9 Things everyone should know

Equity

"Why rent when you can own? Paying rent is just throwing money away. If you buy a house, you'll be building equity." This is a classic real estate agent's argument to convince renters to become homeowners. Does it make sense? For the real estate agents, definitely. When a property changes hands, the agents for both the buyer and the seller collect several percent of the the selling price. That's their business so they are interested in generating the greatest number of sales possible both by convincing new buyers to enter the market and by convincing existing owners to "trade up". The first question I want to look at is what your position as a potential buyer should be.

Let's begin by trying to understand what "building equity" means. The idea is simple. In discussing the INTEREST APPROXIMATION (1.8.12), we saw that the actual periodic payment D you make on a loan is somewhat greater than the periodic interest $B_0 \cdot p$ on the initial loan balance B_0. (By the SIMPLE INTEREST FORMULA (1.1.6), the interest in one period is $I = p \cdot A \cdot T = p \cdot B_0 \cdot 1 = pB_0$.) For example on a loan of $B_0 = \$100,000$ at 7.5% interest which is paid monthly (so $p = .00625$), we'd owe interest of \$625 after one month. The difference $D - B_0 \cdot p$ is the part of the *first* payment which is used to reduce the outstanding balance on the loan. Note that while the interest owed is the same whatever the *term* of the loan — in our example, \$625 — this difference *will* depend on the term. We saw in PROBLEM (1.8.4) that a loan with a 30 year term the payment $D = \$699.21$ and hence the difference $D - B_0 \cdot p = \$74.21$ while for a loan with a 15 year term we have a payment of $D = \$927.01$ and a difference $D - B_0 \cdot p = \$302.01$.

In any case, at the end of the first period, your outstanding balance B_1 has decreased a bit: the reduction in balance is $B_0 - B_1 = (D - B_0 \cdot p)$ so $B_1 = B_0 - (D - B_0 \cdot p)$. In the second month you incur periodic interest of $B_1 \cdot p$. Since B_1 is a bit smaller then B_0, this interest is a bit smaller than the periodic interest due in the first month, so the difference $D - B_1 \cdot p$ between what you pay and the interest you owe is a bit bigger. This difference is also the reduction in your outstanding balance in the second month so $B_1 - B_2$ is a bit bigger than $B_0 - B_1$. This process continues. Each payment you make reduces your outstanding balance a bit more, and the amount of the reduction gets bigger and bigger every payment. Equity is the name for these reductions.

Equity (1.9.1) The difference $E_i = B_0 - B_i$ between the initial balance B_0 of a loan and the intermediate balance B_i after i payments is called your *equity* after i months. The equity increases with each payment made and the size of the increase also grows from payment to payment.

We tend to think of equity as a form of savings or investment particularly with respect to mortgage loans against real estate. Let's assume that *the value of the property on which the mortgage is held is equal to the initial balance of the mortgage at all times*: this is wildly unrealistic but we'll ignore this for now. Assuming this, then the equity E_i is the amount of extra cash you could realize if you sold the property and paid out the outstanding balance after i periods. You'd get back the initial balance B_0 as the sale price, pay the bank the outstanding balance B_i and be left with the equity $E_i = B_0 - B_i$ in your hot little hand. In particular, at the end of the term of the mortgage, when the outstanding balance $B_T = 0$, your equity is equal the value B_0 of the property.

Mortgage based equity was traditionally an important component of overall savings in the United States, particularly for households approaching retirement age. You'd buy a house in your 20's or early 30's, pay off the 30 year mortgage at roughly age 60 and the equity in the house would be your "nest egg" for retirement. Two things have altered this fact. The first is the increased role of targeted retirement funds and pensions. The second is the increased mobility of American families. It is this second effect which I'd like you to understand.

To do so, we'll need a formula for the equity E_i after i periods in terms of the basic quantities B and D. We found a formula for E_1 above: $E_1 = B_0 - B_1 = (D - B_0 \cdot p)$. We could also have found a formula for E_2 above: the reduction in balance in the second month was $D - B_1 \cdot p$ so $E_2 = E_1 + (D - B_1 \cdot p)$ and since $B_1 = B_0 - (D - B_0 \cdot p)$ and $E_1 = (D - B_0 \cdot p)$,

$$E_2 = (D - B_0 \cdot p) + \Big(D - (B_0 - (D - B_0 \cdot p)) \cdot p\Big).$$

Did that go by a bit quick? Not to worry because it is pretty clear that even if we understand E_2 in this way, things are going to be too complicated to get a general formula. We can see that each payment reduces the balance a bit more but the exact amount of the reduction in the i^{th} payment depends on the values of the reductions in *all* the preceding payments. It looks hopeless.

