Economic Catastrophe Bonds: Inefficient Market or Inadequate Model?

HAITAO LI\textsuperscript{a} AND FENG ZHAO\textsuperscript{b}

ABSTRACT

Coval, Jurek, and Stafford (2009, CJS hereafter) claim that senior CDX tranches, which resemble economic catastrophe bonds, are overpriced relative to index options. We show that this result is due to their problematic calibration procedure and restrictive model assumptions. A simple correction of the calibration procedure reduces the overpricing of the most senior tranche by 80%. Moreover, an extremely parsimonious model with essentially no free parameters proposed by Hull and White (2004) can price CDX tranches and index options well. Therefore, it is difficult to conclude that CDX tranches are mispriced simply because CJS cannot price them.

JEL Classification: C11, E43, G11

Keywords: CDX index and tranches, copula model, economic catastrophe risk, fat tail.

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ABSTRACT

Coval, Jurek, and Stafford (2009, CJS hereafter) claim that senior CDX tranches, which resemble economic catastrophe bonds, are overpriced relative to index options. We show that this result is due to their problematic calibration procedure and restrictive model assumptions. A simple correction of the calibration procedure reduces the overpricing of the most senior tranche by 80%. Moreover, an extremely parsimonious model with essentially no free parameters proposed by Hull and White (2004) can price CDX tranches and index options well. Therefore, it is difficult to conclude that CDX tranches are mispriced simply because CJS cannot price them.

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**Keywords:** CDX index and tranches, copula model, economic catastrophe risk, fat tail.
The influential paper of Coval, Jurek, and Stafford (2009) published in the *American Economic Review* investigates “the risk and pricing implications of structured finance activities.”¹ CJS argue that senior CDO tranches “resemble economic catastrophe bonds—bonds that default only under severe economic conditions.” Classical asset pricing theory suggests that “securities that fail to deliver their promised payments in the ‘worst’ economic states will have low values, because these are precisely the states where a dollar is most valuable.” However, CJS argue that “investors in senior CDO tranches are grossly undercompensated for the highly systematic nature of the risks they bear” and should be able to “earn roughly four to five times risk compensation” with equivalent economic exposure in index options. CJS argue that “this discrepancy has much to do with the fact that credit rating agencies are willing to certify senior CDO tranches as ‘safe’ when, from an asset pricing perspective, they are quite the opposite.”

The CJS paper has been widely cited in the literature as definitive evidence of mispricing of CDOs. For example, Faltin-Traeger, Johnson, and Mayer (2010) state “Coval et al. (2009b) conclude that buyers of CDOs accepted much lower yields than buyers of economic catastrophe bonds with similar risks.” Mackenzie (2011) state that “Coval, Jurek, and Stafford (2009) argue that ... the prices of corporate CDO tranches were too high by the standards of asset-pricing theory, and the ratings-spreads convention ... seems to be the cause: it led market participants unwittingly to compare instruments with high systematic risk (senior CDO tranches) to instruments with similar default probabilities but lower systematic risk (corporate bonds). Their article is thus a beautiful demonstration of a convention shaping patterns of prices and creating what is (if modern asset-pricing theory is correct) a very large and very persistent inefficiency.”

The dramatic mispricing of senior CDO tranches documented in CJS is striking given that their empirical analysis mainly focuses on tranches based on the CDX.NA.IG index, one of the most liquid credit derivatives. As pointed out by Collin-Dufresne, Goldstein, and Yang (2010, CGY hereafter), “traders in the CDX market are typically thought of as being rather sophisticated. Thus, it would be surprising to find them accepting so much risk without fair compensation.” While it might be difficult to price certain subprime CDOs due to short historical data and opaque underlying collaterals,² the collaterals of CDX tranches are investment grade corporate bonds of 125 large companies, which are transparent and easy to understand. In fact, recent papers by

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¹For distinction, direct quotations from CJS (2009) and other related papers are in italics.
²See Hull and White (2010) on pricing of CDOs created from mortgages.
Longstaff and Rajan (2008) and others have shown that CDX index and tranches are consistently priced under reduced-form models, and conclude that these securities are reasonably efficiently priced.\(^3\)

