THE DARK SIDE OF LIQUIDITY CREATION:

LEVERAGE AND SYSTEMIC RISK

by

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ABSTRACT:
This paper exposes a fundamental tension between the micro-prudential objective of subjecting banks to greater discipline through debt markets and the macro-prudential objective of containing systemic risk. We start with the familiar result that banks are illiquid due to the inability of bankers to credibly pre-commit to asset choices. Bank debt can reduce this illiquidity by disciplining bankers with the threat of premature liquidations. These liquidations are ex post inefficient in the absence of systematic shocks to asset values. To reduce some of this inefficiency, the discipline of equity can be used by incenting the manager through equity-based compensation. However, this reduces the bank’s liquidity, so some debt is socially optimal in bank capital structures. When we introduce systematic shocks to asset values, another aspect of bank leverage emerges—the liquidation of a bank’s assets leads creditors of other banks to update their priors on these common shock affecting asset values. This can give rise to contagion and liquidations throughout the system that can be either efficient or inefficient. Thus, individual-bank liquidity creation induced by the disciplining role of bank debt has the benefit of generating more information about common asset-value shocks, but it also comes at the cost of greater systemic risk. This increase in systemic risk is not fully internalized by banks in choosing privately optimal levels of leverage. We then consider two different information regimes for lender of last resort (LOLR) interventions. In the first regime, the LOLR has no information about the idiosyncratic conditions of banks or the systematic asset value shock. In this case, the LOLR can diminish the incidence of contagion and thereby reduce systemic risk by agreeing to bail out liquidation-threatened banks, but this carries the misfortune of eliminating creditor incentives to intervene in banks and liquidate efficiently. The analysis points to the desirability of ex-ante capital requirements as an alternative to ex-post LOLR interventions. In the second regime, we permit the LOLR to learn about the systematic shock and show that now contagion can be eliminated without affecting market discipline by adopting a selective intervention policy.

JEL: G21, G28, G32, G35, G38

KEYWORDS: micro-prudential regulation, macro-prudential regulation, market discipline, contagion, lender of last resort, bailout, capital requirements
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“Any observed statistical regularity will tend to collapse once pressure is placed upon it for control purposes”.

Goodhart (1975)

I. INTRODUCTION

In ensuring that the risk of the financial system as a whole stays at “prudent” levels, regulators are tasked to meet two forms of regulatory challenges. One is microprudential regulation, which needs to ensure that risk-taking at the individual bank level is not excessive. The other is macroprudential regulation, which seeks to contain the systemic risk that banks may be excessively exposed to collective failure. To date, these two forms of regulation have been typically dealt with in isolation of each other, especially in policy debates. Microprudential regulation aims to contain the distorted incentives of banks to make choices that maximize the value of bank shareholders’ risk-shifting (or asset-substitution) options, especially in the presence of regulatory put options like deposit insurance. Macro-prudential regulation, on the other hand, focuses on containing the risk of events like systemic capital and liquidity shortages, manifesting as fire sales and the freezing up of asset markets. Macro-prudential regulation also examines ways in which regulatory interventions like bank bailouts and lender-of-last-resort (LOLR) facilities can prevent (or engender) such occurrences and contain (or aggravate) their adverse impact. But since both forms of regulation ultimately seek to enhance financial system stability, a natural question that arises is: what are the microfoundations that possibly link these two forms of regulation? In this paper, we show that not only microprudential and macroprudential regulation affect each other, but that in fact there is a fundamental tension between the two.

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1 There is a long history of academic research on micro-prudential regulation. Merton (1977) aptly recognized the isomorphic correspondence between deposit insurance and common stock put options. An important implication was that, given deposit insurance, a bank has an economic incentive to invest in riskier assets and choose relatively low amounts of capital in its capital structure. This means regulatory monitoring of individual banks is necessary to control excessive risk taking designed to exploit deposit insurance.
Let us explain. Calomiris and Kahn (1991) and Diamond and Rajan (2001) provide a theoretical rationale for financial institutions to be highly levered – leverage increases market discipline and thereby enhances bank loan quality and/or liquidity creation.² This notion is also codified in bank regulation with market discipline being one of the three pillars of Basel II (the other two being regulatory monitoring and capital requirements). This argument about the market discipline of debt is concerned primarily with the attenuation of bank-specific risks.

The recent financial crisis has caused many to assign culpability to the leverage choices of individual financial institutions. Specifically, they argue that very high financial leverage, especially short-term leverage, induced banks to engage in illiquid and risky lending as well as securities activities that resulted in the widespread failures of these institutions (see e.g., Acharya, Schnabl and Suarez (2009), Adrian and Shin (2010), Diamond and Rajan (2009), Goel, Song and Thakor (2011), Mian and Sufi (2010), and Shleifer and Vishny (2010)). There appears to be an emerging acceptance of the fact that increases in leverage seem to increase the systemic risk, or the collective fragility, of financial institutions. Financial crises are typically associated with a few highly-levered banks, initially suffering portfolio shocks that engender capital or liquidity shortages for those banks, with the malaise quickly ensnaring other banks as the crisis deepens.

As a result, bank-specific and systemic risks, and in turn, micro-prudential and macro-prudential regulation, become difficult to separate. In particular, there emerges a somewhat schizophrenic view of the role of leverage. On the one hand, higher leverage means better asset-choices by bank managers and more liquidity when banks are viewed individually. On the other hand, higher leverage also means that the system is more fragile. Faced with circumstances of possible systemic failure, regulatory

² Calomiris and Kahn (1991) were the first to formally argue that monitoring by uninsured depositors can result in a bank manager who is making imprudent asset choices being exposed to the threat of a bank run, and that this can induce the manager to shy away from such asset choices. Diamond and Rajan (2001) note that banks invest in assets that are inherently illiquid due to the inability of bank managers to credibly pre-commit to certain actions, and that the threat of a run by uninsured creditors can make these pre-commitments credible, thereby improving liquidity creation by banks. Acharya and Viswanathan (2011) develop this point in a model where financial intermediaries can switch to riskier assets after borrowing and short-term debt with strong control rights ensures ex-ante liquidity by containing this agency problem.
interventions can play a role in the *ex post* reduction of fragility. However, it is also precisely in these circumstances that the discipline of leverage on *ex ante* asset choices is compromised and the lines between microprudential and macroprudential regulation begin to become blurred.

The underlying linkage between leverage, *ex ante* liquidity creation, and *ex post* systemic risk raise some fundamental questions that we address in this paper.

First, what is the role of bank leverage *vis a vis* equity capital in affecting the bank’s *ex ante* liquidity and portfolio risk? Second, how does maximizing *individual* bank liquidity (a micro-prudential regulation concern) affect *systemic risk* (a macro-prudential regulation concern)? Third, is there a rationale for a lender of last resort (LOLR) intervention, and if yes, under what circumstances? Fourth, how does the LOLR affect bank leverage, and what are the implications of this for microprudential regulation? That is, when does the LOLR affect (interfere) with the market discipline role of leverage and what are its (perhaps unintended) consequences?

To address these questions, we develop a model of an uninsured bank whose manager has asset-choice flexibility. The bank is *a priori* illiquid because the manager cannot credibly pre-commit to the right asset choices given his personal preference for a private-benefit project. The bank’s *ex ante* liquidity is measured by the financing it can raise by issuing claims against its terminal cash flows. This financing can be any mix of debt and equity. We permit *both* debt and equity to discipline the bank manager to create *ex ante* liquidity, but this discipline is different depending upon whether it is imposed by debt or equity. Debt disciplines the bank manager by the credible threat that there will be liquidation in some interim states, conditional on interim cash-flow realizations. Equity disciplines the bank manager by providing compensation-based incentives to the manager to select the efficient project. However, since the incentives provided by equity involve payments from *ex post* cash flows, they reduce the *ex ante* liquidity of the bank relative to raising debt. In other words, debt enables the bank to discipline the manager without sacrificing cash flows that can be pledged to financiers. Offsetting this *ex ante* advantage is the fact that leverage leads to liquidation of the bank in some states, and this liquidation can be *ex-post* inefficient. The bank’s privately optimal capital structure is determined in the model by the
tradeoff between the \textit{ex-ante} efficiency of leverage relative to equity in the provision of incentives to bankers and the expected ex post cost of inefficient liquidations induced by leverage.

Bank asset portfolios are then allowed to suffer systematic shocks to value that are observed by some of each bank’s creditors but not commonly observed by creditors across banks. This means that the (interim) liquidation decision made by the creditors of a bank can be due to either bank-specific information or information about the systematic shock. Since not all creditors of a bank receive information about the systematic shock, but they can observe the liquidation decisions of other creditors, they learn from each other’s decisions and update their beliefs about the systematic shock. Their learning is noisy, however, because of the commingling of information about idiosyncratic and systematic risks in any bank’s observed liquidation. This can give rise to contagion effects as those creditors of a bank that possess \textit{no} adverse idiosyncratic or systematic risk information about the bank, may choose nonetheless to liquidate their bank at the interim date based solely on observing the liquidations of other banks. Note that with the systematic asset-value shock, liquidations are \textit{not} always ex-post inefficient since they are sometimes in response to creditors observing a negative shock to asset value that falls below liquidation value due to the shock. However, there are instances when contagion can lead to ex-post inefficient liquidations. Thus, one dark side of leverage-based liquidity creation is the attendant systemic risk (when contagious liquidations are inefficient), and the higher the leverage of banks, the greater the systemic risk.

We solve for the bank’s privately-optimal capital structure in the presence of the systematic asset-value shock. Assuming that the regulator’s objective is to maximize the value of the entire banking industry,\footnote{In our model, this objective is equivalent to maximizing the banking industry’s aggregate liquidity.} we derive conditions under which the regulator’s optimal level of leverage for a bank is above the bank’s private optimum and the conditions under which it is below. A divergence between the regulatory and private optima arises because, in choosing its own capital structure, an individual bank internalizes neither the valuable information about the systematic shock conveyed to other banks by its own leverage and creditor-led liquidation (a positive externality) nor the higher likelihood that another bank may be inefficiently liquidated when it is liquidated (a negative externality). We establish
conditions under which the privately-optimal bank leverage will be too high relative to the regulatory optimum. One of these conditions is that the probability of the systematic shock is low enough. Since asset values will be higher when the likelihood of an asset impairment shock is lower, this suggests that the regulator may wish to impose ex ante (countercyclical) capital requirements on banks that are binding during high-asset-valuation periods.