What we need is another way to think about E_i. The basic formula $E_i = B_0 - B_i$ provides just this. We know B_0 just equals B. So, if we can find a simple formula for the outstanding balance B_i we can use it to get a simple formula for E_i. We have just such a formula: the PRESENT AMORTIZATION FORMULA (1.8.1)! The trick making this do the work is to ask, "What would the bank be willing to accept as a substitute for the lump sum balance B_i *at the end of the* i^{th} *period*?" We know one answer: the remaining $(T - i)$ periodic payments of \$$D$ which we would have to make if we did *not* pay of the balance after i periods. In other words, the balance B_i is the answer to the question, "How big a loan can I take out at a periodic interest p if I am willing to make $T - i$ periodic payments of \$$D$?" Thus, we can think of B_i as the *initial* balance of a loan with payment \$$D$, periodic rate p and *shortened term* of $T - i$ periods. The PRESENT AMORTIZATION FORMULA (1.8.1) tells that this initial balance is

$$B_i = D\frac{(1 - (1 + p)^{-(T-i)})}{p}$$

and hence that

$$E_i = B_0 - B_i = B - D\frac{(1 - (1+p)^{-(T-i)})}{p}.$$

This is another of those cases where it is much easier to remember the idea than to learn a new formula. After all, we really just used the PRESENT AMORTIZATION FORMULA (1.8.1).

BALANCE AND EQUITY PRINCIPLE (1.9.2) The intermediate balance B_i outstanding after exactly i payments have been made on a loan with initial balance B, payment D, periodic rate p and term T periods equals the *initial balance* of a loan with payment D, periodic rate p and *shortened term* $T - i$ periods. The equity E_i after i payments is just the difference between the initial balance B and the i^{th} intermediate balance B_i.

We can put this more informally: to get the balance after i payments, just shorten the term by i periods. Because we're using a formula we already understand, we can jump right in and work equity and balance problems. Here's the method.

METHOD FOR FINDING BALANCE AND EQUITY (1.9.3)

Step 1: We start by using the METHOD FOR SOLVING PRESENT OR LOAN AMORTIZATIONS (1.8.3) to find the payment amount D on the loan as usual — often, we will already know this and so can skip this step.

Step 2: Use the PRESENT AMORTIZATION FORMULA (1.8.1) with this payment and the *same* periodic rate to compute the intermediate balance B_i remembering to replace the original term T with the number of payments $T - i$ *not yet made* on the loan. In other words, we just *subtract* the number i of payments *made* from T: remember that i is in units of *payments* or periods not units of *years*. If we're asked for a balance after z years we'll have to convert the term of z

years to an equivalent number i of periods or payments.

Step 3: Compute the intermediate equity E_i by subtracting the intermediate balance B_i from the initial balance B.

Example (1.9.4) Here's a typical example of how we use this. Let's find the outstanding balance and equity on a \$100,000 mortgage at interest of 7.5% with a term of 30 years at the end of 5 years.

Solution

Step 1: Finding the payment for this mortgage was part a)i) of Problem (1.8.4) so we can just borrow the values from solution given there. As usual $m = 12$ and we saw that $p = .0075$, $T = 360$ and $D = \$699.21$.

Step 2: Over the first 5 years of the mortgage we will make $i = 5 \cdot m = 60$ payments, leaving $T - i = 360 - 60 = 300$ payments to go, so the formula

$$B_i = D\left(\frac{1-(1+p)^{-(T-i)}}{p}\right) \quad \text{becomes} \quad B_{60} = \$699.21\left(\frac{1-(1+.00625)^{-300}}{.00625}\right) = \$94,616.83.$$

Step 3: The desired equity E_{60} is given by $E_{60} = B - B_{60} = \$100,000 - \$94,616.83 = \$5,383.17$.

Self-Test

Here are some exercises for you to try.

Problem (1.9.5) Find the outstanding balance and equity on a \$100,000 mortgage at interest of 7.5% with a term of 15 years at the end of

a) 5 years.

b) 10 years.

Solution

Step 1: Here we need to figure out the payment first. We again have $B = \$100000$, $m = 12$ and $p = .00625$ but now $T = 15 \cdot 12 = 180$ so

$$D = B\left(\frac{p}{1-(1+p)^{-T}}\right) = 100000\left(\frac{.00625}{1-(1+.00625)^{-180}}\right) = \$927.01.$$

Step 2: Over the first 10 years of the mortgage we will make $i = 10 \cdot 12 = 120$ payments, leaving $T - i = 180 - 120 = 60$ payments to go, so the intermediate balance we want is

$$B_{120} = \$927.01\left(\frac{1-(1+.00625)^{-60}}{.00625}\right) = \$46,262.72.$$

Step 3: The desired equity is $E_{120} = B - B_{120} = \$100,000 - \$46,262.72 = \$53,737.28$.

c) 15 years. (No calculator allowed!)