While CJS interpret their results as evidence of mispricing of senior CDX tranches, the well-known joint test problem of market efficiency suggests that the CJS model could also be mis-specified. In this paper, we re-examine the pricing of CDX tranches using the same data under the same modeling framework of CJS. We show that the conclusion of CJS that senior CDX tranches are overpriced is due to their problematic calibration procedure and restrictive model assumptions. A simple correction of the calibration procedure can reduce the average overpricing of the senior (15-30) tranche, which most closely resembles economic catastrophe bonds among the five tranches that CJS consider, by 80% from 16.2 bps to 3.3 bps! Moreover, an extremely parsimonious model with essentially no free parameters proposed by Hull and White (2004) can price CDX tranches and index options well. We emphasize that we do not come up with the Hull and White model after a long and exhaustive “model mining” of the literature. Instead, the Hull and White model is like an “off-the-shelf” model that has been known in the literature for years and is just one of many alternative models that have been developed in recent years for CDO valuation. As a result, its excellent pricing performance makes it difficult to conclude that CDX tranches are mispriced simply because the CJS model cannot price them!

The rest of the paper is organized as follows. In Section I, we introduce the CJS model and relate it to the copula model of Li (2000). In Section II, we identify a problem with the calibration procedure of CJS, which introduces significant biases in CDX tranches prices. In Section III, we provide compelling evidence that the CJS model is so mis-specified that it cannot capture CDX tranches prices even in sample. We also show that the Hull and White model can price CDX tranches and index options well. Section IV concludes.

1 The CJS Model and Its Copula Representation

In the CJS model, the (log) asset and market returns under the physical measure \(\mathcal{P}\) are given by

\[
\ln \left( \frac{A_{i,t}}{A_{i,t}} \right) = \mu_{i,t}^\mathcal{P} r + \beta_{i,a} \sigma_m \sqrt{r} Z_m + \sigma_{i,e} \sqrt{r} Z_{i,e},
\]

\(^3\)For discussions of reduced-form approach to CDO pricing, see also Duffie and Garleanu (2001).
\[ \ln \left( \frac{M_T}{M_t} \right) = \mu_{m,t}^P \tau + \sigma_m \sqrt{\tau} Z_m, \]

where \( \tau = T - t \), \( \mu_{m,t}^P \) and \( \mu_{m,t}^P \) represent the expected asset and market return, respectively, \( \beta_i,a \) is the asset CAPM beta, \( \sigma_i,e \) is the idiosyncratic asset volatility, \( \sigma_m \) is the market volatility, and \( Z_m (Z_i,e) \) represents the systematic (idiosyncratic) factor under \( P \). CJS define the systematic factor as

\[ m_\tau = \ln \left( \frac{M_T}{M_t \exp ((r_f - \delta_m) \tau)} \right). \]

The implementation of the CJS model requires the risk-neutral distribution of the systematic factor \( m_\tau \) and estimates of \( \left\{ \ln \left( \frac{D_i}{A_{i,t}} \right) , \beta_i,a, \sigma_i,e \right\} \), where \( D_i \) represents the default boundary for firm \( i \).

While CJS develop their model under the structural approach of Merton (1974), their model also has a standard copula representation under the risk-neutral measure \( Q \). Given the total asset volatility \( \sigma_i,a = \sqrt{\beta_i,a \sigma_m^2 + \sigma_i,e^2} \),

\[ X_{i,\tau} (t) = \frac{\ln \left( \frac{A_{i,t}}{X_{i,t}} \right) - \mu_i^Q \tau}{\sigma_i,a \sqrt{\tau}} = \sqrt{\rho_i} Z_m + \sqrt{1 - \rho_i} Z_i,e, \]

where \( \mu_i^Q \) is the expected return of firm \( i \) and \( \bar{Z}_m \) represents the systematic factor under the risk-neutral measure \( Q \), and

\[ \rho_i = \frac{\beta_i^2 \sigma_m^2}{\beta_i,a \sigma_m^2 + \sigma_i,e^2} \]

measures the fraction of the total variance of asset return explained by the systematic factor (aka the variance ratio). Moreover, \( \rho_i \) is the copula correlation, since the correlation between \( X_i \) and \( X_j \) is \( \sqrt{\rho_i \rho_j} \). In their footnote 10, CJS (2009) acknowledge that “Since our model relies on option prices to characterize the distribution of the common factor under the pricing measure, it can also be thought of as an option-implied copula model.”