Faced with the prospect of contagion arising from a bank’s liquidation, a lender of last resort (LOLR) can step in with a liquidity infusion that effectively bails out the bank, prevents inefficient liquidations (and unfortunately, efficient liquidations as well), and forestalls a contagion in the form of a system-wide liquidation of banks. We consider the unintended consequences of such LOLR under two information regimes. First, we assume that the LOLR can observe neither the idiosyncratic interim financial condition of the bank nor the systematic asset-value-impairment shock, and adopts an unconditional bailout policy. We show in this case that the presence of the LOLR can destroy all ex-ante market discipline of debt as creditors, who anticipate ex-post bailouts, and have no incentives to engage in privately-costly monitoring. This, in turn, causes a complete evaporation of the asset-value information generated by creditor liquidations. Somewhat paradoxically, capital structures of banks become irrelevant. However, since all market discipline of leverage is lost, the ex-ante liquidity of banks declines relative to that available in the absence of the LOLR. In other words, the LOLR eliminates ex-post bank liquidations and reduces system-wide fragility; the price of this ex-post liquidity created by LOLR shows up as lower *ex-ante* liquidity and the loss of valuable information about asset values, creating a tension between the micro-prudential and macro-prudential regulation of banks.

In one sense, our analysis highlights Goodhart’s (1975) Law in that the market discipline of debt collapses once leverage-based liquidation risk is relied upon by regulators to undertake LOLR interventions. This would – at least to some extent – not be the case if regulatory interventions were based on regulatory intelligence about bank solvency over and above market signals, rather than just based on market signals (such as creditor-led liquidations). This suggests that the LOLR may be more effective if accompanied by information generation by central banks or bank regulators that is
independent of the information generated by bank creditors.\textsuperscript{4} Our analysis can also be viewed as an illustration of the Lucas Critique (1976) in that the macroprudential effects of the LOLR cannot be predicted without accounting for how individual institutions and investors will change their behavior (monitoring by creditors and recapitalization by bank owners, in our model) in response to the change in policy resulting from the introduction of the LOLR.

We then examine a second information regime in which the LOLR continues to be unable to observe any bank’s interim (idiosyncratic) financial condition (which its creditors can observe), but can observe whether the systematic asset-value impairment shock has occurred. In this case, a selective bailout policy, wherein the LOLR bails out banks only if asset values are systematically impaired, eliminates liquidation contagion but tolerates inefficient liquidations triggered by idiosyncratic, bank-specific information possessed by creditors.

The rest of the paper is organized as follows. Section II provides a brief summary of related literature. Section III develops the model. Section IV contains the analysis of the basic model and shows how leverage helps create both liquidity and systemic risk. Section V examines the role of the LOLR in controlling or aggravating systemic risk. Section VI concludes. All proofs are in the Appendix.

II. RELATED LITERATURE

Our paper is related to many strands of the literature. On the topic of micro-prudential regulation of banks, the role of leverage in imposing market discipline on banks has been recognized in numerous papers, as mentioned earlier. See, for example, Calomiris and Kahn (1991), Dewatripont and Tirole (1994)\textsuperscript{5}, Diamond and Rajan (2001), and Acharya and Viswanathan (2011). Like these papers, we also show how bank leverage, through its disciplining role, enhances bank liquidity. However, in contrast to these papers, we also allow bank equity to discipline the manager, albeit through a different channel,

\textsuperscript{4} An important exception is if the regulators require bank creditors to be segregated by priority and rely on information signals generated by non-contagious or run-prone liabilities of banks, as analyzed by Hart and Zingales (2009).

\textsuperscript{5} Dewatripont and Tirole (1994) also discuss how shareholders can affect managerial incentives in banks.
namely compensation. This allows us to examine the tradeoff between disciplining the bank through leverage and disciplining it through equity.

In this respect, the manner in which equity discipline works in our model is different from the way it works in various other papers where high equity capital deters asset-substitution moral hazard (see, for example, Bhattacharya, Boot and Thakor (1998) for a review of this literature). Acharya, Mehran and Thakor (2011) have recently pointed out that this role of equity in deterring asset-substitution incentives also produces a tension between having leverage and equity in a bank.

There are also various papers on systemic risk in banking that are related to our work. One strand of research focuses on effects of risk-sharing on systemic risk. Shaffer (1994) and Winton (1999) illustrated the point that “pooling (diversification) elevates joint failure risk”. Wagner (2010) shows that while diversification reduces the risk of an individual bank, it increases systemic risk. Another strand of research has highlighted that the presence of the LOLR can make more likely the very state of correlated capital and liquidity shortages that the LOLR is trying to avoid ex post (see e.g., Acharya and Yorulmazer (2007), Acharya (2009), Farhi and Tirole (2009), Acharya, Mehran and Thakor (2011), and Kane (2010)). In contrast to these papers, we show that systemic risk can arise – even in the absence of regulatory safety nets – due to contagious runs upon liquidation of individual banks. Our key point, however, is that the safety nets designed to address such contagion can destroy the very reason for banks to have leverage in the first place. What is more, they can in fact delay the recognition of early warnings about crises and lead to delayed but deeper crises. Finally, Diamond and Rajan (2011) also highlight that ex post forbearance reduces ex-ante liquidity creation by banks and recommend that the central bank adopt a policy that raises interest rates in good times to counter the effect of forbearance.

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6 Brunnermeier and Sannikov (2010) also show that risk sharing by individual institutions within the financial sector can amplify systemic risk.

7 Contagion can also arise due to interconnectedness rather than systematic risk exposures. Caballero and Simsek (2010), for instance, argue that increased complexity due to greater interconnectedness among individual banks can generate endogenous risk and crises. See also Allen, Babus and Carletti (forthcoming). Moreover, financial innovation incentives can also generate systemic risk. See Thakor (2012).
III: THE MODEL

This section develops the basic model and the contracting opportunities. Consider an economy in which all agents are risk-neutral and the riskless rate is zero. There are three dates $t = 0, 1, 2$. There are banks run by managers that choose loan portfolios (or investments) at $t=0$ that generate payoffs at $t = 1$ and $t = 2$. There are two types of loans (or projects), both of which require liquidity (investment) $I$ at $t = 0$. If this liquidity need is met at $t = 0$, then the loan generates a random cash flow $x$ at $t = 1$, with density function $g(x)$ and cumulative distribution $G(x)$. The support of $g$ is $[0, x_{\text{max}}]$. Each bank is initially owned by shareholders who have no liquidity of their own, so they seek to sell debt and equity claims against the bank in a competitive capital market as to maximize the total value of the bank and hence the revenue raised. Out of this revenue, the manager’s initial wage is paid and the chosen loan portfolio (“loan” henceforth) is financed. The rest is consumed by the initial owners.

A. Types of Loans

The two types of mutually-exclusive loans available to the bank are the “good” loan and the “private-benefit” loan.\(^8\) The good loan produces a cash flow at $t = 2$ of $Hx$ with probability $q \in (0, 1)$, and zero with probability $1 - q$. The private-benefit loan produces a cash flow at $t=2$ of $Hx$ with probability $p \in (0, q)$, and zero with probability $1 - p$ at $t = 2$. Both loans have the same density function of the date-1 cash flow, $x$, accruing to the bank. The bank’s manager gets a private benefit of $B$ per unit of $x$, i.e., a total benefit of $Bx$ at $t = 2$. We assume that the private-benefit loan is socially inefficient relative to the good loan:

$$qH > [pH + B].$$  

(1)

Moreover, the good loan is socially efficient:

$$qHE(x) > I,$$

(2)

\(^8\) The model could also potentially be written in terms of a risk-shifting or asset-substitution moral hazard, instead of a private benefit project, as in Acharya and Viswanathan (2011) or Acharya, Mehran and Thakor (2011).
where \( E(x) \) is the expected value of \( x \). The good loan and the private-benefit loan are mutually exclusive. Moreover, the bank manager makes an initial loan choice at \( t = 0 \) but can costlessly switch projects at \( t = 1 \).

We assume that all cash flows are pledgeable but we will show that they will not always be pledged in equilibrium due to the provision of incentives to bankers. The bank can raise the required initial liquidity via debt or equity whose features we describe next. Moreover, the incumbent bank manager possesses unique human capital to manage either of the two loans. Transferring the management of the loan to another manager substantially reduces loan value.

**B. Debt and Equity Contracts and Assets Portfolio Liquidity**

If the bank raises debt financing at \( t = 0 \), it is assumed that the debt contract contains a covenant whose violation at \( t = 1 \) permits the creditors to take control and demand full repayment at \( t = 1 \). Asking for full repayment could result in the bank being liquidated at \( t = 1 \) if its interim cash flow is insufficient to make the repayment. Such “accelerated repayment” clauses are standard in debt contracts. If the bank is not liquidated at \( t = 1 \), then it will continue until \( t = 2 \). Another way to think about this is to view the debt contract as having a one-period maturity, so that the creditors decide at \( t = 1 \) whether to renew funding for an additional period after the debt matures or simply collect whatever the bank can repay at \( t = 1 \) and deny renewal of funding.

The face value of debt (promised repayment to creditors) is \( F \). The date-1 cash flow is pledged to the creditors up to this face value of \( F \), so the remaining repayment to them at date 2 (in case the bank is not liquidated at date 1) is \( \max(F-x, 0) \). It is assumed that the liquidation value of the bank at \( t=1 \) is dependent on the realized date-1 cash flow \( x \). This liquidation value is \( Lx \). We assume that

\[
q_H > L > p_H + B. \tag{3}
\]

That is, liquidation is better than the private-benefit loan under the manager, but worse than the good loan. One way to interpret the liquidation value is to view it as the value of the loan if its management is transferred to another manager.
If equity is used instead of debt, it is assumed that shareholders have no control rights at $t = 1$ other than over the cash flow $x$. That is, shareholders cannot “withdraw” their equity investment in the firm at $t = 1$ by liquidating the bank. We just take this feature of the equity contract relative to debt as a given. That is, our purpose is not to endogenously derive debt and equity as (constrained) efficient contracts in a security-design setting. However, if the shareholders wish, they can provide incentives for the manager to choose the value-maximizing loan by giving him a suitably chosen share $\theta \in (0,1)$ of the shareholders’ payoffs (that is, residual payoffs at date-1 and date-2 after payments to creditors). It is assumed that the manager values this ownership, which represents a claim of future cash flows, at a discount factor $\rho \in (0,1)$ of the value assigned by the shareholders. The valuation divergence between the shareholders and the manager could arise from fundamental disagreement (e.g. Boot, Gopalan and Thakor (2006), and Van den Steen (2010) or simply risk aversion or managerial myopia. It reflects the fact that the manager values his future equity-based compensation less than his up-front fixed wage.