Problem (1.9.6) Find the outstanding balance and equity on a \$100,000 mortgage at interest of 7.5% with a term of 30 years at the end of

a) 10 years.

b) 15 years.

c) 20 years.

d) 25 years.

Stop for a moment and stare at your answers to these problems before going on. They are striking in several ways.

The first worked example describes a scenario which the author has seen played out many times by friends. A young family buys a house with a 30 year mortgage (usually they cannot afford the higher payment which would go with a shorter term) and then moves after 5 years (because of a job change or a divorce, to buy a bigger house etc.). Note that the equity such a family accumulates over the first 5 years they pay the mortgage before reselling the house is only about $5,000. Closing costs on the purchase of a house of this value — closing costs are the commissions and fees you pay to the real estate agent, banks, title search agency, and lawyers, are likely to be at least this large. In other words, such a family will not build up enough equity to recover its initial closing costs in buying the house — not to speak of what the yield on this outlay might have amounted to over the 5 years period nor of the closing costs involved in selling the house at the end of the term.

The second worked example might describe a family who can afford to take out a mortgage with a term of 15 years and then stay in the house for 10 years before moving. This is different in two ways. The first difference is that the term is only half as long (15 years instead of 30). This means a higher payment but the extra amount is surprisingly small. If we ignored interest, we'd need a payment twice as big to pay off a loan in half the time. The effect of compounding is to compress this gap strikingly: the actual difference of about $228 a month is less than 25% of the 15 year payment. The second difference in this example is that the family stays twice as long. We'd expect both factors to lead to a larger equity. One guess might be to double the equity from $5,000 to $10,000 to account for doubling the payments made and then add 25% to account for the higher payment: this would give equity of $12,500 or so. The actual equity is more than *four times* this guess and more than *ten times* what the first family would accumulate!

What are we missing? The answer is that we are making a simple mistake in two ways. Instead of comparing the payments for the two mortgages we should be comparing the amount of each payment which *remains* after deducting interest owed. These are the amounts that yield equity. We saw above that for the first payment the remainder was \$74.21 for the 30 year mortgage and \$302.01 for the 15 year mortgage. The remainder after we remove the interest in the 15 year mortgage is *four times* the remainder in the 30 year mortgage. So we can improve our guess and say that the second family should have 8 times the equity of the first — a factor of two from the number of payments and a factor of 4 for the size of the remainder.

So, why is the actual multiple closer to 10 than 8? For the same reason in another guise. Another way to say that the equity on the 15 year mortgage is growing faster is to say that the outstanding balance is shrinking faster. Therefore, the interest due each month on the outstanding balance is also dropping faster making the difference between the two remainders at the ends of the respective periods is even bigger. After 5 years the balance outstanding on the 30 year mortgage will be \$94,616.83 so the interest owed in the 61st month will be \$591.36 and the remainder in this payment will be \$699.21 − \$591.36 = \$107.85. The balance after 120 months on the 15 year mortgage is \$46,262.72 so the interest owed in the 121st month will be \$289.14 and hence the remainders in this month will be \$927.01 − \$289.14 = \$637.87. The ratio of the two remainders thus ends up at 6 not 4. On the average this ratio should be about 5 which with the factor of 2 for the term gives us the factor of 10 we see in the equities.

To really get a feel for how equity build up, a picture is worth a thousand words. Each of the graphs in Figure (1.9.7) plots the number of payments made on the *x*-axis against the equity built up after that many payments on the *y*-axis. In both graphs, the interest rate is 7.5% but on the left I have used a term of 30 years while on the right the term is 15 years. Actually, I have plotted the equity as a *fraction* of a initial loan balance *B* which is why the ticks on the *y*-axis go from 0 (no equity) to 1 (no outstanding balance). For example, after 60 payments on the 30 year mortgage the graph is at height about .05: on a balance of \$100,000 I would have equity of about

$.05 \cdot \$100,000 = \$5,000$. Likewise, after 120 payments on the 15 year mortgage the graph is at height about .54: on a balance of \$100,000 I would have equity of about $.54 \cdot \$100,000 = \$54,000$. Remember that you can use the magnifier tool in **Acrobat** to zoom in on the graph.