The copula representation of the CJS model provides an interesting economic interpretation for the copula model of Li (2000), which has been widely regarded as a pure statistical model without much economic content. More important, it makes it much easier to identify potential problems with the implementation procedure and assumptions of the CJS model. While CJS treat the systematic and idiosyncratic components of asset returns separately, one key input for accurate pricing of CDX tranches is the variance ratio \( \rho \). In the next section, we show that the calibration procedure of CJS introduces significant biases in \( \rho \) and consequently the spreads of senior CDX tranches.
Problematic Calibration Procedure

In this section, we show the way that CJS calibrate leverage, beta, and idiosyncratic volatility is problematic and introduces significant biases in CDX tranches prices. Specifically, assuming homogeneous firms, on each day, CJS estimate \( \{ \ln \left( \frac{P}{X_{t,i}} \right), \beta_i, \sigma_i \} \) by matching the CDX index spreads, the average equity beta (which equals 1), and the average pairwise equity correlation of the 125 firms between 2003 and 2007 (which equals 0.2).\(^4\)

Under the assumption of homogeneous firms, pairwise correlation equals the copula correlation or the variance ratio \( \rho \). In line 86 of the subroutine BondPricer.m that CJS posted on the Web site of the *American Economic Review*, we see the following relation is one of the equations that CJS use to back out \( \sigma_i(t) \) for each \( t \):

\[
0.2 = \frac{\beta^2 \sigma^2_{\text{ATM}}(t)}{\beta^2 \sigma^2_{\text{ATM}}(t) + \sigma^2_i(t)},
\]

where \( \sigma_{\text{ATM}}(t) \) is the implied volatility of five-year ATM S&P 500 index option. However, since the distribution of the systematic factor \( m_r \) backed out from index options is non-Gaussian, \( \sigma^2_{\text{ATM}}(t) \) is generally smaller than the true variance of \( m_r \), \( \sigma^2_m(t) \), which can be obtained by numerical integration of the distribution of \( m_r \).\(^5\) Since CJS use the actual distribution of \( m_r \) in pricing CDX tranches, the actual \( \rho \) used to price CDX tranches in the CJS model is much higher than 0.2.

In Panel A of Figure 1 we provide time series plots of \( \sigma_{\text{ATM}}(t) \) and \( \sigma_m(t) \), the latter is obtained through numerical integration of the probability density of \( m_r \) backed out by CJS from S&P 500 index options. We see clearly that \( \sigma_{\text{ATM}}(t) \) is smaller than \( \sigma_m(t) \) throughout the entire sample. In Panel B of Figure 1, we plot the actual \( \rho \) CJS use to price CDX tranches, which is calculated by using the values of \( \beta_i \) and \( \sigma^2_i(t) \) obtained by CJS but replacing \( \sigma^2_{\text{ATM}}(t) \) by \( \sigma^2_m(t) \) in (1). Even though CJS intend to keep \( \rho \) at 0.2, because of the wrong variance for the systematic factor in (1), the actual \( \rho \) used in their pricing exercise is much higher than 0.2.

It has been widely recognized in the literature that the copula correlation \( \rho \) is crucial for pricing CDX tranches. In Figure 2, we plot the relation between the spreads of both junior and senior tranches and \( \rho \), where the parameters of the systematic and the idiosyncratic factors are taken as averages of the CJS estimates. It is clear that the spreads of the equity tranche

\(^4\)We ignore explicit dependence on \( i \) in most of the notations due to the homogeneous firms assumption.
\(^5\)It has been widely recognized in the VIX literature that ATM implied volatility does not represent the actual forward-looking volatility of the index, see Carr and Wu (2009) and references therein.
relatively senior CDX tranches are extremely sensitive to $\rho$. As a result, CJS seriously exaggerate the magnitude of the pricing errors of all CDX tranches.

Table 1 reports the actual spreads of the CDX tranches, and spreads predicted by the original CJS model and the CJS model with $\rho$ fixed at 0.2.\footnote{We price CDX tranches using the same parameters as in CJS for the systematic and idiosyncratic factors but keep $\rho = 0.2$.} We see clearly that the original CJS model cannot price all CDX tranches. For the 15-30 senior tranche, the average model spread is 24.8 bps, about three times of the actual average spread, which is 8.6 bps.\footnote{Our calibration of the CJS model gives slightly different tranche spreads than those reported in CJS (2009), as we allow defaults to occur every quarter before maturity and use the actual number of bonds in the CDX index, which is 125. CJS (2009) assume defaults occur only at maturity and there are an infinite number of bonds in the index.} However, by fixing $\rho$ at 0.2, the average model spread is reduced to 11.9 bps, which is much closer to the actual average spread. For the 7-10 and 10-15 tranches, although the model spreads at $\rho = 0.2$ are still much higher than the actual spreads, they are about 25 bps smaller than that of the original CJS model, which are nontrivial given that the actual average spreads for the two tranches are 18 and 39 bps, respectively.