We assume that a monitoring technology is available to creditors that would enable them to discover the bank’s loan choice at a cost $c > 0$. If they dislike this loan choice, they can ask the bank’s manager to change it. If the manager refuses, and financiers are creditors, they can deny renewal of funding at $t = 1$ for the second period. This lack of funding renewal may force the bank to liquidate in order to meet its repayment obligation. Faced with this prospect, the bank manager may agree to switch loan choice. It follows immediately that the bank’s creditors will refuse to renew funding at $t = 1$ if they discover the manager has not chosen the loan they desire. If the bank can fully pay off creditors from its cash flow at $t = 1$ and still continue, it will choose to do so. But if this cash flow is insufficient, then the creditors’ threat of liquidation will have bite and a loan change will occur.
In contrast, if shareholders were to expend \( c \) in monitoring effort and discover a loan choice they did not like, they have no credible threat that they can make to force a change. This is because equity has no finite maturity and shareholders cannot ask for their investment to be returned at \( t = 1 \).\(^9\)

This disciplining role of debt (but not equity) via a withdrawal threat is familiar from the previous work of Calomiris and Kahn (1991), and Diamond and Rajan (2001). The very nature of the debt contract facilitates monitoring-based market discipline that is not possible with equity (see also Hart and Moore (1995)). The fact that creditors can deny renewal of funding at \( t = 1 \)—through either a covenant trigger that leads to a demand for accelerated repayment on two-period debt or a denial of renewal of one-period debt—is also similar to repo market funding drying up for financial institutions during the recent crises.\(^10\)

C. Lack of Pre-Commitment

The first-best is achieved if the manager could make a pre-commitment at \( t = 0 \) that he will invest in the good loan and not switch at \( t = 1 \) to the private-benefit loan. However, we assume that such commitment is not credible other than through incentives provided by debt and equity.

D. Managerial Compensation

The manager’s reservation wage is \( W > 0 \). This can be paid with any combination of a fixed wage and incentive compensation. The fixed wage is \( W_f \) and the equity-based compensation is \( W_e \). The fixed wage is paid up-front at \( t=0 \) and \( W_e \) is paid at \( t=2 \).

E. Systematic Shock to Asset Values

We assume that asset values are subject to a systematic shock, represented by the realized value of an underlying state variable, \( \xi \), that is experienced at \( t = 1 \). With probability \( \beta \in (0,1) \), a systematic asset-value impairment shock is realized as \( \xi = \xi_e \). This shock lowers the value of \( H \) (for both types of

\(^9\) Moreover, because of the nature of the equity contract, shareholders have inherently weaker incentives than creditors to liquidate the bank. Continuing to gamble for resurrection (rather than liquidating) is always more attractive for equity than for debt.

\(^10\) When short-term financiers in the repo market became aware of adverse information about financial institutions, the short-term funding they had provided dried up. This aspect of wholesale funding of banks has also recently been examined by Huang and Ratnovski (2011), though they focus on its negative side-effects.
loans) to $H^- < H$, with $q H^- < L$. With probability $1 - \beta$, the state variable is $\xi = \xi_{th}$, and asset value is now higher at $H^+ > H$, with $\beta H^+ + [1 - \beta] H^+ = H$.

**F. Observability**

We assume that the interim cash flow $x$ of each bank is not observable to any party other than the bank’s shareholders, creditors and the manager. Moreover, the manager’s loan choice is not directly observable to anyone but the manager himself, unless financiers intervene and are able to enforce a different loan choice.

The realized value of the systematic shock to a bank’s asset value is not observed with certainty by the bank’s creditors. The probability that the bank’s creditors will observe the systematic-shock signal $\xi$ is $\gamma \in (0,1)$. Conditional on a particular $\xi$, the random variable $\gamma$ is independent and identically distributed across banks in the economy. The realization of the systematic shock $\xi$ affecting a bank and the bank’s cash flow $x$ are privately observed (if at all) only by that bank’s manager, creditors and shareholders. However, the liquidation of a bank by its creditors is commonly observed by all agents.

The sequence of events in the model is summarized in Figure 1.

--- FIGURE 1 ABOUT HERE ---

**IV. ANALYSIS OF THE MODEL**

First, we present an analysis of the model developed in the previous section by considering the relative advantages of debt and equity financing when there is no systematic component to the asset value shock.

We begin with an analysis of the events at $t = 1$, and then we turn to the events at $t = 0$.

**A. Analysis of the Model without the Systematic Asset Value Shock: Managerial Incentives and Creditors’ Liquidation Decision at $t = 1$**

Consider first the case in which the bank finances with all equity at $t = 0$. To induce the manager to select the good loan at $t = 1$, shareholders must provide the manager with equity that represents a
fraction $\theta \in (0,1)$ of ownership in the bank. The value of $\theta$ must satisfy the incentive compatibility (IC) constraint point-wise at $t = 1$ for every realized $x$.\(^{11}\)

$$\rho \theta qHx \geq \rho \theta pHx + Bx.$$ 

Since this IC constraint is binding in equilibrium, we can write the optimal value of $\theta$, call it $\theta^*$, as:

$$\theta^* = B[H\Delta]^{-1},$$

where $\Delta = [q - p]\rho$. That is, equity provides incentives to the manager via a payoff-contingent compensation contract. To meet the manager’s reservation wage, a fixed wage component may also be necessary.

The following result is immediate from our assumptions

**Lemma 1:** *If the manager is given a fixed wage, $W_f$, it is optimal for the shareholders to pay the fixed wage up-front at $t = 0$ rather than at a future date.*

The intuition is straightforward. A dollar of compensation at $t = 0$ is valued by the manager at a dollar, but a claim on a dollar of future compensation is valued by the manager at $\rho$. Thus it is cheaper for the shareholders to pay the manager his fixed wage up-front.

Next we turn to the case in which the bank has financed with debt at $t = 0$. Having collected $x$ from the date-1 payoff, creditors will assess their payoffs from liquidation and continuation (when they enforce the efficient loan choice) as follows:

- **Liquidate:** $\min(Lx, F - x)$
- **Continue:** $q\min(Hx, F - x)$

We now have the following result:

**Lemma 2:** *In absence of the systematic shock $\xi$, the bank will adopt a debt contract in which the creditors set the covenant violation trigger at $F - \xi[\Delta]$, so that if $x < F - \xi[\Delta]$ creditors expend $c$ to discover the manager’s loan choice at $t = 1$ and enforce a different loan choice if they so desire. If*

\(^{11}\) The reason why the IC constraint must hold point-wise for every $x$ realized at $t=1$ is that the manager makes his project choice at $t=1$ after observing $x$. 

$x \geq F - c[\Delta]^{-1}$, creditors do not expend $c$ to investigate, and unconditionally allow the manager to continue with the loan. Conditional on having expended $c$, creditors adopt the following liquidation/continuation policy:

- Continue if $x \geq \lambda F$;
- Liquidate if $x \in (\lambda F, F - \frac{c}{\Delta})$;

where $\lambda = q[q + L]^{-1}$; All liquidations by creditors are (ex-post) inefficient. When $x \geq F - c[\Delta]^{-1}$, shareholders provide incentives to the manager by giving him ownership $\theta^*$ of the bank’s terminal payoff, where $\theta^*$ is given by (4).

The different cases are pictorially depicted in Figure 2. The intuition is that when $x$ is very high, unconditional continuation is optimal for the creditors at $t = 1$ because their claim is covered out of just the date-1 cash flow, and it does not matter to them what project the bank invests in. In these states in which creditors are paid off out of the date-1 cash flow, shareholders provide the necessary project-choice incentives for the continuation by awarding the manager an ownership share of the bank. That is, when $x$ is high, compensation incentives provided by equity replace the monitoring discipline provided by debt. When $x$ is very low, creditors cannot be paid off fully from the date-1 cash flow, but they prefer to continue rather than liquidate the bank. This is because the liquidation payoff $Lx$ is so low that it is better for creditors to take a chance on a higher payoff $qHx$ by continuation. For intermediate values of $x$, liquidation is optimal for creditors because they get a sufficiently high certain payoff of $Lx$ so that given the “risk aversion” induced by concavity of the debt contract, it does not pay for the creditors to gamble on a risky continuation payoff.

--- FIGURE 2 ABOUT HERE ---

This analysis reveals the pros and cons of debt financing. With debt financing, the manager will not have to be provided a compensation incentive over the entire range of $x$, as is the case when only...
equity is used. These incentives need to be provided with debt only when \( x > F - \frac{c}{\Delta} \). Thus, debt reduces the region over which compensation-based incentives must be provided from \([0, x_{\text{max}}]\) to \( \left[F - \frac{c}{\Delta}, x_{\text{max}}\right] \).

This is the benefit of debt financing. Since performance-based compensation is disliked by the bank manager, it induces a concomitant and compensating upward adjustment in the manager’s fixed wage, which then reduces the initial maximum liquidity the firm can raise via external financing at \( t = 0 \). This \textit{ex ante} liquidity reduction is avoided by debt financing. The disadvantage of debt financing is that it creates a region \( \left[2F, F - \frac{c}{\Delta}\right] \) over which the bank is (\textit{ex-post}) inefficiently liquidated. In what follows, we shall assume that \( c \) is arbitrarily small by letting \( c = 0 \) because doing so reduces notation without qualitatively affecting the analysis.

