 = 216$. Further since $r = 3.9\%$, the periodic rate $p = \frac{r}{100 \cdot m} = \frac{3.9}{100 \cdot 12} = .00325$. The final sum $S = S_{216}$ I want to reach is \$120,000. The only thing I don't know is how big a deposit D to make every month.

Let's just leave this as an unknown for now and try to understand how the money in the account builds up. To do so, I'll use S_i to stand for the sum at the end of i months. Let's compute the first few intermediate sums S_i (remember this is the amount in the account at the end of i months) and see whether we can spot a pattern.

After one month, all that's in the account the first deposit D which I have just made so

$$S_1 = D\,.$$

That was easy. The second month isn't much harder. The ONE PERIOD PRINCIPLE (1.2.3) says that adding the interest earned in the second month to the sum S_1 just multiplies it by the magic factor $(1+p)$ to $S_1(1+p)$ to which we add the second deposit D to get the second month's sum

$$S_2 = D + (1+p)S_1 = D + (1+p)D = D\,(1 + (1+p))\,.$$

(Wondering why I factored out the D and then did *not* rewrite the $(1 + (1+p))$ as $2 + p$? Hindsight! This helps makes the final pattern easier to spot as you'll see in a moment.)

The third month isn't much harder either. Again, the ONE PERIOD PRINCIPLE (1.2.3) says that adding the interest earned in the third month to the sum S_2 just multiplies it too by the magic factor $(1+p)$ to $S_2(1+p)$ to which we add the third deposit D to get the third month's sum

$$S_3 = D + (1+p)\,S_2 = D + (1+p)D(1 + (1+p)) = D\left(1 + (1+p) + (1+p)^2\right)\,.$$

It's not too hard to see the pattern which is emerging here. Passing from one month's sum to the next is always the same. Suppose that our sum after i months is S_i. Then the ONE PERIOD PRINCIPLE (1.2.3) says that adding the interest earned in the next — $(i+1)^{\text{st}}$ — month to the sum S_i just multiplies it by the magic factor $(1+p)$ giving $S_i(1+p)$ to which we add the next or $(i+1)^{\text{st}}$ deposit D to get the sum S_{i+1} at the end of $i+1$ months:

$$S_{i+1} = D + S_i\,(1+p)\,.$$

Moreover, it's easy to see that the final expanded formula for S_i to which this leads is

$$S_i = D\left(1 + (1+p) + (1+p)^2 + \cdots + (1+p)^{i-2} + (1+p)^{i-1}\right)\,.$$

All the formulas begin with a common factor of D which multiplies a sum in which all but the first two terms are powers of the magic factor $(1+p)$ and the highest exponent which appears is 1 less than the number i of months we are working with. We can actually think of *all* the terms as powers of $(1+p)$ by remembering that for any positive base x, we have $x^0 = 1$ and $x^1 = x$. Taking $x = (1+p)$, this lets us write the last formula as:

FIRST SUM FORMULA (1.6.5) $$S_i = D\left((1+p)^0 + (1+p)^1 + (1+p)^2 + \cdots + (1+p)^{i-2} + (1+p)^{i-1}\right)\,.$$

We can check our guesses for both patterns by plugging the FIRST SUM FORMULA (1.6.5) into the equation relating S_i and S_{i+1} above:

$$\begin{aligned}
S_{i+1} &= D \qquad +(1+p)S_i \\
&= D\cdot 1 \qquad +(1+p)D\left((1+p)^0 + (1+p)^1 + (1+p)^2 + \cdots + (1+p)^{i-2} + (1+p)^{i-1}\right) \\
&= D(1+p)^0 \qquad +D\left((1+p)^1 + (1+p)^2 + \cdots + (1+p)^{i-2} + (1+p)^{i}\right) \\
&= D\left((1+p)^0 + (1+p)^1 + (1+p)^2 + \cdots + (1+p)^{i-2} + (1+p)^{i}\right)
\end{aligned}$$

Again the highest power of $(1 + p)$ is i which is one less than the number of months in S_{i+1} so we have verified the prediction of the FIRST SUM FORMULA (1.6.5).

