11 Saving to spend

Now we are ready to pull out all the stops and combine savings and loans amortizations in a single problem. You might think that I'd have to make us some artificial situation to come up with such a problem but you'd be wrong. Your retirement planning is a perfect example. In considering retirement planning so far, we have always had as our goal *saving* a certain lump sum of money in the at retirement: this was always a future value amortization. That's OK if you are a Midas who just wants to count his gold but what most people want when they retire is an *income*. The real point of accumulating that lump sum in your retirement account is to be able to finance such an income. The mechanics of doing so are just like those of a mortgage or present value amortization. The only difference is that *you're the bank*: you hand over the lump sum (like the bank handing the homeowner that initial mortgage balance) when you retire and then sit back and receive a series of monthly payments (analogous to those the bank gets from the homeowner) which represent your retirement income. It's only rarely that simple. It is possible to use your retirement fund to buy what's called a *fixed term annuity* which promises you a fixed monthly payment for a fixed term. But most people are afraid — or better, hopeful — that they'll outlive that term and be left without any income. Such people usually buy a *life annuity* which promises a fixed monthly payment as long as you live. This involves merging a whole series of amortizations with different terms – after all, the term of a life annuity is not fixed but depends on how long you live. Thus, the price of such an annuity has to take into consideration how likely you are to die at various ages — the fancy term for this is mortality — and somehow merge the various present values into a single price. Moreover, these are just the simplest wrinkles. You might want an income until both you and your spouse die (a *joint life annuity*) and you might want the annuity to take into account inflation or rises in the cost of living. The complications which arise have spawned an entire profession: actuaries specialize in the mathematics and statistics of this type of calculation. Moreover, the yield on such annuities is generally fairly low. If you can tolerate a bit more risk, you can do better by continuing to invest your retirement fund yourself and paying yourself an income out of the fund. If you do this, you'd like to be sure that the checks are not going to suddenly start bouncing when you are 73. When the time comes, I'd suggest getting professional advice. But we already know enough to make calculations which can help you in planning when you are younger. I'd like to look at few examples of these to close the chapter.

Example (1.11.1) Here's an example which will illustrate the basic idea. Right now, I am 35 and just starting my retirement planning. I have found an insurance company which offers fixed term annuity accounts which earn 5.1% a year (compounded monthly as usual) and I have decided that when I am 65 I'd like to be able to buy such an annuity with a term of 25 years and monthly payment of $2500. I plan to finance this purchase with the proceeds of a retirement account into which I will make monthly payments over the intervening 30 years. With this long term time horizon, I am prepared to invest this account fairly aggressively — that is, in higher yielding but risker securities — and I think it is prudent to plan on an average return of 9% over this period. The basic question I need to answer now is, "How much do I need to deposit every month?

What are we dealing with here? Two separate amortizations. My deposits into the retirement account are a savings or future amortization: I know the term is 30 years (so $T = 360$) and I am assuming the rate will be 10% (so $p = .0075$) but I know neither the sum which is my goal nor the monthly deposit I will make. This means we are missing two ingredients in the Future Amortization Formula (1.6.9): S and D and so I can't use it to find out either. So we'll put this aside for now. The annuity I plan to buy represents the other amortization. It's a loan or present value amortization in which, as remarked above, I'm the bank and the insurance company plays the role of the mortgage holder who makes regular monthly payments. I know the term of this annuity is 25 years (so $T = 300$), the rate r is 5.1% (so $p = .00425$) and the monthly payment D is $2500. Thus I can use the Present

Amortization Formula (1.8.1) to find the balance B I'll need to pay the insurance company. We find,

$$B = D\left(\frac{1-(1+p)^{-T}}{p}\right) = \$2500\left(\frac{1-(1+..00425)^{-300}}{.00425}\right) = \$423,419.45\,.$$

Now comes the only new point. The balance B that I'll need to *purchase* the annuity is the same thing as the sum S that I'll want to have *saved* in my retirement account when I'm 65: so $B = S = \$423,419.45$. Now we *do* know enough to use the Future Amortization Formula (1.6.9) to find the balance to find the deposit D I'll need to make into the retirement account. We find that

$$D = S\left(\frac{p}{(1+p)^{T}-1}\right) = \$423,419.45\left(\frac{.0075}{(1+.0075)^{360}-1}\right) = \$231.28\,.$$

I need to deposit \$231.28 a month.