**B. The Bank’s Optimal Capital Structure Without the Systematic Asset Value Shock**

For simplicity, it will be assumed now that \( g(\bullet) \) is uniform on \([0, x_{\text{max}}]\). Let \( W_e(F, \theta) \) be the manager’s equity compensation as assessed by the shareholders, and let \( W_f(F, \theta) \) be the fixed wage. Both are functions of the face value of the bank’s debt and the fractional ownership \( \theta^* \) given to the manager. Thus,

\[
W_e(F, \theta^*) = \theta^* \left[ E(x - F) + qHE(x) \mid x \geq F \right]
\]

where \( E(\cdot) \) is the expectation operator and \( \theta^* \) is given in (4). The manager’s valuation of his equity compensation, denoted \( W_m(F, \theta^*) \), is:

\[
W_m(F, \theta^*) = \rho \theta^* \left[ E(x - F) + qHE(x) \mid x \geq F \right]
\]

Then, assuming a competitive market among banks for hiring managers, it can be shown that the manager’s reservation utility constraint will be binding in equilibrium so that the manager’s fixed wage is:

\[
W_f(F, \theta^*) = \overline{W} - W_m(F, \theta).
\]
Given the liquidation and continuation regions identified in Lemma 2, we can write the bank’s optimal capital structure as the solution to the following problem of maximizing the total value, \( V(F) \), of the bank:

\[
\text{Max} \, V(F) = E(x) + qH \int_0^{x_F} \frac{x}{x_{\max}} \, dx + \int_{x_F}^{x_{\max}} \frac{Lx}{x_{\max}} \, dx + qH \int_{x_F}^{x_{\max}} \frac{x}{x_{\max}} \, dx - W_e(F, \theta^* ) - W_e(F, \theta^* )
\]  

(10)

It is convenient to define an upper bound on the manager’s private benefit as:

\[
\bar{B} = \frac{\Delta [1- \lambda^2] \left[ qH - L \right]}{q \left[ 1- \rho \right]}
\]  

(11)

We now obtain the following result.

**Proposition 1**: Assume \( B < \bar{B} \). Then the bank’s optimal (value-maximizing) capital structure includes \( F^* \in (0, x_{\max}) \) where:

\[
F^* = \frac{x_{\max}}{\left[ 1 - \lambda^2 \right] \left[ qH - L \right] \left[ (1 - \rho) \theta^* \right] - \left[ qH - 1 \right]}
\]  

(12)

The condition that \( B \) not be too large is needed because otherwise the need for debt discipline is so large that the bank goes to a corner optimum of all debt. As long as this is not the case, there is an interior value of leverage, \( F^* \), that maximizes the value of the bank. This optimal value balances the cost of inefficient liquidation due to leverage against the benefit of leverage in disciplining the manager without the dissipative cost of incentivizing him with equity.

The next result provides comparative statics on \( F^* \).

**Corollary 1**: \( F^* \) is increasing in the manager’s private benefit, \( B \), and decreasing in the manager’s discount factor for equity valuation, \( \rho \).

This result is intuitive. As \( B \) increases, the need for debt discipline grows and leads to higher optimal leverage. As the manager’s discount factor, \( \rho \), increases, using equity compensation to incentivize him becomes less expensive, so optimal leverage declines.
C. Analysis of the Model with the Systematic Asset-Value Shock: The Creditors’ Liquidation Decision at \( t = 1 \)

Thus far, we have examined the monitoring and liquidation decisions of creditors and the contracting decision of banks when asset values are not subject to the systematic asset-value shock. We now include this systematic shock, so that one bank’s liquidation can convey information about the shock to another bank.

Imagine there are two banks in the economy and conditional on the systematic shock \( \xi \), each bank is faced with identically and independently distributed (i.i.d.) shocks. In this two-bank economy, we also need to specify how the banks observe each other’s liquidation outcomes and how this affects the liquidation decisions of creditors of each bank. We model this as a two-stage game. First, each bank’s creditors announce, simultaneously with the creditors of the other bank, whether they will liquidate the bank based solely on their own information about the bank. In this state, neither bank’s creditors have access to the decision of the other bank’s creditors. Second, after having observed each other’s first-stage liquidation outcomes, each bank’s creditors decide whether they wish to liquidate in the second stage (provided they did not liquidate in the first stage). In particular, bank \( i \)’s creditors have no incentive to decide not to liquidate in stage 2 if they decided to liquidate in stage 1.\(^{12}\) But they may decide to liquidate in stage 2 even if they announced in stage 1 that they would not liquidate, if they observe the creditors of the other bank liquidate. We assume (and derive condition below) such that if a bank is liquidated, the inference about \( \xi \) for the creditors of the other bank is sufficiently adverse that they liquidate too.

Note that conditional on the creditors observing \( \xi = \xi^* \), the creditors will liquidate if

\[
q_{H^*} < L, \tag{13}
\]

which we assumed earlier. To figure out the sufficient condition for our assumption that the creditors of bank \( i \) will liquidate in stage 2 if they observe the creditors of bank \( j \) liquidating, let us calculate the posterior belief of creditors of bank \( i \) (if they observe the creditors of bank \( j \) liquidating) concerning the

\(^{12}\) There are no strategic manipulation incentives in the model. In particular, there is nothing to be gained for the creditors in one bank from liquidating or not liquidating a bank in order to strategically manipulate the behavior of creditors in another bank.
aggregate shock being adverse. At this stage, we need to introduce some notation that is useful for examining the inference problem from the perspective of bank $i$. Define for bank $j$:

$$\delta^j = \Pr \left[ x \in (\lambda F_j, F_j) \right] = \frac{F_j - \lambda F_j}{x_{\text{max}}} = \left[ 1 - \frac{\lambda}{x_{\text{max}}} \right] F_j,$$

(14)

$$\delta^i = \beta [1 - \gamma] [1 - \delta^j],$$

(15)

$$\delta^j = [1 - \beta] \gamma + [1 - \gamma] [1 - \delta^j].$$

(16)

Note that $\delta^j$ is simply the probability that bank $j$'s $x \in (\lambda F_j, F_j)$ and hence it is liquidated by its creditors solely due to the realization of its own cash flow.

Next, $\delta^i$ is the joint probability that the adverse asset-value shock was realized (the probability that $\xi = \xi_i$ is $\beta$), neither bank observed the shock (probability $(1 - \gamma)^2$), and bank $j$ did not liquidate due to a cash-flow realization $x \in (\lambda F_j, F_j)$ (the probability that $x \notin (\lambda F_j, F_j)$ is $1 - \delta^j$).

Similarly, $\delta^j$ is the joint probability that the favorable asset-value shock was realized (probability $1 - \beta$), and either it was observed by the bank (probability $\gamma$) or it was not observed (probability $1 - \gamma$) and bank $j$ did not liquidate due to its own $x \in (\lambda F_j, F_j)$ (which has probability $1 - \delta^j$). Thus, $\delta^i + \delta^j$ is the probability that bank $i$ is not liquidated when its own $x \in (0, \lambda F_j)$, i.e., the probability that it is not liquidated when its own interim loan cash flow is low enough to not warrant liquidation on the basis of that cash flow.

In calculating the posterior belief that the aggregate shock is adverse, what we are assuming is that the bank in question (bank $i$) did not receive the signal about the shock, decided not to liquidate in stage 1 but observed the other bank (bank $j$) liquidating in stage 1. Then bank $i$ forms the following posterior belief about $\xi$:

$$\Pr \left( \xi = \xi_i \mid \text{bank } j \text{ announced liquidation in stage 1} \right)$$

$$= \frac{\Pr \left( \text{bank } j \text{ liquidated in stage 1} \mid \xi = \xi_i \right) \Pr \left( \xi = \xi_i \right)}{\Pr \left( \text{bank } j \text{ liquidated in stage 1} \right)}$$
\[
\begin{align*}
&= \left( \frac{\gamma \lambda F_j [x_{\text{max}}]^{-1} + \delta^j}{\left[ \gamma \lambda F_j [x_{\text{max}}]^{-1} + \delta^j \right] \beta + [1-\beta] [\delta^j]} \right) = \left[ \frac{\beta \gamma \lambda F_j}{\beta \gamma \lambda F_j + F_j [1-\lambda]} \right] \\
&= \frac{[1-\lambda] [1-\gamma] \beta}{[1-\lambda] [1-\gamma] - (1-\lambda)} = \hat{\beta}.
\end{align*}
\]

Note that this probability is positive only if bank \( i \) did not observe \( \xi = \xi_i \) in the first stage. Further \( \hat{\beta} > \beta \).

To understand the numerator of (17), note that, conditional on \( \xi = \xi_i \), bank \( j \) can be liquidated in one of two cases: (i) if its creditors observe \( \zeta \) (which has probability \( \gamma \)) and bank \( j \)'s cash flow \( x \leq \lambda F_j \) (which has probability \( \lambda F_j [x_{\text{max}}]^{-1} \)), or (ii) bank \( j \)'s cash flow realization \( x \in (\lambda F_j, F_j) \) (which has probability \( \delta^j \)). Hence, the probability of being in one of these two states is \( \lambda F_j [x_{\text{max}}]^{-1} + \delta^j \). This probability is multiplied with \( \beta \), the probability that \( \xi = \xi_i \). In the denominator, there is an additional term, \( [1-\beta] \), which is the probability that bank \( j \) is liquidated due to its \( x \in (\lambda F_j, F_j) \) even when \( \xi = \xi_i \) (which has probability \( [1-\beta] \)).

Also, for bank \( i \)'s creditors to decide to liquidate (only) after having observed bank \( j \)'s creditors liquidations, we need two conditions:

**Condition 1:** Creditors will not unconditionally liquidate before observing \( \xi \):

\[
q \left[ \beta H^- + [1-\beta] H^+ \right] > L
\]

which implies:

\[
qH > L \quad (18)
\]

which we have assumed throughout.

**Condition 2:** It pays for the creditors of bank \( i \) to liquidate if they observe liquidation by the creditors of bank \( j \):

\[
q \left[ \hat{\beta} H^- + [1-\hat{\beta}] H^+ \right] < L 
\]

(19)
where \( \hat{\beta} \) is defined in (17). Define \( \bar{L} \) as the value of \( L \) at which (19) holds as an equality, so that the inequality in (19) will hold for all \( L > \bar{L} \).

We now have:

**Proposition 2:** For a given leverage of bank \( j, F_j \), the (unconditional) ex ante probability of liquidation of bank \( i \) is increasing in bank \( i \)'s leverage, \( F_i \). Moreover, fixing its own leverage, \( F_i \), the unconditional probability of liquidation of bank \( i \) is increasing in bank \( j \)'s leverage, \( F_j \).

The intuition can be seen by observing that higher leverage of bank \( j \) makes it more likely that bank \( j \) will be liquidated since it reduces the probability that bank \( j \)'s interim cash flow, \( x \), will exceed its leverage \( F_j \). Wherever bank \( j \) is liquidated, bank \( i \)'s creditors noisily infer that state \( \xi = \xi_j \) is likely to have occurred, and liquidate bank \( i \).