Therefore, a simple correction of the calibration procedure of CJS reduces the average overpricing of the senior 15-30 tranche, which most closely resemble economic catastrophe bonds among the five tranches, by 80% from 16.2 bps to 3.3 bps. This result raises serious doubts on the robustness of the CJS finding.

### 3 Restrictive Model Assumptions

The other two important ingredients of the CJS model are the systematic and idiosyncratic factors. While the idiosyncratic factor is Gaussian, the systematic factor is calibrated from five-year S&P 500 index options based on the following hyperbolic tangent function for the implied volatility

$$\sigma(x, \tau) = a + b \tanh(-c \ln x) \quad (a > b > 0),$$

where $x$ denotes moneyness and $\tau$ is time to maturity. The probability density of the systematic factor is obtained by the approach of Breeden and Litzenberger (1978) by assuming a Black and Scholes (1973) option pricing formula with the above implied volatility function. CJS claim that

\textit{within this class of implied volatility functions, we are unable to find a set of option prices that...}
can jointly price the CDX and CDX tranches...Based on this, we conclude that our model permits only two interpretations of the data: either CDO tranches are mispriced or both index options and corporate bonds are mispriced.”

However, an alternative interpretation of the result is that the distributional assumptions of the systematic and idiosyncratic factors of the CJS model are too restrictive to capture CDX tranches prices. The way CJS implement their model is like an out-of-sample analysis, where they use \( \{ \ln \left( \frac{D_i}{X_i} \right), \beta_a, \sigma_a \} \) calibrated from historical equity returns and systematic factor from index options to price CDX tranches. To test whether the volatility function in (2) is overly restrictive, we give the CJS model the maximum flexibility to fit tranches prices. Specifically, we conduct an in-sample analysis where we choose \( \rho \) and the parameters of the implied volatility function to match the spreads of all CDX tranches each day. Under the null hypothesis of a correctly specified model and efficient tranches market, the pricing errors from this in-sample fit should be small. Column 5 of Table 1 shows that the model still has relatively large pricing errors. For the 3-7, 7-10, and 10-15 tranches, the average model spreads are about twice of the actual average spreads.

However, to conclude that the CDX tranches market is inefficient, one has to show that no reasonable model can capture tranches prices. Next we show that an extremely simple and parsimonious model can price CDX tranches well. The model we consider is the well-known double-\( t \) model of Hull and White (2004), which allows both the systematic and idiosyncratic factors to follow Student-\( t \) distribution with degree of freedom of 4. Each day we use the double-\( t \) model with \( \rho = 0.2 \) to price of all the tranches and index options.\(^8\) We emphasize that all the parameters of the double-\( t \) model have been fixed before the model is applied to the data. Column 6 of Table 1 shows that the double-\( t \) model can fit most tranches spreads quite well. Figure 3 provides time series plots of actual spreads and spreads predicted by the CJS and the double-\( t \) model, where for the CJS model, the variance ratio \( \rho \) and the parameters of the systematic factor are taken from CJS (2009). While the CJS model exhibits huge pricing errors for all tranches during the entire sample, the double-\( t \) model can price these tranches quite well.

Table 2 shows that the percentage pricing errors under the double-\( t \) model for out-of-the-

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\(^8\)The pricing performance for tranches is similar when the double-\( t \) model is fitted to only tranches prices. While we could back out the degrees of freedom of the systematic and idiosyncratic factors from prices of tranches and index options each day, the improvements in pricing performance are not very dramatic, highlighting the robustness of our results.
money options (moneyness from 0.7 to 0.9) are less than 5% and for near- or at-the-money options (moneyness of 0.95 and 1) are about 10%. Given that percentage bid-ask spreads for long-term index options range between 5 to 10%, the double-t model can price index options reasonably well. To better capture the volatility skew in index options, we introduce negative skewness in the systematic factor of the double-t model. Specifically, we allow both the systematic and idiosyncratic factors to follow a skewed-t distribution of Hansen (1994) with degree of freedom of 4 and a skewness parameter of -0.5. Therefore, the skewed-t model has the same number of parameters as the double-t model. As shown in Column 7 of Table 1 and Column 5 of Table 2, the skewed-t model has an excellent fit for index options and tranches.