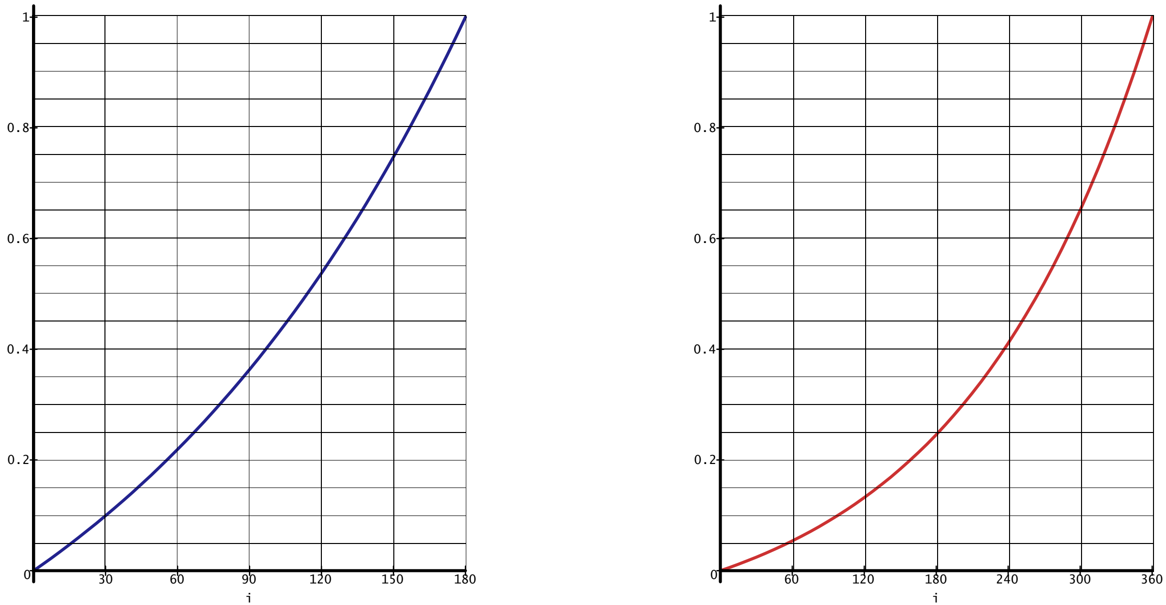

FIGURE (1.9.7) EQUITY AS A FRACTION OF LOAN BALANCE VERSUS PAYMENTS MADE

Problem (1.9.8) Check your answers to Problem (1.9.6) and Problem (1.9.5) against the graphs.

Challenge (1.9.9) Is it really OK to ignore the loan balance? More precisely, the way I drew the graph above implicitly claims that the equity accrued as a fraction of initial balance only depends on the rate r and term T of the loan and the number i of payments made, not on the initial balance B.
a) First, check this by computing a few such fractions for a loan of \$200,000 at 7.5% interest and comparing with known answers for a loan of \$100,000. (You can save half the work if you use as examples a loan with a 30 year term after $i = 60$ payments are made and one with a 15 year term after $i = 120$ payments are made.)
b) Next, use Present Amortization Formula (1.8.1) to show that the fractions are the same for any two balances B and B' if r, T and i are the same.
c) Finally, explain why the principle of Equality of dollars (1.1.4) implies that the fractions in the graph are independent of the initial loan balance.

Let me make a point about these checks. The check in a) looks easiest — it's just a standard concrete balance problem — but is the most work. The check in b) is a bit scary — you've got to write down the quotient of two medium sized formulas — but when you do the B's just cancel. But the conceptual argument in c) is the easiest of all. Moral: an ounce of inspiration is worth a pound of perspiration.

What conclusions can we draw from these graphs? First, staying 10 years won't help the family with the 30 year mortgage much. Their equity will be just over a tenth of the value of their mortgage after 10 years — actually about .13 — so on a balance of \$100,000 they will only have accumulated about \$13,000 in equity. They will need to keep their house for over 20 years before they build up equity of \$50,000.

Problem (1.9.10) Use the Method for finding balance and equity (1.9.3) to check that after 22 years this family still owes a touch more than \$50,000 on their mortgage.

Not many people stay in the same house this long today. So, we conclude that if you need to take out a 30 year mortgage to finance a home purchase, you should probably *not* expect to build up much equity before moving. Of course, you *might* stay put for 30 years and build up $100,000 in equity but while this was common thirty or forty years ago, you'd be a rare exception if you did this today.