Recall that in the previous analysis conducted in the absence of the systematic asset-value shock \( \xi \), any liquidation by a bank’s creditors was ex-post inefficient (see Lemma 2). That is no longer true, however, when the systematic asset-value shock \( \xi \) is introduced. Now there can be an ex-post efficiency gain from liquidation in the case where the creditors of bank \( i \) do not observe \( \xi \) but liquidate based upon observing the liquidation of bank \( j \) whose creditors observe \( \xi = \xi_j \). Proposition 2 shows that bank \( i \)'s own leverage increases the probability of liquidation of bank \( i \). Thus, an increase in a bank’s leverage contributes to ex-post efficiency in such states. Hence, with the introduction of the systematic asset-value shock, leverage-induced liquidity creation has both a *dark side* (liquidations that are sometimes inefficient) and a *bright side* from an ex-post standpoint.\(^\text{13}\) It is the dark side that contributes to a leverage-engendered increase in systemic risk.

Note also that we have assumed that creditors in different banks are distinct. What if say a single creditor could take diversified positions in all banks? On the one hand, this will eliminate inefficient

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\( ^\text{13} \) We have conducted this analysis for the two-bank case to convey these ideas as transparently as possible. The \( n \)-bank case, with \( n > 2 \), is qualitatively similar, although it permits a weakening of the restrictions on the exogenous parameters that are needed for liquidation contagion.
liquidations because there is no contagion to worry about. On the other hand, efficient liquidations will also decline because the probability of receiving information about the systematic shock will be limited to the probability that one bank will receive this signal.

D. The Value of the Levered Bank and Optimal Leverage at $t = 0$

We can now move to $t = 0$ and examine the bank’s optimal capital structure decision with the systematic asset-value shock. Now bank $i$’s value function can be written as:

$$V(F_i) = E(x) + \int_{x_{min}}^{x_{max}} \left( \frac{Lx}{x_{max}} \right) dx + qH \int_{F_i}^{x_{max}} \left( \frac{X}{x_{max}} \right) dx$$

$$+ [\delta_i H^- + \delta_j H^+] q \int_{0}^{x_{max}} \left( \frac{X}{x_{max}} \right) dx + [1 - \delta_i - \delta_j] L \int_{F_i}^{x_{max}} \left( \frac{X}{x_{max}} \right) dx$$

$$- W_i(F_i, \theta^*) - W_i(F_i, \theta^*)$$

(20)

To understand (20), note that its structure is similar to (10). The first term, $E(x)$, is the expected value of the interim cash flow.

The second term is the liquidation by creditors of bank $i$ that occurs when $x \in [0, \lambda F_j]$, and the value is $Lx$.

The third term refers to the state $x \in (F_i, x_{max}]$. In this state, there is no liquidation or creditor monitoring of bank $i$, so the manager is incentivized by shareholders with an equity compensation contract. The value of the bank is the third term and the equity compensation shows up in the sixth term (recall that the seventh term is the fixed compensation).

To understand the fourth term, note that $1 - \delta_i - \delta_j$ is the probability of liquidation of bank $i$ that is not based on bank $i$’s realization of $x$ when $x \in [0, \lambda F_j]$, but based on that of bank $j$, and $\delta_i + \delta_j$ is the probability of no liquidation of bank $i$, when $x \in [0, \lambda F_j]$. As explained earlier, $\delta_i$ is the joint probability of $\xi = \xi_i$ and no liquidation of bank $j$, and $\delta_j$ is the joint probability of $\xi = \xi_j$ and no liquidation of bank $j$. Note also that if $\xi = \xi_i$ is realized, the expected value of date-2 cash flows is $qH^-$ per unit of $x$ and if $\xi = \xi_j$ is realized, this value is $qH^+$ per unit of $x$. This explains the fourth term.
The fifth term refers to the liquidation of bank $i$ that occurs because creditors observe the liquidation of the other bank $j$, i.e., this is liquidation that is not based on bank $i$’s own $x$. This occurs with probability $1-\delta^i - \delta^j$ when for bank $i$ we have $x \in [0, \lambda F_i]$, and the liquidation payoff is $Lx$.

It is intuitive that bank $i$’s value is increasing in $\delta^i[H^+ - L]$ since this quantity represents the expected net gain in value from avoiding (ex post) inefficient liquidation when $\xi = \xi_h$. Moreover, bank $i$’s value is decreasing in $\delta^i[L - qH^+]$ because this term represents the expected net gain from incurring (ex post) efficient liquidation when $\xi = \xi_l$.

Finally, note that the leverage of the other bank, $j$, is contained in the probabilities $\delta^i$ and $\delta^j$. Formally, let $V_i(F_i, F_j)$ denote the value of bank $i$ given the leverage of the two banks $F_i$ and $F_j$. Then the best-response of bank $i$ in terms of its leverage choice $F_i^*(F_j^0)$ maximizes $V_i(F_i, F_j^0)$ for a given leverage of bank $j$, $F_j^0$. The Nash equilibrium of private leverage choices $(\overline{F}_i, \overline{F}_j)$ satisfies the fixed-point problem $\overline{F}_i = F_i^*(\overline{F}_j)$ and $F_j = F_j^*(\overline{F}_i)$.

It is convenient to introduce the following definition:

$$\hat{B} \equiv \Delta \frac{qH - L - \lambda^2 \left\{ \delta^i \left[ H^+ - L \right] - \delta^i \left[ L - H^- \right] \right\}}{[1-\rho]q}$$

(21)

$\hat{B}$ is similar to the $\bar{B}$ defined in (11). Note that $\hat{B}$ depends on the leverage of bank $j$, it is a constant from the standpoint of bank $i$. Then, we obtain the following characterization of best-responses and the symmetric Nash equilibrium:

**Proposition 3:** Assume $B < \hat{B}$. Then, in any Nash equilibrium, bank $i$’s best response, $F_i^*(F_j^0)$, at $t=0$ to bank $j$’s (privately-optimal) leverage, $F_j^0$, is:

$$F_i^*(F_j^0) = \frac{1}{qH - [1-\rho]H^1 - \lambda^2 \left\{ \delta^i \left[ H^+ - L \right] - \delta^i \left[ L - H^- \right] \right\}} \left[ \frac{1-\rho}{\theta^* x_{\max}} \right]$$

(22)
where $\delta_1^i$ and $\delta_2^i$ are functions of $F_j^o$. In a symmetric Nash equilibrium, each bank’s privately-optimal leverage is:

$$F_j = F_j(F_j^o) = F_j^o(F_j^o) = \mathcal{F}_j = F_j^o = \frac{\sqrt{a^2 + 4b - a}}{2b}$$

(23)

where $a = \left\{qH - [1 - \rho] \theta' [qH - 1] L - \lambda^2 [H - L] [1 - \lambda^2 + 2\beta \gamma - \beta \gamma^2] \right\}$$

$$b = \left\{\frac{\lambda^2 [H - L] [1 - \gamma] [1 - \beta \{2 - \gamma\}] [1 - \lambda]}{(1 - \rho) \theta' [x_{\text{max}}]} \right\}$$

Moreover, $F_j^o \in (0, x_{\text{max}})$.  

The condition that the manager’s private benefit not be too large is for the same reasons as discussed earlier. The proposition characterizes the best response of a bank in terms of its choice of leverage (given the leverage of the other bank) that maximizes the value of the bank and hence its $\text{ex ante}$ liquidity. The proposition asserts that $F_j^o$ is a unique symmetric Nash equilibrium outcome. Note that each bank’s leverage choice has to be examined as part of a Nash equilibrium because each bank’s leverage affects the other bank’s expected payoff (formally through $\delta_1^i$ and $\delta_2^i$). While multiple Nash equilibria are possible, each bank’s best response in any equilibrium must satisfy (22). While (22) expresses each bank’s leverage as a function of the other bank’s leverage, (23) expresses the symmetric Nash equilibrium choice purely as a function of exogenous parameters (recall $\theta'$ is expressed purely as a function of exogenous parameters via (4)).

The following corollary establishes comparative statics on $F_j^o$.

**Corollary 2:** The privately optimal leverage $F_j^o$ is increasing in the manager’s private benefit, $B$, and the likelihood, $\gamma$, that the systematic state is visible to bank creditors, and decreasing in the probability, $\beta$, of systematic asset-value impairment shock, and the managerial discount factor, $\rho$.

The intuition for the impact of $B$ on $F_j^o$ is similar to that in our earlier discussion. As (22) clarifies, an increase in $B$ increases $\theta'$ and the illiquidity arising due to equity-based compensation, and
debt becomes more attractive. The intuition for \( \gamma \) is straightforward in that as debt becomes more informed about the nature of the systematic shock, inefficient liquidations by creditors are fewer and deadweight costs due to debt become lower.

The intuition for the impact of \( \beta \) is as follows. The main role of leverage is to discipline the manager via the threat of bank liquidation. While bank liquidation by creditors, based on having observed a signal that asset value is impaired, also requires the presence of some leverage, the probability that the creditors of bank \( i \) will observe this signal is unaffected by the leverage of bank \( i \). When \( \beta \) increases, it becomes more likely, \textit{ceteris paribus}, that the creditors of bank \( i \) will liquidate based on observing a liquidation of bank \( j \). This contagion effect makes debt less attractive ex ante. Finally, as the manager’s discount for equity-based compensation less, that is, the discount factor for risk-based compensation goes up, debt becomes relatively less attractive to equity financing.

Next, we examine how the Nash equilibrium of privately-optimal leverage choices compares with the regulatory optimum. The regulator’s optimal leverage choice \((F_i, F_j)\) maximizes the sum of the two bank values taking account of the externalities of bank leverage on other banks. That is, the regulatory problem is to maximize \(V_i(F_i, F_j) + V_j(F_i, F_j)\), whose first-order conditions are given by

\[
\frac{\partial V_i}{\partial F_i} + \frac{\partial V_j}{\partial F_i} = 0, \quad \frac{\partial V_i}{\partial F_j} + \frac{\partial V_j}{\partial F_j} = 0.
\]  

In contrast, the private optimum satisfies \(\frac{\partial V_i}{\partial F_i} = 0\) and \(\frac{\partial V_j}{\partial F_j} = 0\). For the rest of the analysis in this section, we will assume the symmetry condition \(L - H^- = H^+ - L\), and that \(\beta \leq 0.5\).