SECOND APPROACH (1.6.6) There's another way to think about the FIRST SUM FORMULA (1.6.5) which sometimes comes in handy. (Having an second way to think about any problem is always a good thing because often something which looks difficult or messy from one point of view becomes very simple when we use the other). If we track that very first deposit through the calculation it shows up in the sum for S_i as the term $D\,(1+p)^{i-1}$. This term is what the COMPOUND INTEREST FORMULA (1.2.4) would give for the future value after $i - 1$ periods of an amount $A_0 = D$ at a periodic interest rate of p. A moment's thought reveals that this is not an accident: that first payment was made at the end of the first month so at the end of the i^{th} month it's been earning interest for $i - 1$ months and has accrued to $D\,(1+p)^{i-1}$. The same is true for all the other terms in the sum: the term $D\,(1+p)^{i-2}$ corresponds to the future value of the second deposit which has earned $i - 2$ months of interest, and so on down to the terms $D\,(1 + p)^1$ and $D\,(1 + p)^0$ which correspond to the last two deposits which have earned one month and 0 months of interest respectively. In other words, we can reach the formula *either* by computing successive sums as we did above *or* by tracking deposits individually to the end of some period and then adding.

Great! I have two ways to reach a formula for the final sum S in my daughter's college fund because $S = S_{216}$, the intermediate sum after 216 months. There's just one problem with this formula as it stands. The sum of powers of $(1 + p)$ in the formula for S_i has i terms. This means that to use it to figure out what will my sum will be after 216 months, I'd need to compute and sum 216 powers of $(1 + p)$. No thanks. I might not finish before my daughter's 18^{th} birthday. When mathematicians are faced with making long computations they are usually just as unhappy as most of you. The difference is that, instead of giving up or plowing away, a mathematician's reaction is to ask: "If I think hard and come up with the right clever idea, can't I find some way to get the answer to this calculation without doing all the work?" The power of mathematics is that when you do think hard the answer is surprisingly often, "Yes!".

In this case, the clever idea is an easy one based on difference of squares factorization, $(x+y)(x-y) = x^2 - y^2$. If we let $x = 1$, this becomes $(1+y)(1-y) = 1 - y^2$. Why aren't there more terms? If we expand out, we see that $(1+y)(1-y) = 1(1-y) + y(1-y) = 1 - y + y - y^2 = 1 - y^2$. The two y's cancel because of the opposite signs. The clever idea has two parts. First, let's think of $(1+y)$ as $y^0 + y^1$ as we did above and view the factorization as

$$(y^0 + y^1)(1-y) = 1 - y^2 .$$

Second, let's ask what happens add a few more powers of y. Adding a y^2, we find

$$(y^0 + y^1 + y^2)(1-y) = y^0(1-y) + y^1(1-y) + y^2(1-y) = y^0 - y^1 + y^1 - y^2 + y^2 - y^3 = 1 - y^3 .$$

There are 2 cancellations this time and we still have only two terms in the final right hand side. Adding a y^3, we find

$$(y^0+y^1+y^2+y^3)(1-y) = y^0(1-y)+y^1(1-y)+y^2(1-y)+y^3(1-y) = y^0-y^1+y^1-y^2+y^2-y^3+y^3-y^4 = 1-y^4 .$$

This time there are 3 cancellations this time and we still have only two terms in the final right hand side. Once again, a pattern is clear. The only thing that changes when we add another power is that the exponent of y on the final right hand side goes up by 1. If we go up to the j^{th} power — you'll see in a moment why I want to use j here and not i as above — what we find is:

$$\begin{aligned}
&(y^0 + y^1 + \cdots + y^{j-1} + y^j)(1-y) \\
=&y^0(1-y) + y^1(1-y) + \cdots + y^{j-1}(1-y) + y^j(1-y) \\
=&y^0 - y^1 + y^1 - y^2 + \cdots + y^{j-1} - y^j + y^j - y^{j+1} \\
=&1 - y^{i+1} .
\end{aligned}$$

Does this look a bit familiar? I hope so. This is the basis of a famous formula called the GEOMETRIC SERIES FORMULA (1.6.7) which I hope you have already seen. A *series* is a fancy word for a sum in which the terms follow a predictable pattern and a *geometric series* is one in which the pattern is to take higher and higher powers of a common base. (If you think *power* series would be a better name so do I, but this terminology is too standard for us to fight.) To get the formula, we just divide both sides by $(1 - y)$ leaving just the series on the left side.