A second conclusion we can draw is that even if you can afford to take out a 15 year mortgage, you need to stay for most of the term to build substantial equity. After 5 years, your equity fraction will be about .22 and after 10 (as we've seen) it's about .53. In other words, you create roughly 4 times as much equity in the last 10 years as the first 5 and almost as much in the last 5 years as you do in the first 10.

Don't get me wrong. I have nothing against buying a home per se. It *is* sometimes cheaper than buy than to rent even when you include closing costs, insurance, maintenance, taxes and so on and even if you are not planning to stay in the place you are buying "forever". One important factor that I haven't mentioned is that mortgage interest is often deductible from your income for federal tax purposes. But, if you are concerned about money, this is the calculation you should be making: which is cheaper *all costs included*. Buying does not turn out to be cheaper very often. It's true that there are intangible values to owning your home — no landlord, you can fix it up any way you want, ... — the whole "It's our very own place" factor; or, the kind of home you want may be impossible to rent. But these work both ways: if the roof starts leaking, *you* have to fix it; if you want to move, you have to find a buyer. You have to decide how much these intangibles matter to you. My advice is simply to have a clear idea of relative costs to balance these other factors against. And, unless you can afford a mortgage with a short term and expect stay put for most of that term, don't make "building equity" one of those intangibles.

PROJECT (1.9.11) This project involves a lot of calculation so you might want to work with some friends and divide up the grunt work.

a) Another common term for mortgages is 20 years. Compute the equity built up in a $100,000 mortgage at 7.5% at two year intervals. Plot these on a graph like those above. How do the three graphs compare?
b) Interest rates also affect how equity grows. Make plots like those above assuming terms of 15 and 30 years but changing the interest rate to 6% and to 9%. Discuss how higher or lower rates seem to affect the growth of equity.

Major Project (1.9.12) Which is cheaper in your area, renting or owning? Imagine that you have an income of $45,000 a year and that mortgage interest rates are 7.5%.
a) Most banks have a rule that your mortgage payment can not be greater than 28% of your income. How much can you afford to spend on a house if you do not have any money for a down payment? (In real life, you are usually required to include costs like taxes in the 28% figure. Moreover, if you do not have a down payment of 20% of the purchase price, this percentage would be reduced somewhat and the interest rate raised. We'll ignore such facts.)

b) Write down specifications for a home you'd "like" to buy (number of beds and baths, amenities etc.). Then find current ads for several homes of this type in your area which are for rent and for sale. Compute the *total* cost renting or of owning each property for 3 years, for 5 years and for 10 years. Be sure to include costs like closing costs, maintenance, insurance, utilities and taxes in your calculations in addition to rent or mortgage costs. In budgeting like this, the federal mortgage interest deduction is significant so be sure to include it in your calculation. Remember you must stay within the mortgage budget from a). You may need to research some of these costs with a realtor and or a bank in your area.

c) Wait, I can hear some of you saying "Real estate is booming". It's true. I have so far completely ignored the possibility that the value of your home will *increase* while you own it. When you sell, this increased value is like "extra" equity you have accumulated. Of course, prices may drop and you may wind up with less or even "negative" equity. Repeat the calculation of part a) (including the same costs) but incorporating changing rents and real estate values in your area. To do this you will need to find data for homes in your area which are for sale today and were for rent or sale 3, 5 and 10 years ago so you can calculate these changes. To make it easier to find examples, we'll remove the ceiling on your mortgage budget. To be fully fair to renting, you should also include a down payment. For simplicity, suppose that the buyer was required to make a 20% down-payment on each house when you purchased it while the renter had this money to invest in the stock market over the same period. Use the S&P 500 index to measure the renter's yield. Who comes out ahead now over each of the three terms?

"Money talks, nobody walks"

The title of this subsection is an old used car dealer's slogan. "I'll do anything to sell this car". If you think the salesman wants the money in your pocket, you're wrong. You don't have to have *any* money in your pocket now because "I'll finance your car with no money down". The money the dealer wants is the money that is *going* to be in your pocket in the next few years. If he can get you to take out a loan to pay for the car, he'll make a sale and that is what he really wants. Of course, there is a substantial risk. You have his car and he has only your word that you'll make all the payments. What if you default? That's easy: buyers are charged such a high interest rate that the even after figuring in the losses involved in repossessing from those who do not pay up the loans offer an excellent yield. In fact, in most cases, the car dealer will immediately sell your loan to a collection agency. He receives less than the face value of the loan but has that loquacious money in his hand. In other words, he has achieved his goal and sold a car — that's his job. The collection agency has the difference between the face value and the sale price of the loan as an extra margin to compensate it for delinquent borrowers as it collects the payments —- that's its job. You have the car — that was your goal. It seems like everyone comes out ahead. What's the catch?