Then, we obtain the following intuitive result (focusing on the symmetric case as before):
Proposition 4: The regulator’s problem of maximizing $V_i + V_j$ is a convex optimization problem with a unique global optimum. In the (symmetric) Nash equilibrium involving banks $i$ and $j$, the privately-optimal leverage $F^o$ exceeds the socially optimal leverage $\hat{F}$, where

$$\hat{F} = \sqrt{\frac{a^2 + 4(b+c) - a}{2(b+c)}}$$

(26)

where

$$c = \frac{\lambda^2[1-\lambda][1-\gamma][1-\beta(2-\gamma)]}{2[1-\rho] \theta^* [x_{max}]^2} > 0$$

This proposition captures the essence of our main point highlighting the systemic risk induced by leverage-based creation of liquidity. As discussed earlier, the main role of leverage is to discipline the manager. However, as bank $i$’s privately-optimal leverage increases, its negative impact on the value of bank $j$—which derives no benefit from better discipline at bank $i$—also increases; recall that this negative impact arises from bank $j$ being inefficiently liquidated when bank $i$ is liquidated only because of an adverse idiosyncratic cash flow realization. Thus, the regulatory optimum drops below the private optimum. The reason why the condition $\beta \leq 0.5$ is needed is that when $\beta$ is low, the social value of liquidations based upon creditors observing the liquidations of other banks is also low. This is because the likelihood of the asset-value impairment shock is low, so the value of learning about this shock through another bank’s liquidation is also low. Since higher bank leverage leads to a higher liquidation probability, the social value of higher leverage declines as the value of learning about the systematic shock through liquidations declines due to a reduction in $\beta$. The inefficiency associated with contagious liquidations remains unaffected, however. That is, since the privately-optimal leverage choice of a bank does not internalize the inefficient contagious liquidations of other banks induced by its own leverage, the bank leveres up more than is desirable from the regulator’s viewpoint.

This is the “dark” side of leverage-based liquidity creation by individual banks. Having leverage at the bank level is a desirable objective from a micro-prudential standpoint but socially costly due to its
pervasive implications for macro-prudential outcomes. Formally, it is the case that socially optimal leverage $\hat{F}$ has the same comparative statics properties as for the privately optimal leverage $F^*$ in Corollary 2. Nevertheless, what matters for efficiency is the difference between the two $(F^* - \hat{F})$ and this difference has similar comparative statics as in Corollary 2. That is, the difference between the privately optimal leverage $F^*$ and the socially optimal leverage $\hat{F}$ is increasing in the manager’s private benefit, $B$, and the likelihood, $\gamma$, that the systematic state is visible to bank creditors, and decreasing in the probability, $\beta$, of systematic asset-value impairment shock, and the managerial discount factor, $\rho$.

These comparative statics on inefficiency of privately optimal leverage are difficult to prove analytically but illustrated graphically in Figure 3 for a numerical example (and found to be robust across a range of parameter values we have tried). In the numerical example, we choose as baseline parameters $q = 3/4, p = 1/2, B = 0.4, \rho = 0.8, \gamma = 0.6, x_{\text{max}} = 4, H = 7/4, L = 1, H^+ = 1.5, H^- = 0.5, \beta = 1/8$.

This is one parameter constellation that ensures that various restrictions assumed throughout the model are all jointly satisfied.

--- FIGURE 3 ABOUT HERE ---

The intuition for the inefficiency of privately optimal leverage when the likelihood of the negative asset-impairment shock is sufficiently small (as in the condition for Proposition 4) is as follows. Consider for instance the effect of private benefit parameter, $B$. This makes leverage privately more attractive, but as each bank increases its leverage to minimize on compensation costs due equity financing (which are higher for higher $B$), it ignores the externality through contagion of increasing leverage on the other bank. A decrease in the discount factor of the manager on incentive-based compensation produces a similar effect. And, increase in the visibility of systematic asset shock to creditors – that is, an increase in the information generation capacity of creditors – makes debt privately more attractive but the private choice again ignores the contagion-based externality imposed on the other bank.
Thus private incentives to lever up banks are in general inefficient, so that from a regulatory standpoint, there is a possible justification for a role for ex-ante capital requirements or a leverage requirement that bank leverage not exceed $\hat{F}$. It is important to point out that the optimal capital level of the banks in our model is not 100%, or conversely leverage is not zero. This is because just as leverage financing of banks in the model has deadweight costs for the system due to inefficient contagion from one bank’s liquidation to the other bank’s liquidation, there is also the deadweight cost of equity-based financing, in particular, that equity financing requires incentivizing managers through performance-based compensation but that this entails overall higher compensation cost to the firm as managers discount incentive-based compensation more than incentive-based compensation. Another way of seeing this is that in the off-equilibrium calculation where the bank is entirely financed by equity outside, the compensation required to be paid to the bank manager to ensure correct incentives may be so high that the bank may have no ex-ante liquidity in the first place, that is, it may not be economically viable. Bank leverage in contrast has the benefit of disciplining the banker in a more incentive-efficient manner, implying that there is some benefit to bank leverage even in the socially optimal case (in addition to the fact that bank leverage also sometimes generates efficient information about the systematic shock).

In what follows, we explore the tension between privately-optimal and socially-optimal bank leverage further by studying how ex-post regulatory interventions, such as bailouts or lender of last resort policies, can potentially reduce the dark side of bank leverage by containing contagious liquidations. However, if these interventions are conducted purely based on market information, e.g., observation of liquidation risk of banks, then they can – as an unintended consequence – interfere with the bright sides of bank leverage, namely unlocking liquidity and generating valuable information about asset-value shocks.

V. THE EFFECT OF LENDER OF LAST RESORT

Given that some of the contagious liquidations are inefficient, there may be a role for regulatory intervention in the form of lender of last resort (LOLR) actions. The LOLR could step in and bail out banks threatened by liquidation. In particular, if the LOLR observes a threatened liquidation that may trigger a contagion of liquidation on the other banks, it steps in and buys out the creditors of threatened
banks. The purpose of the following discussion is to examine the consequences of such LOLR intervention. Our analysis proceeds in two steps. First, we assume that the LOLR operates under rather severe informational constraints, and show that the desire to avoid a collapse of the industry in all states of the world and thereby reduce systemic risk has adverse consequences for the leverage decisions of banks and their ex ante liquidity. Second, we permit the LOLR to have access to additional information (at a cost) and show that there are circumstances in which the LOLR can improve on the market outcome.

A. Loss of market discipline and information role of debt in presence of an informationally-constrained LOLR

The key assumption we make in this analysis is that the LOLR operates under a serious informational constraint – it does not observe the $x$ of any bank and hence cannot distinguish between liquidations based on the realization of $x$ observed by bank’s own creditors and those based on the observed liquidations of other banks. The problem with the regulatory resolution of a threatened bank is that unless the LOLR backstops creditors, information about there being potentially an adverse asset-value shock is likely to leak out. In this case, the bailout by the LOLR serves no purpose at all since all other banks’ creditors will infer that a liquidation was imminent, and contagion will set in.\(^{14}\) Knowing this, creditors who can threaten liquidation at $t=1$ will demand that the LOLR pay them $F-x$ in full after they have collected the date-1 cash flow $x$. This means that creditors can threaten liquidation at $t=1$ in every state and receive $F-x$ in addition to $x$. Anticipating this, they would have no reason to engage in privately-costly monitoring, and all the market discipline of leverage is lost. Of course, the ex ante pricing of leverage will reflect the fact that it is riskless, so creditors will not ex ante get a “free lunch”. However, the entire burden of disciplining the manager now falls on the compensation contract and the amount of bank leverage is irrelevant. The value of the bank then is:

$$V_{LOLR} = E(x) + qH\int_{x_{\text{min}}}^{x_{\text{max}}} \left(\frac{x}{x_{\text{max}}} \right) dx - \bar{W} \left[1 - \rho \right] \theta \left(\int_{x_{\text{min}}}^{x_{\text{max}}} \left(\frac{x}{x_{\text{max}}} \right) dx + qH\int_{x_{\text{min}}}^{x_{\text{max}}} \left(\frac{x}{x_{\text{max}}} \right) dx\right).$$  \hspace{1cm} (27)$$

We now have the following result:

\(^{14}\)With dispersed creditors, it may be difficult to prevent this information from leaking out.
**Proposition 5:** When the LOLR is present to bail out banks at $t = 1$ and prevent liquidations, the bank’s capital structure becomes irrelevant, and for $c > 0$ sufficiently small, the bank’s ex ante liquidity is lower with the LOLR than without.

The intuition for capital structure irrelevance is straightforward. If we remove the disciplining role of debt and make interim liquidations impossible, then we have neither a benefit nor a cost associated with leverage. Hence, the Modigliani and Miller (1958) indifference theorem applies.\(^{15}\) The reason why the LOLR causes the bank to have lower ex-ante liquidity is that the disciplining role of leverage is important for enhancing the bank’s liquidity ex ante. When that role is lost, so is some ex-ante liquidity. Note, however, that this is not a total loss of liquidity as in some models where the absence of the disciplining role of leverage means that the loan financed by the bank becomes completely illiquid. The compensation-based incentives provided by equity do provide ex ante liquidity, but it is not as much as is available with the optimal combination of debt and equity in the absence of the LOLR.

This is somewhat of a paradox. The role of the LOLR is to increase the liquidity in the system. And yet the presence of the LOLR reduces the ex-ante liquidity of banks. Thus, the “price” of having a system flush with ex-post liquidity provided by the LOLR is lower ex-ante liquidity.

A further cost of having the LOLR intervene with bailouts is that the financial system stops generating any valuable signals about asset values that are produced via creditor liquidations absent the LOLR. Hence, the presence of the LOLR eliminates the ability to liquidate assets early and prevent further asset-value erosions from continuations when $\xi = \xi_r$. In some cases, such signals may be especially valuable because they are “early warnings” of bigger problems down the road, an issue we explore further below.