GEOMETRIC SERIES FORMULA (1.6.7) For any integer $j \geq 0$, $y^0 + y^1 + \cdots + y^{j-1} + y^j = \dfrac{1 - y^{j+1}}{1 - y}$.

We can use this to eliminate the long string of powers and additions from the FIRST SUM FORMULA (1.6.5). The sum in this formula which gets multiplied by D is just the sum we get on the left side of the GEOMETRIC SERIES FORMULA (1.6.7) if we substitute $y = (1 + p)$, $j = i - 1$. (See why I used the j? Trying to substitute $i - 1$ for i is one of the most surefire ways I know to get confused and make errors. I planned ahead so we could avoid doing this.) Therefore, we can replace the sum in the FIRST SUM FORMULA (1.6.5) by what we get on the right hand side of the GEOMETRIC SERIES FORMULA (1.6.7) when we make the same substitutions. Of course if $j = i - 1$ then $j + 1 = i$ and if $y = (1 + p)$ then $(1 - y^{j+1}) = (1 - (1 + p)^i)$ and $(1 - y) = (1 - (1 + p)) = -p$ so what we get is

SECOND SUM FORMULA (1.6.8)

$$S_i = D\left(\frac{1 - (1 + p)^i}{-p}\right) = D\left(\frac{(1 + p)^i - 1}{p}\right).$$

By setting $S = S_{216}$, this gives me a formula relating the final sum S in my daughter's college fund to the deposit D that I make and the periodic interest rate p. But there's really nothing special about this example except that we fixed the term T to be 216 periods. So, let's write down the general formula before seeing what it implies for my daughter's college fund.

FUTURE AMORTIZATION FORMULA (1.6.9) If a deposit is made at the end of each of T periods into an account which earns compound interest at a periodic rate p, then the final sum S in the account at the end of the T^{th} period and the amount D of each deposit are related by

$$S = D\left(\frac{(1+p)^T - 1}{p}\right) \quad \text{and} \quad D = S\left(\frac{p}{(1+p)^T - 1}\right).$$

Working with the future amortization formula

Note how amazingly simple the FUTURE AMORTIZATION FORMULA (1.6.9) is. This means that, although we had to huff and puff a fair bit to get to them, applying it is a piece of cake.

EXAMPLE (1.6.10) We did all the work above: if my account earns 3.9% for 18 years compounded monthly, then $p = .00325$ and $T = 216$. If I want to have a final sum S of \$120,000, then the equation for D tells me I need to deposit

$$D = S\left(\frac{p}{(1+p)^T - 1}\right) = \$120000\left(\frac{.00325}{(1+.00325)^{216} - 1}\right) = \$384.05$$

every month. That's a big expense but it is still a sum I can think about including in my monthly budget (especially as such deposits generally are not taxed).

We'll come back to consider other ways to use the FUTURE AMORTIZATION FORMULA (1.6.9) in a moment but first let's formalize what we did above with a method. As always, the first two steps are the same (find m and use it to get p and T) and the third just involves plugging values into the formula.

METHOD FOR SOLVING FUTURE OR SAVINGS AMORTIZATIONS (1.6.11)

Step 1: Determine the periods in the problem (that is, the units in which the term is measured) and the value of m, the number of periods per year.

Step 2: Use the INTEREST RATE CONVERSION FORMULA (1.1.11) to find the periodic interest rate p from the nominal interest rate r and the TERM CONVERSION FORMULA (1.1.14) to find the term T *in periods* from the term in years y.

Step 3: Apply the appropriate FUTURE AMORTIZATION FORMULA (1.6.9) to find whichever of the the deposit D and the sum S is to be determined.

SELF-TEST

Here are a few exercises for you to try which involve what are generally called *sinking funds*. These are accounts into which businesses make regular deposits which accumulate towards the purchase of some high-ticket item. The purpose, as with my daughter's college fund, is to spread out the corresponding expense over the term of the fund and avoid having a large charge on the books in any accounting period. I have worked a few of the examples.

PROBLEM (1.6.12) An insurance company wants to make monthly deposits into an account which interest compounded monthly to fund the purchase of a computer which will cost $135,000. How much should each deposit be if:
a) the account has a nominal rate of 5% and the payments are made over 5 years?

Solution

Step 1: The periods are months so $m = 12$.

Step 2: $p = \dfrac{r}{100m} = \dfrac{5}{100 \cdot 12} = .0041\dot{6}$ and $T = my = 12 \cdot 5 = 60$.