The catch comes over the next few years when you make also those easy monthly payments. (Ever noticed how *all* monthly payments are easy?) In this section, I'd like to compare what the costs of such a loan are to those of saving to buy the car. The principles we'll discover apply to many other kinds of consumer credit loans: for furniture, electronics, appliances and so on.

Example (1.9.13) Let's say the car you want has a sale price of $4,800. The salesman is likely to ask whether you can afford to pay $150 a month. "I think so", you say. "Fine", he answers, "We can arrange for you to make 48 easy monthly payments of just $149.93 a month". What interest rate are you being charged? If you are like most people, you have no idea. There are two ways to find out. The easy one is to look at the finance contract. Down in the small print somewhere you'll find the interest rate because it's a legal requirement to put it there: it's 21.5%. In the next section, The Hunt and peck method (1.10), we'll learn how to find find the interest rate given the balance, payment and term.

Now let's ask what you'd need to do to save for the car and pay cash. I claim that if you put away $90 a month for 48 months you'll have the same $4800. Before we check this, let's note the main point. If we use the loan payment as a reference, then this means that by saving the money to borrow the car instead of borrowing it, I can reduce my monthly payment by about 40%. If I use the savings deposit as a reference the difference is even more striking: I have to pay fully *two-thirds* again as much to borrow for the car as to save.

Another way to get a feel for the difference between saving and borrowing is to ask how long you'd have to make monthly deposits of $150 a month into a savings account to accrue $4800. I claim that it would take about 30 months. In other words, you have to make the same payments for an extra *year and a half* if you borrow the money. For short term investing of this type, safety is more important than yield. The last problem shows that even a very high yield shortens the term by only a couple of months. But to get a high yield you'll have to risk substantial losses and if these occur during such a short term they will delay reaching the goal considerably.

Before we go on, let's check the numbers in EXAMPLE (1.9.13).

PROBLEM (1.9.14) Show that the monthly payment on a loan with a 48 month term at interest of 21.5% is $149.93.

PROBLEM (1.9.15) Check the savings deposit amount given above. Of course, one ingredient is missing: the interest rate.
a) Show that at a rate of 5.3% the monthly deposit needed to accumulate $4800 in 48 months is $89.99.
b) Paradoxically, here the rate is not critical because the term of the amortization is so short. Show that at rates of 2% or 10% the monthly deposit you will need to make is within 10% of $90.
c) Compare the potential plusses and minuses of various kinds of investments for this kind of a short-term savings goal. What would you choose?

PROBLEM (1.9.16) a) Show that at a rate of 5.3% a monthly deposit of $150 for 30 months accumulates to $4800 to the nearest dollar.
b) Here the rate you get on your savings matters even less because the term of the amortization is even short. Show that at rates of 1% to 12%, the number of months you need to deposit $150 to accumulate $4800 changes by no more than 2.

Did I hear an objection? I hope so. I have slurred over two issues in EXAMPLE (1.9.13). Perhaps, *I* should have been a used car salesman. Let's discuss them in turn. The basic moral of the example is that you pay a very heavy penalty for borrowing to make a consumer goods purchase like that new car — one amortized over a term of a few years at a high interest rate — instead of saving for the purchase. The main source of this penalty is that high interest rate.

On the other hand, in PROBLEM (1.9.15) and PROBLEM (1.9.16) we checked that the amount and/or term for which you'd have to save are relatively little affected by the rate your savings earn. Why does the rate matter for the loan but not for the savings? It's *not* that there is some basic difference between savings and loans as the following problem lets you check.

PROBLEM (1.9.17) a) Check that the monthly payment needed to amortize a loan of \$4800 over 48 months at an interest rate of 6% is \$112.73.
b) Show that at rates of 2% or 10% the monthly deposit you will need to make is within 10% of this amount.

What these problems illustrate is that for both savings and loan amortizations, when the *term is short* and the *interest rate is fairly low* (say less that 10%), then the deposit D is relatively insensitive to the the interest rate. It's basically the final sum (if you're saving) or the initial balance (if you're borrowing) divided by the number T of payments. This is exactly correct if the interest rate is 0%. If you make T payments of $\$D$ and no interest is involved they amount to a sum of $\$S = T \cdot \$D$ or pay off a loan of $\$B = T \cdot \$D$. In EXAMPLE (1.9.13), this zero interest deposit D is just $\$100 = \dfrac{4800}{48}$. For each extra percent of interest paid, $\$D$ changes by a few dollars. PROBLEM (1.9.15) a) and PROBLEM (1.9.17) b) suggest that D changes about \$2 for each percent change in the nominal rate. When we save D goes down because we're being paid the interest: at 5.3%, D went down about \$10 to \$90. Likewise, D goes up when we borrow and pay the interest: it went up \$12 to \$112 when we were charged 6% interest. But in both cases we stay close to the zero-interest deposit of \$100.