---

\(^{15}\) Now, any distortion in favor of debt such as tax deductibility of interest rate payments can lead the financial sector to lever up in a significant manner. Also, as argued by Acharya and Yorulmazer (2007), Farhi and Tirole (2009), and Acharya, Mehran and Thakor (2011), there would be an incentive to increase the systematic risk of projects, if that were a choice variable in our model, and banks would prefer to fund these correlated projects with leverage to “loot” the LOLR (assuming bank equity is not bailed out in case of joint failures).
B. Some Remarks on Possible Private Contracting Solutions and Recent Regulatory Reform Proposals

In this section, we examine private-contracting solutions to the problem of correlated failures that would obviate the need for ex post bailouts by the LOLR. We will examine these generically, and focus on contingent capital (debt securities that are mandatorily convertible into equity under pre-specified stress conditions). Contingent capital proposals have been discussed mainly as regulatory initiatives – and some variants do involve the conversion of subordinated debt into equity to be triggered by regulatory action – but they typically do not (and in fact are proposed precisely to not) involve regulatory bailouts (e.g. see Flannery (2005) and Squam Lake (2009)). This means it would not be difficult to imagine these being implemented via private contracting, even though their design may originate with the regulator. That is, in the analysis below, we focus on schemes that are ex post privately optimal for the bank and its financiers. Of course, in practice, there may be circumstances in which social and private optima may diverge, so that regulator “activism” may be necessary to implement some schemes. We do not consider those here.

In essence, the contingent claims proposals are debt-equity swaps. We will restrict ourselves to “voluntary exchanges” in our analysis, i.e., exchanges in which the creditors are willing to participate. Mathematically, this is equivalent to the bank’s shareholders being willing to buy out the creditors at a price that makes them indifferent between liquidating and not liquidating the bank. Note that, from the shareholders’ perspective, this is necessary only when $x \in [0,F]$ and creditors wish to liquidate. A key point is to recognize that the shareholders will be cognizant of the altered incentives of the bank manager when the creditors’ claims are bought out by the shareholders and hence converted into equity. In particular, creditor monitoring disappears in those states. So, the shareholders will now have to use compensation-based incentives to motivate the manager. Whether the shareholders wish to buy out the creditors in the liquidation states now depends on a comparison of what they have to pay the creditors to buy them out ($L$) and their own net payoff after recalibrating managerial incentives. Thus, we have:
Lemma 3: If the manager’s private benefit, $B$, is sufficiently large, then the shareholders will not wish to buy out the creditors when they threaten to liquidate the bank.

The intuition is that converting debt into equity has a cost, namely that creditor monitoring has to be forgone. This forces reliance on compensation-based incentives that equity deploys, which diminishes the net terminal payoff available for shareholders. The more expensive the compensation-based incentive schemes, the lower the net terminal payoff available to shareholders, and hence the less attractive it is for shareholders to buy out the creditors.

Of course, if contingent capital was mandated by regulatory fiat, then the shareholders will have no choice but to accept the conversion of debt into equity. Nonetheless, the discipline of bank debt will be lost post-conversion, so the analysis sheds light on the reluctance of banks to endorse contingent capital proposals.

C. Regulatory Minimum Capital Requirements

Our analysis shows that during times that $\beta$, the probability of a systematic asset-value impairment, is low, privately-optimal leverage exceeds the regulatory optimum. These will also be the times when bank asset values are relatively high since a low $\beta$ corresponds to higher expected future asset values. This suggests that a relatively high regulatory minimum capital requirement during high asset-valuation periods may be desirable since it could make banks hold capital that is above privately-optimal levels and at or near the regulatory optimum. When $\beta$ is high and asset values are consequently low, it may be advisable to lower regulatory capital requirements, but they may not be binding because the privately-optimal leverage may be below the regulatory optimum, in which case banks will voluntarily hold more capital than the regulatory requirement. Hence, the analysis in this paper is supportive of countercyclical capital requirements.

Clearly, such a regime would not be costless. Even countercyclical capital requirements will reduce ex ante liquidity creation during high-asset-valuation periods when they are binding. But, if the costs of systemwide failures is high enough, then the benefit of reducing systemic risk through higher
capital requirements may more than offset the cost of lower ex ante liquidity. Moreover, having more
capital injected into the banking system via higher capital requirements will shift more of the burden of
corporate governance from debt to equity, with the attendant benefit of lower systemic risk.

One of the goals of the Dodd-Frank Act is to control systemic risk. A focus will be on identifying
and monitoring Systemically Important Financial Intermediaries (SIFIs). Our analysis points out that an
important complement to this would be countercyclical capital requirements, since systemic risk increases
with leverage.

D. The LOLR With Expanded Information Access

Suppose the LOLR can, at a cost $\psi > 0$, obtain information about the systematic asset-value-
impairment shock with probability 1 at $t=1$. One can interpret this as the LOLR setting up a new
regulatory body like the Financial Stability Oversight Council (FSOC) to gather information germane to
systemic risk and monitor risk across different parts of the economy. We continue to assume that the
LOLR cannot observe the $x$ of any bank. The goal of the LOLR is simply to contain systemic risk, i.e.,
prevent contagion. The following result indicates that a regulatory policy of selective intervention that
tolerates some bank failures can achieve this goal without distorting asset choices or leverage decisions
away from the equilibrium without LOLR intervention.

**Proposition 6:** Suppose the LOLR precommits to an intervention policy such that any bank whose
creditors threaten liquidation is bailed out by having the LOLR promise the bank’s creditors in private
negotiations that they will be paid in full at $t=2$ if they allow the bank to continue only if the LOLR has
learned at $t=1$ that bank asset values are impaired, and letting the bank be liquidated by the creditors at
$t=1$ if the LOLR has learned that there is no asset value impairment. Assuming that the bailout
negotiation is secret and the creditors of no other banks learn that particular bank was bailed out, the
probability of contagion is driven to zero, and no bank’s creditors liquidate the bank when they observe
another bank being liquidated. Moreover, each bank chooses debt of

$$
\tilde{r}^* = \frac{\lambda_{\max}}{\left\{ [1-\hat{\lambda}^2][qH^\omega - L](1-\rho)\theta \right\}^{-1} - [qH^\omega - 1]}
$$

(28)
where $H^* = \beta H^- + (1-\beta)H^-$ \hfill (29)

and also chooses the good loan. The market discipline of both debt and equity remain qualitatively the same as in the case in which the LOLR does not bail out any banks and there is no asset-value impairment shock.

This proposition says that the LOLR can eliminate systemic risk by guaranteeing that any bank that is on the brink of being liquidated due to the systematic shock will be bailed out by the LOLR. This can either be a standard bailout or something akin to the Troubled Asset Relief Program (TARP), wherein the government steps in and agrees to purchase the (devalued) assets of banks at a price exceeding market value. Of course, implementing such a scheme requires that the LOLR be able to have access to information about the systematic shock, and this may prove to be quite expensive in practice (high $\psi$). It is also crucial that the LOLR’s bailout is a secret negotiation with the bank in question. This means that even though the systemic asset value impairment shock may have been realized, the LOLR will permit unassisted continuation of banks whose creditors do not threaten liquidation.

The other part of this intervention is that the LOLR avoids any bailout if the creditors’ liquidation threat does not coincide with an impairment in bank asset values. That is, any liquidation due to an $x$ realization being in the liquidation range is allowed to proceed. Note, however, that common knowledge of such in intervention policy means that any liquidation that is publicly observed noiselessly reveals that asset values are not systematically impaired, so there is no contagion. Moreover, because such liquidations are allowed to occur, all the market discipline of debt and equity that was present in the case without the LOLR and the systematic shock is resurrected. This is why $\hat{F}^*$ in (28) has the same functional form as the $F^*$ in (12); the only difference is that $H$ is replaced by $H^*$.

The LOLR thus avoids contagion by giving banks a put option on the value of their assets, but this option has value only if it is exercised at $t=1$. If they do not threaten liquidation at $t=1$ (because they do not receive a signal that asset values have declined), but there has actually been a decline in asset values, then their loss at $t=2$ will not be covered by the LOLR. This suggests that it might pay for the
creditors to threaten liquidation wherever \( x \in [x_{\text{min}}, \lambda F] \), even when they have not received an asset-value impairment shock. If the LOLR agrees to bail the bank out (because the systematic shock has, in fact, occurred), fine. If not (because asset values are not truly impaired), then creditors can withdraw their liquidation threat. That is, there may be “frivolous” liquidation threats in order to “game” the LOLR. To avoid this, the LOLR can stipulate penalties on creditors for liquidation threats that are not carried out in the event the LOLR refuses to bail out.

Avoiding contagion like this and simultaneously restoring market discipline comes at a cost. Specifically, the LOLR prevents efficient liquidations by bailing out banks when the systematic shock indicates they should be liquidated, and permits inefficient liquidations based on idiosyncratic \( (x) \) risk to proceed; recall that, absent the systematic shock, any liquidation based on \( x \) is inefficient.

**VI. CONCLUSION**

In this paper, we re-examined the role of bank leverage as an instrument of liquidity creation. We exposed a fundamental tension between the micro-prudential goal of encouraging the market discipline of banks via greater uninsured leverage in their capital structure and the macro-prudential goal of containing systemic risk. While higher bank leverage creates stronger creditor discipline at the individual bank level, it leads to greater systemic risk induced by contagious runs when creditors liquidate banks. This invites ex-post LOLR intervention to bail out “failing” banks from being liquidated by creditors and thereby to maintain the interim liquidity and continuity of banks. The consequences of this LOLR intervention depend on the LOLR’s information and the nature of the intervention. When the LOLR has no information about whether systematic asset-value impairment has occurred, the intervention is unconditional and the disciplining role of bank leverage is lost, valuable information about asset-value impairment is not generated, and ex ante bank liquidity actually declines. In fact, the more vigilant the LOLR is in reacting to early warnings by intervening with bailouts of failing banks, the worse is the problem. We also examined private-contracting alternatives to LOLR intervention such as contingent capital or debt-for-equity swaps. Such schemes can eliminate the problem of lack of private incentives to
avoid contagious liquidations. But they also sacrifice the market discipline of debt. When the LOLR can acquire information about the systematic shock and condition intervention based on that, contagion can be eliminated without sacrificing market discipline.

In this context, higher regulatory minimum capital requirements that reduce systemic risk and decrease the expected social cost of complete industry collapse may be preferable to ex-post LOLR and contingent capital arrangements. While ex ante bank liquidity may be diminished by higher capital requirements, some bank failures would be tolerated ex post and the resulting system may be less prone to systemic risk. Indeed, our analysis suggests that improving the governance that can be imposed on banks by shareholders may be a more robust – even though not much discussed – intervention to reduce excessive bank reliance on leverage for liquidity.
APPENDIX

Proof of Lemma 1: Obvious from the discussion in the text.