Step 3: We know the sum $S = \$135000$ so we plug into find the deposit

$$D = S\left(\frac{p}{(1+p)^T - 1}\right) = 135000\left(\frac{.0041\dot{6}}{(1+.0041\dot{6})^{60} - 1}\right) = \$1985.12.$$

b) the account has a nominal rate of 8% and the payments are made over 3 years?

c) the account has a nominal rate of 3% and the payments are made over 5 years?

Problem (1.6.13) A University wants to make annual deposits of \$10000 into a sinking fund on which interest account is compounded annually to fund the purchase of a statue to commemorate a distinguished faculty member. How much can they afford to pay for the statue if

a) the account has a nominal rate of 4.8% and the payments are made over 4 years?

Solution

Step 1: The periods are years so $m = 1$.

Step 2: $p = \dfrac{r}{100m} = \dfrac{4.8}{100 \cdot 1} = .048$ and $T = my = 1 \cdot 4 = 4$.

Step 3: Here we know the deposit amount $D = \$10000$ so we plug into find the final sum

$$S = D\left(\frac{(1+p)^T - 1}{p}\right) = 10000\left(\frac{(1+.048)^4 - 1}{.048}\right) = \$42,973.27.$$

b) the account has a nominal rate of 3.2% and the payments are made over 3 years?

c) the account has a nominal rate of 6.6% and the payments are made over 7 years?

How can we check such amortization calculations? Basically, both the SIMPLE INTEREST APPROXIMATION (1.3.3) and the CONTINUOUS APPROXIMATION (1.3.11) can be souped up for use in checking amortizations. The latter is actually more straightforward. We simply approximate the exponential $(1+p)^T$ in the FUTURE AMORTIZATION FORMULA (1.6.9) by the slightly larger exponential $e^{(p\cdot T)} = e^{\left(\frac{r}{100}\cdot y\right)}$. In the formula for D where this appears in the denominator and we are now dividing by a larger quantity, we get an approximation slightly smaller than the exact value.

FUTURE AMORTIZATION: CONTINUOUS APPROXIMATION (1.6.14) S is a bit less than $D\left(\frac{e^{\left(\frac{r}{100}\cdot y\right)}-1}{p}\right)$.

This check lacks one feature of the CONTINUOUS APPROXIMATION (1.3.11): there's still a periodic rate p in each formula. If you forgot to convert the nominal rate when using the FUTURE AMORTIZATION FORMULA (1.6.9), you'll probably use r for p here too. Fortunately, the different numerator will lead to a different answer and let you catch your mistake. You may also notice that I haven't colored it: this is one formula which you don't really need to learn. You can make the necessary approximations if you just remember to use the CONTINUOUS APPROXIMATION (1.3.11) to replace the $(1+p)^T$.

EXAMPLE (1.6.15) Let's check the calculation in EXAMPLE (1.6.10). Here we had $r = 3.9\%$ and $y = 18$ years and since we were compounding monthly $m = 12$ and $p = .00325$. Our D was \$384.05 so the sum S of \$120,000 should be a bit smaller than

$$D\left(\frac{e^{\left(\frac{r}{100}\cdot y\right)}-1}{p}\right). = 384.05\left(\frac{e^{(.039\cdot 18)}-1}{.00325}\right) = \$120,270.78\,.$$

and it is. You can check a sum calculation the same way.

PROBLEM (1.6.16) Use the continuous approximation to check your answers to problems PROBLEM (1.6.12) and PROBLEM (1.6.13).

As with SIMPLE INTEREST APPROXIMATION (1.3.3), the simple interest approximation to an amortization calculation is both better and worse. It's better because the calculation is easier — you can often do it in your head — and experience shows that we're much likelier to make an easy check than a hard one — but worse because it's less accurate. (It's like the difference between a cheap point-and-shoot camera you can put in your pocket and reflex camera with lots of lenses. The reflex camera takes much better pictures but if you leave it in the hotel room because the case is so heavy, so what.)