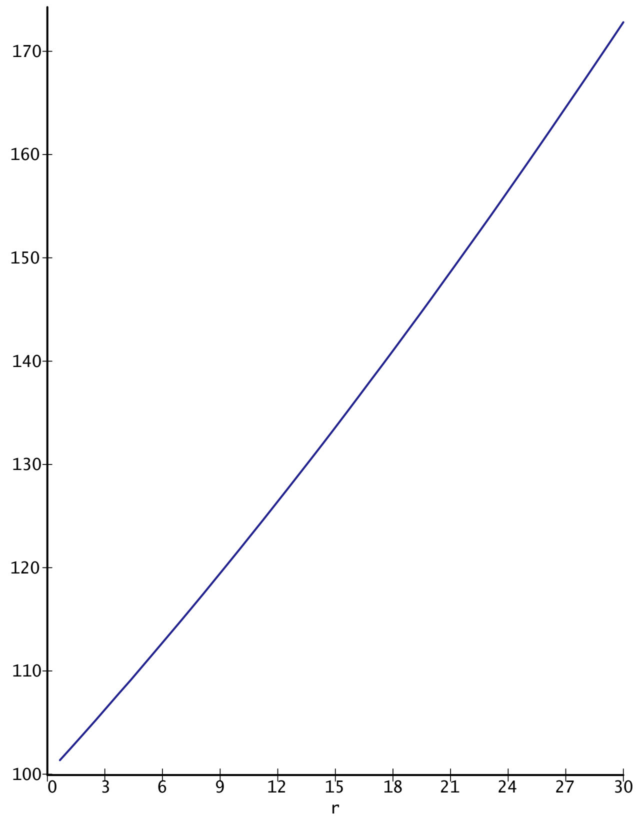

FIGURE (1.9.18) PAYMENT $\$D$ VERSUS NOMINAL RATE r IN EXAMPLE (1.9.13)

Pictorially what I am claiming is that if we plot D against the nominal rate r we'll see something that looks a lot like a straight line with slope 2. FIGURE (1.9.18) is the proof. The graph curves up a bit — the slope is closer to 3 than to 2 at the right side — but at a glance looks straight. Saying the rate is low means we're on the left side of the graph where the deposit is close to \$100.

But this is *not* true if *either* of the italicized assumptions fails to hold. We have already seen lots of examples of how sensitive to interest rates the final sums of long term amortizations can be: the answers to part b) of PROBLEM (1.6.20) are excellent ones. (By the way, the answers to part a) of this problem give another example of how over a short term this need not be so). In the example of the used car loan, the term is short but the interest rate on the loan is no longer "fairly low". It's big enough to make a substantial difference even over a short term. We have moved out to the right side of FIGURE (1.9.18) where the deposit is no longer close to \$100.

Consumer loan rates — rates on credit card balances, loans for consumer goods like furniture, home electronics etc, and in general loans made by people in order to sell you something generally have rates in the range 18%-24%. Such rates are high enough to substantially affect the deposit D need to amortize a given balance or sum even over short terms. This applies whether saving or borrowing. The following problem shows, for example, that if you could earn 20% on your savings you could reduce the \$90 deposit in PROBLEM (1.9.15) by more than 25%.

PROBLEM (1.9.19) Suppose you could find a savings account that earned 20% interest. Check that the monthly payment needed to amortize a loan of \$4800 over 48 months would be \$66.07 and that depositing \$150 a month into this account you would accrue slightly over \$4800 in 26 months. Explain the why the reduction in the deposit relative to that in PROBLEM (1.9.15) is more substantial than the drop in the term relative to that in PROBLEM (1.9.16).

So the first clarification that needs to be made about EXAMPLE (1.9.13) is that it's borrowing at high consumer loan interest rates which is pernicious not the act of borrowing per se. A good thing too, since we wouldn't want to have to save to save for 25 years — meanwhile paying rent — to buy a house. (Note, however, that there *are* countries in which this is more or less what you have to do to buy a house). The difference in a mortgage is the substantially lower rate as you can confirm in the following problem which asks what would happen if mortgages had rates like consumer loans.