One potential issue with the CJS model is that the tails of their systematic and idiosyncratic factors are not fat enough to capture extreme market movements. The pioneering works of Mandelbrot (1963) and Fama (1965) have shown that stock return distributions exhibit power tails and rather than exponential tails of the commonly used Gaussian distribution. The importance of fat-tailed distributions for modeling financial time series and option pricing have been widely recognized in the literature (see, e.g., Bollerslev (1987), Carr and Wu (2003), and Wu (2006) among others). Though the systematic factor of the CJS model has a skewed distribution, it exhibits only exponential tails, which cannot generate as extreme outcomes as the power tails of the Student-t distribution. Fat-tailed distributions in both systematic and idiosyncratic factors are extremely important for pricing CDOs. In the standard Gaussian copula model, $\rho$ is the only variable that determines the spreads of all tranches. The high $\rho$ that is needed to price the most senior tranche leads to pricing errors for the more junior tranches. With a fat-tailed systematic factor to capture catastrophe risks, $\rho$ needs not to be as high to price senior tranches. In the meantime, at the level of $\rho$ that is needed to price more senior tranches, the spread of the equity tranche tends to be too low. A fat-tailed idiosyncratic factor increases the default risk of individual firms and the spread of the equity tranche.

Compared to the huge pricing errors of the CJS model, the excellent fit of both tranches and index options by the double-t model is amazing. This is especially so given that the double-t model shares the same structure as the CJS model (with only different distributional assumptions on

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9To obtain a skewed-t model, Hansen (1994) introduces an extra parameter $\lambda$ to the t distribution, which ranges from -1 to +1 and determines the skewness of the distribution. Negative skewness requires a $\lambda$ from -1 to 0. We ex ante pick the middle number, -0.5, in our model. Skewed-t distribution has been used in studies such as Christoffersen and Langlois (2011) to model equity returns.
the systematic and idiosyncratic factors) but with essentially no free parameters!\textsuperscript{10} CGY (2010) develop a structural model with stochastic volatility, stochastic dividend yield, and correlated jumps in market return, volatility and dividend yield, and show that the model can price index options and CDX tranches reasonably well. In contrast, the parsimonious double-$t$ model shares the same modeling framework of CJS, which makes the comparison more direct.

While we do not claim that the double-$t$ model is a perfect model for CDO pricing, our results raise serious concerns about the robustness of the CJS finding. If the CJS model cannot capture CDX tranches prices even given the maximum flexibility in fitting the data, and if the double-$t$ model, which slightly modifies the assumptions of the CJS model and has essentially no free parameters, can capture the tranches prices well, then it is difficult to conclude that the tranches are mispriced simply because the CJS model cannot price them!

4 Conclusion

In this paper, we revisit an important issue raised by the influential study of Coval, Jurek, and Stafford (2009). Based on a Merton structural model under the CAPM framework, CJS argue that senior CDX tranches are overpriced relative to S&P 500 index options because their model cannot reconcile the prices of these two markets. We carefully examine the limitations of the CJS model from several different perspectives and provide compelling evidence that (i) the calibration procedure of CJS is problematic and (ii) the assumptions of the CJS model are too restrictive to capture CDX tranches prices. On the other hand, we show that an extremely parsimonious model with essentially no free parameters proposed by Hull and White (2004) can price CDX tranches and index options well. Therefore, the conclusion of CJS that CDX tranches are mispriced seems to be premature, and there is no evidence that the tranches market is inefficient.

\textsuperscript{10}The extra number of parameters the CJS model has over the double-$t$ model is huge since each day the CJS model requires extra number of parameters to fit index options.
REFERENCES


Table 1: Actual and Model Tranches Spreads

We report the average daily spreads for the 3-7, 7-10, 10-15 and 15-30 tranches and the daily average upfront fee for the 0-3 tranche. We report the upfront fee for the 0-3 tranche because the conversion to spreads is model-specific. The spreads are in basis points and the upfront fee is in percentage. The CJS model is implemented using the original parameters in CJS (2009). The CJS with $\rho = 0.2$ is implemented using the original parameters of the systematic factor in CJS (2009) and setting the variance ratio at 0.2. The CJS in-sample is implemented by choosing the variance ratio and the parameters of the systematic factor to match tranches spreads. The double-t model is implemented with $\rho = 0.2$. The degree of freedom of both the systematic and idiosyncratic factors of the double-t equals 4. In the skewed-t model, the systematic and idiosyncratic factors follow the same skewed-t distribution of Hansen (1994) with degree of freedom of 4 and skewness parameter of -0.5. The skewed-t model is implemented with $\rho = 0.2$.