The idea is very simple. First add up all the deposits getting an amount $A = T \cdot D$, then add some simple interest to this. The only question is how much simple interest to add. All the deposits earn simple interest for differing numbers of periods so we really ought to add a different amount of interest to each. The problem with doing so is that you get a formula more complicated than the one you are trying to check. The solution is to group pairs of deposits moving inwards from the start and end of the term. The first deposit earns $T-1$ periods interest and the last 0 periods. Together the two earn a total if $T-1$ periods interest and an average of $\frac{T-1}{2}$ periods. The second deposit earns $T-2$ periods interest and the second last 1 period of interest. Again, the total is $T-1$ periods and the average is $\frac{T-1}{2}$ periods. The third deposit earns $T-3$ periods interest and the third last 2 periods of interest. Again, the total is $T-1$ periods and the average is $\frac{T-1}{2}$ periods. So the pattern is that an average deposit earns $\frac{T-1}{2}$ periods of interest. To keep things simply, we replace this with $\frac{T}{2}$ periods which is an average term of $\frac{y}{2}$ years. Now using years as periods, the periodic rate is $\frac{r}{100}$ so the SIMPLE INTEREST

FORMULA (1.1.6) says the interest earned should be roughly $\frac{r}{100} \cdot A \cdot \frac{y}{2}$. If we then add the amount A, we should get a total $A + A\frac{r}{100}\frac{y}{2} = A\,(1 + \frac{r}{100}\frac{y}{2})$ somewhat smaller than the actual final sum S — smaller because we are ignoring the effect of compounding. Note that I didn't collect the two fractions because I think it's easier to use the formula when we think of them separately. In fact, this is another formula where it's better to remember the idea — an average deposit earns interest for half the term — than the formula. The example following the formula illustrates this.

FUTURE AMORTIZATION: SIMPLE INTEREST APPROXIMATION (1.6.17) S is larger — possibly quite a bit larger — than $T \cdot D\left(1 + \frac{r}{100}\frac{y}{2}\right)$.

EXAMPLE (1.6.18) Let's recheck the deposit $D = \$384.05$ I calculated in EXAMPLE (1.6.10). Here we had $T = 216$, $r = 3.9\%$ and $y = 18$ years so so $\frac{r}{100} = .039$ and $\frac{y}{2} = 9$. Our final sum S should be somewhat greater than

$$T \cdot D\left(1 + \frac{r}{100}\frac{y}{2}\right) = 216 \cdot 384.05(1 + .039 \cdot 9) = \$112,071.93$$

and it is. As we learned to expect in APPROXIMATING COMPOUND INTEREST (1.3), this approximation is cruder than the previous one. However, by making it a bit cruder still, we could use it without getting our calculator out. In a class, I'd say something like this. "Well, 216 times 384.05 is about 200 times 400 or 80,000. And, 3.9% times half of 18 is about 4 times 9 or about 36% — lets say 40%. So add $\frac{2}{5}^{\text{ths}}$ of 80,000 which is 32,000 to get \$112,000. That's a bit less than the sum of \$120,000 so I'm happy." The point to notice is that I was happy to replace 216 by 200 or 3.9 by 4 or 36 by 40 to simplify the arithmetic. I know this approximation isn't going to give me a lot of decimals of accuracy anyway. I just want to make sure that I didn't do something stupid — which I do all the time, just a bit less often than most of you, I hope. If I did, my approximation and my answer would probably be off by a factor of 2 or 200, I'd be worried and I'd go look for my mistake; here they're within 10% of each other which is reasonable agreement.

Problem (1.6.19) Use the simple interest approximation to check your answers to Problem (1.6.12) and Problem (1.6.13).

Problem (1.6.20) You make monthly deposits of $200 into a retirement account.
a) How much will you have in the account at the end of 3 years if the account earns interest of
 i) 2%?
 ii) 6%?
 iii) 10%?
b) How much will you have in the account at the end of 30 years if the account earns interest of
 i) 2%?
 ii) 6%?
 iii) 10%?
c) Discuss which approximation is appropriate for checking your answers to a)ii) and which is appropriate for checking your answers to b)ii). Then carry out these checks.

Any smokers out there? Like to quit? Here's some motivation.

Problem (1.6.21) Let's suppose that cigarettes cost $2 a pack and that you smoke a pack a day. We'll call this a monthly expense of $60 on cigarettes. Suppose you are 21 now and that you quit smoking and put that $60 a month into a retirement account which invests in stocks and which yields 8% every month. If you continue investing your cigarette money until you are 65, show that you will have over $290,000 in the account! Exactly how much will you have?