Problem (1.9.20) What is the monthly payment on a mortgage with a balance of $100,000 and an interest rate of 20% if the term is
a) 30 years?
b) 15 years?

What's the second clarification? Suppose we ignore the high interest rate charged on consumer loans and imagine they are available at, say, 6%. Look at the answers to Problem (1.9.15) and Problem (1.9.14): for the same amount and term and for our hypothetical interest rate of 6%, the savings payment is about 20% less than the loan payment. Is this a second factor — besides the high rate on consumer loans — which argues for saving to buy rather than borrowing to buy? No!. The reason is a key fact which I ignored all through Example (1.9.13). You've probably spotted it long ago. The borrower has that car *now* and through the four years of the loan. The saver has to wait until the end of the 4 year period to buy the car. The use of the car during those intervening 4 years, like the use of money over a period of time has a value and the higher loan payment reflects the fact that the borrower acquires this use while the saver does not.

As a consequence, all my statements as to how much more it was costing the borrower than the saver are exaggerated: they all failed to take account of the fact that what the borrower is buying — a car today — is worth somewhat more than what the saver is buying — a car four years from now — and he should expect to pay somewhat more. My basic point, however, stands. The lion's share of premium that the borrower pays is attributable to the high interest rate charged and not to the greater value of what is purchased. Here's final striking way to view this difference. Suppose that both the borrower and the saver buy a $4800 car every 4 years. The borrower gets his first car today, pays off the loan at $150 a month over 4 years, and repeats this cycle. The saver takes the bus for 4 years and puts away $90 a month for a car and $60 a month for his retirement (both at 6% interest). He then pays cash for his car but continues to deposit $90 a month so he can pay cash for his next car in 4 years and to save $60 a month for his retirement. What are the plusses and minuses? Both are out $150 a month. For the first 4 years, the borrower has a car but the saver does not. Thereafter, they both drive the same car. Eight cars later the saver has a retirement account with over $90,000 in it. The borrower has nothing!

Problem (1.9.21) Show that after 36 years of making monthly payments of $60 a month into a retirement account which earns 6% interest, you'll have a sum of $$91,495.13.

Here are a few more problems to illustrate the principles in this discussion.

"No interest and no payments until June 1!" This, or something like it, is a come-on often used in selling electronics, furniture etc. In the next problem, we analyze how such offers work and what their effect is. Payments on these loans generally come at the start of each period but we'll pretend they come at the end to avoid having to introduce more formulas.

Problem (1.9.22) Let's suppose that what you want is to buy that big-screen TV for Christmas. Scanning the newspaper on December 1, you see an ad for that big screen TV you been hankering for, "This weekend only: $2599 or just 36 easy payments of $99 a month. Plus no interest and no payments until June 1!". In Problem (1.10.21), we'll see that the hidden interest rate in this offer is 21.8%.
a) Check that this is the correct rate. Then check that at 6% interest, you'd have to put away about $66 a month

for the same 36 month period.
b) What does "No interest and no payments until June 1!" mean. If you think it means that you can make the first of your 36 $99 payments on June 1, go to the back of the class. If you look at the small print in the loan contract, you'll see three things. First, you are not obliged to pay anything until June 1. Second, you may, if you can and wish, pay off the *entire* $2599 before June 1 and you will not owe the store anything. But you're not obliged to do anything but make that first $99 payment. So far so good. Third —and here's the kicker — on July 1, the store will add 5 months interest (from December 1 to May 1) on whatever part of the balance you have not paid off to that balance. Suppose that you're like most people and pay off nothing before June 1. Show that your outstanding balance on June 1 will be $2,843.81.
c) Show that it will take you not 36 but 41 easy monthly payments of $99 to pay off this debt.
d) How much would you have to save each month in an account which earned 3% interest to accumulate $2599 in 41 months?

Problem (1.9.23) You lose your job and to cover your expenses while looking for a new one use a credit card which charges a nominal rate of 17.65% on outstanding balances to pay most of your bills. By the time you find a new job get your financial life sufficiently organized to confront your outstanding balance, it has run up to $17,610. You cut up the card and go to work to pay this off.
a) Show that when you start the additional interest you owe each month on the debt is $259.01.
b) Show that if you pay off this debt at $500 a month it will take you about 50 months to amortize it, if you can only afford $400 a month it will take 71 months (that's almost six *years*) and if you can only afford $300 a month it will take 136 months (over 11 years).
c) What if you can only afford to pay off $250 a month?
d) If you can't afford to pay more than $300 a month, how much more debt could you assume before you'd never

be able to repay?