To emphasize, let's restate the new idea here. We have a pair of related amortizations: a future or savings amortization in which we assemble a sum of money which we then use to finance a present or loan or annuity amortization. (Of course, both amortizations live in the future, the "present" one lives further in the future than the "future" one and the "loan" is really a purchase: we are using all these terms in the conventional sense established earlier in the chapter). If we know everything but the final sum S of the savings amortization, the initial balance B of the loan amortization and one other quantity, then we can use a two step process like that above to completely describe both amortizations by identifying B and S. Either, as above, we know everything about the loan or annuity except the balance B and can solve for this using the Present Amortization Formula (1.8.1), or, as above we know everything about the savings amortization and can solve for S using the Future Amortization Formula (1.6.9). Then, we use the fact that $B = S$ to use the other formula to determine the remaining missing piece of information about the other amortization (this was the deposit of \$231.28 above). Here are some problems for you to try.

PROBLEM (1.11.2) Recalculate the payment I'll need to make in my retirement account keep all the values of EXAMPLE (1.11.1) except that we assume that
a) my annuity has a yield of 3% and my retirement account has a yield of 5%. (Here I am asking, "What's the worst that can happen?".)
b) I am 45 years old and want to retire and buy a 30 year annuity when I am 60.
c) I want the annuity to pay \$3000 a month.

PROBLEM (1.11.3) a) Suppose that I am 45 years old and starting a retirement account. Based on my current income, IRS will only let me put \$275 a month into this account tax free.
If I think the account will have a yield of 10%, what monthly payment can I expect to get from a 25 year annuity yielding 4.5% which is purchased with the sum in the account when I am 65?
b) How does the answer to a) change if I am 25 today?
c) How do the answers to a) and b) change if I expect a yield of 7.5% on both the retirement account and the annuity?

PROJECT (1.11.4) What fundamental reality has been totally ignored in this entire chapter? Hint: it's not death. Right, taxes! We have discussed all kinds of financial planning decisions which in real life are critically affected by tax considerations without ever mentioning this issue. In this project, I'd like you to pick a few of the examples we have looked at and to try to understand how tax issues should affect your thinking about them. Here are a three suggestions but feel free to pick others if they interest you.
a) Mortgage interest. The *interest* you pay on a mortgage on your primary residence may generally be deducted

from your income for federal tax purposes. Discuss how this affect the relative desirability of owning versus renting. Illustrate your general discussion with a concrete scenarios involving low, middle and upper income taxpayers.

b) Retirement income. The Federal Government encourages you to save for retirement by allowing you to deduct many forms of retirement savings from your income for federal tax purposes. However, when you spend these savings at retirement, they — and any gains realized from them — are subject to taxes. How might this affect your planning for retirement? Are there situations when you might want to pay taxes on income before investing it for retirement.

c) Capital gains. Your profits on many investments like stocks and bonds are subject to tax *when realized*. For example, if you bought 100 of a stock for $10 a share and it is now selling at $20 a share you have a paper profit of $1000. If you sell the stock and realize the profit, it will be subject to capital gains tax. You will not, however, be taxed if you continue to hold the stock. In effect, the government is allowing you to invest that profit without paying taxes on it but only in the stock you already own. Discuss how this affects the real yield of investment strategies. Illustrate your discussion with comparative scenarios. For example, how much higher a before-tax yield must you be able to achieve via a strategy which involves selling assets every 6 months to match the after-tax yield of a strategy in which you hold assets for an average of a decade?