<table>
<thead>
<tr>
<th>Tranche</th>
<th>Actual (bps)</th>
<th>CJS (bps)</th>
<th>CJS $\rho=0.2$</th>
<th>CJS in-sample (bps)</th>
<th>Double-t $\rho=0.2$</th>
<th>Skewed-t $\rho=0.2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>15-30</td>
<td>8.6</td>
<td>24.8</td>
<td>11.9</td>
<td>6.9</td>
<td>10.1</td>
<td>10.5</td>
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<tr>
<td>10-15</td>
<td>18.2</td>
<td>86.1</td>
<td>60.0</td>
<td>37.8</td>
<td>24.9</td>
<td>22.3</td>
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<tr>
<td>7-10</td>
<td>39.1</td>
<td>151.9</td>
<td>126.7</td>
<td>91.9</td>
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<td>40.8</td>
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<tr>
<td>3-7</td>
<td>138.0</td>
<td>281.7</td>
<td>276.5</td>
<td>262.5</td>
<td>169.4</td>
<td>140.4</td>
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<tr>
<td>0-3 (%)</td>
<td>34.0</td>
<td>14.6</td>
<td>21.5</td>
<td>26.7</td>
<td>33.1</td>
<td>34.6</td>
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</tbody>
</table>
Table 2: Actual and Model Implied Volatilities

We report the average daily Black-Scholes implied volatilities for the index put options of moneyness 0.70 to 1.00. The CJS model is implemented using the original parameters calibrated in CJS (2009). The double-t model and the skewed-t model are implemented using the distributional assumption on the systematic factor and the parameter $\sigma_m$ is calibrated from OTM put option prices.

<table>
<thead>
<tr>
<th>Moneyness (Strike/Forward)</th>
<th>Actual</th>
<th>CJS</th>
<th>Double-t</th>
<th>Skewed-t</th>
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</thead>
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<tr>
<td>0.70</td>
<td>22.04%</td>
<td>22.06%</td>
<td>21.22%</td>
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<td>0.75</td>
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<tr>
<td>0.85</td>
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<td>20.02%</td>
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<tr>
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<td>19.38%</td>
<td>19.37%</td>
<td>20.32%</td>
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<tr>
<td>0.95</td>
<td>18.76%</td>
<td>18.74%</td>
<td>20.26%</td>
<td>18.94%</td>
</tr>
<tr>
<td>1.00</td>
<td>18.17%</td>
<td>18.13%</td>
<td>20.24%</td>
<td>18.44%</td>
</tr>
</tbody>
</table>
Figure 1: ATM Implied Volatility vs. True Systematic Volatility and Actual Variance Ratio $\rho$

We provide time series plots of ATM implied volatility of five-year S&P 500 index options and the true volatility of the systematic factor, which is obtained through numerical integration of the distribution of the systematic factor backed out from a cross section of index options. We also provide time series plots of the actual variance ratio used in CJS for pricing CDX tranches, which is computed using the true systematic volatility and the calibrated idiosyncratic volatility and asset beta in CJS (2009). The periodic drop in the variance ratio is due to the rollover of the CDX index at two fixed times each year.
We report the relation between CDX tranches spreads and the variance ratio $\rho$ under the CJS model. The parameters of the systematic factor, the CDX implied marginal default probability, and the risk-free zero curve are taken at sample averages. We calculate the spreads of four tranches at different levels of $\rho$. For the 0-3 equity tranche, the price is quoted in terms of up-front fee, whereas for all other tranches, the price is quoted in basis points.
Figure 3: Actual and Model Predicted Tranches Spreads.

We provide time series plots of actual spreads of four CDX tranches and spreads predicted by the CJS model and the double-\(t\) model of Hull and White (2004). For the CJS model, the variance ratio \(\rho\) and the parameters of the systematic factor are taken from CJS (2009). For the double-\(t\) model, the variance ratio \(\rho\) is set to 0.20. The degree of freedom of both the systematic and idiosyncratic factors in the double-\(t\) model equals 4.