Proof of Lemma 2: First, let us consider the intervention policy. If the creditors intervene, they can force the bank manager to invest in the good loan and the creditors’ date-2 payoff, after having been paid \( x \) at date-1, is \( q[F-x] - c \), taking into account the investigation cost. If creditors do not intervene, the manager will select the private-benefit loan, and the creditors’ date-2 payoff will be \( p[F-x] \). Thus, intervention is optimal for the creditors if:

\[
q[F-x] - c > p[F-x]
\]

or

\[
x < F - c \left[ \Delta \right]^{-1}
\] (A-1)

Now, we turn to the liquidation/continuation decision after having invested \( c \). We compare (5) and (6). Since \( L < qH \), there are three cases to consider: (i) \( F-x < Lx \); (ii) \( Lx < F-x < Hx \); and (iii) \( F-x > Hx \).

Case (i): \( F-x < Lx \): In this region, \( x > F/[1+L] \). Thus, the liquidation payoff to creditors is \( F-x \). The continuation payoff is \( q[F-x] < F-x \). Hence, the creditors liquidate the bank.

Case (ii): \( Lx < F-x < Hx \): In this case, \( Lx+x < F \), which means \( x < F/[1+L] \). Moreover, \( F < Hx+x \), which means \( x > F/[1+H] \). So, \( F/[1+H] < x < F/[1+L] \). The liquidation payoff to creditors is \( Lx \), whereas the continuation payoff is \( q[F-x] \). Creditors liquidate whenever:

\[
Lx > q[F-x]
\]

or

\[
x > \lambda F, \quad \text{where} \quad \lambda = \frac{q}{q + L}.
\]

Creditors continue if \( x \leq qF \left[ q + L \right]^{-1} \).
Case (iii): $F - x > Hx$: In this region, $x < F\sqrt{1 + H}$. The liquidation payoff of creditors is $\min(Lx, F - x) = Lx$. Since $x + xH < F$ implies that $x + xL < F$. The continuation payoff of creditors is $qHx$. Since $qH > L$, the creditors continue. Finally, since $qH > L$, all liquidation is inefficient.

Proof of Proposition 1: Using (7), (8) and (9), we can write (10) as:

$$V(F) = E(x) + qHE(x) - qH\int_{x}^{F} \left( \frac{x}{x_{\text{max}}} \right) dx + \int_{x_{\text{max}}}^{F} \left( \frac{Lx}{x_{\text{max}}} \right) dx$$

$$-\theta \left[ E(x - F) + qHE(x) \right]_{x > F} - \bar{W}$$

$$+\rho \theta \left[ qH(x - F) \right]$$

$$= [1 + qH] E(x) + qH\int_{x}^{F} \left( \frac{x}{x_{\text{max}}} \right) dx + \int_{x_{\text{max}}}^{F} \left( \frac{Lx}{x_{\text{max}}} \right) dx$$

$$-\bar{W} - [1 - \rho] \theta \left[ F - \frac{Lx}{x_{\text{max}}} \right] dx + qH\int_{x}^{F} \left( \frac{x}{x_{\text{max}}} \right) dx$$

$$= [1 + qH] E(x) - \frac{[qH - L]\left[ 1 - \lambda^{2} \right] F^{2}}{2x_{\text{max}}} - \bar{W}$$

$$- [1 - \rho] \theta \left[ \frac{[1 + qH]\left[ x_{\text{max}} - F \right] - \lambda^{2}}{2x_{\text{max}}} \right]$$

(A-2)

The optimal $F$, say $F^{*}$, must satisfy the first-order condition:

$$-[qH - L]\left[ 1 - \lambda^{2} \right] + [1 - \rho] \theta \left[ [1 + qH] F^{*} + x_{\text{max}} - 2F^{*} \right] = 0$$

(A-3)

Rearranging (A-3) yields (12).

Next, we verify the second-order condition for a unique global maximum. This condition is that

$$-[qH - L]\left[ 1 - \lambda^{2} \right] + [1 - \rho] \theta \left[ qH - 1 \right] < 0$$

(A-4)

Now let us see what conditions are needed for $F^{*} \in (0, x_{\text{max}})$. For $F^{*} < x_{\text{max}}$, we need:

$$\frac{[1 - \lambda^{2}] [qH - L]}{[1 - \rho] \theta} - [qH - 1] > 1$$

(A-5)

Substituting for $\theta^{*}$ from (4) and rearranging yields:

$$B < \frac{\Delta[1 - \lambda^{2}] [qH - L]}{q[1 - \rho]} = \overline{B}$$

(A-6)
Since the proposition states $B < \bar{B}$, we know $F^* < x_{\text{max}}$.

To ensure that $F^* > 0$, we need
\[
\frac{[1-\lambda^2][qH - L]}{[1-\rho]\theta^*} - [qH - 1] > 0,
\]
which is obviously satisfied for $B < \bar{B}$ (because then (A-5) holds). Moreover, it is also clear that since (A-5) holds, the inequality in (A-4) also holds.

\[\blacksquare\]

**Proof of Corollary 1:** Note that (12) shows that $\partial F^*/\partial \theta^* > 0$. Moreover, from (4) we know that $\partial \theta^*/\partial B > 0$. Thus, $\partial F^*/\partial B > 0$. From (12) it also follows immediately that $\partial F^*/\partial \rho < 0$. \[\blacksquare\]

**Proof of Proposition 2:** The (unconditional) ex ante probability of liquidation by bank $i$’s creditors (see (17)) is
\[
PRB_i \equiv G(F) - G(\lambda F)\left[\delta^i_l + \delta^i_r\right].
\]

Note that
\[
\frac{\partial PRB_i}{\partial F_j} = \left[x_{\text{max}}\right]^{-1} \left[1-\lambda \left[\delta^i_l + \delta^i_r\right]\right]
\]
\[> 0.\]

Similarly,
\[
\frac{\partial PRB_i}{\partial F_j} = \left[x_{\text{max}}\right]^{-1} \left[-\lambda F_j \left\{ \frac{\partial \delta^i_l}{\partial F} + \frac{\partial \delta^i_r}{\partial F} \right\}\right].
\]

Since $\partial \delta^i_l / \partial F_j < 0$ and $\partial \delta^i_r / \partial F_j < 0$, it follows that $\partial PRB_i / \partial F_j > 0$. \[\blacksquare\]

**Proof of Proposition 3:** Using (7), (8) and (9), we can write (20) as:
\[
V(F) = E(x) + \left[ \delta_1 H^- + \delta_2 H^+ \right] \int_0^{x_f} \left( \frac{x}{x_{\max}} \right) dx + \int_0^{x_f} \left( \frac{Lx}{x_{\max}} \right) dx + \left[ 1 - \delta_1 - \delta_2 \right] L \int_0^{x_f} \left( \frac{x}{x_{\max}} \right) dx + qH \int_{\delta_f}^{x_{\max}} \left( \frac{x}{x_{\max}} \right) dx - [1 - \rho] \theta \left\{ \int_{\delta_f}^{x_{\max}} \left( \frac{x - F}{x_{\max}} \right) dx + qH \int_{\delta_f}^{x_{\max}} \left( \frac{x}{x_{\max}} \right) dx \right\}
\]

(A-7)

Simplifying and rearranging terms, we can write (A-7) as:

\[
V(F) = -F^2 / 2x_{\max} \left[ qH - [1 - \rho] \theta'[qH - 1] - \left[ L + \lambda^2 \left\{ \delta_i' \left[ H^+ - L \right] - \delta_i' \left[ L - H^- \right] \right\} \right] \right] + [1 - \rho] \theta' F + \text{constant}
\]

(A-8)

where constant includes all the terms that do not contain \( F \). The first-order condition for the optimal best response of bank \( i \), \( F_i^o \), is (suppressing the argument \( F_i^o \)) is:

\[
-F_i^o \left\{ qH - [1 - \rho] \theta'[qH - 1] - \left[ L + \lambda^2 \left\{ \delta_i' \left[ H^+ - L \right] - \delta_i' \left[ L - H^- \right] \right\} \right] \right\} + [1 - \rho] \theta' x_{\max} = 0
\]

(A-9)

Rearranging (A-9) and simplifying yields (22).

The second-order condition is:

\[
-\left\{ qH - [1 - \rho] \theta'[qH - 1] - \left[ L + \lambda^2 \left\{ \delta_i' \left[ H^+ - L \right] - \delta_i' \left[ L - H^- \right] \right\} \right] \right\} < 0
\]

(A-10)

To ensure \( F_i^o < x_{\max} \), (22) indicates that we need

\[
qH > [1 - \rho] \theta' qH + L + \lambda^2 \left\{ \delta_i' \left[ H^+ - L \right] - \delta_i' \left[ L - H^- \right] \right\}
\]

(A-11)

Using (4) to substitute for \( \theta' \) and rearranging gives us:

\[
B < \frac{\Delta}{[1 - \rho] q} \left[ \left\{ qH - L - \lambda^2 \left\{ \delta_i' \left[ H^+ - L \right] - \delta_i' \left[ L - H^- \right] \right\} \right\} \right] = \tilde{B}
\]

(A-12)

Note that as long as \( B < \tilde{B} \), the second-order condition (A-10) will also be satisfied. Moreover, satisfaction of (A-10) also guarantees that \( F_i^o > 0 \). Thus, \( F_i^o \in (0, x_{\max}) \).
To obtain (23), note that with symmetry across banks \( i \) and \( j \), both banks have exactly the same best response conditional on a given leverage choice by the other bank. Substituting \( F_i^0 \left(F_i^0\right) = F_j^0 \left(F_j^0\right) \) in (22), we can write
\[
F^* = \frac{1}{a + bF^*}
\]  
(A-13)
where \( a \) and \( b \) are defined in Proposition 3. Solving (A-13) as a quadratic equation yields (23). Note that we need \( b > 0 \), which is true if \( \beta < [2 - \gamma]^{-1} \). This is guaranteed by \( \beta \leq 0.5 \).

**Proof of Corollary 2:** Write \( F_i^* \left(F_i^0\right) \) as
\[
F_i^* \left(F_i^0\right) = \frac{x_{\text{max}}}{qH - \left[L + \delta^2 \left[H^+ - L\right] - \delta \left[L - H^-\right]\right] - \left[qH - 1\right]}  
\]  
(A-14)
It is clear that, holding \( F_i^0 \) fixed, \( \partial F_i^* \left(F_i^0\right)/\partial \theta^* > 0 \). Since \( \partial \theta^* / \partial B > 0 \), it follows that \( \partial F_i^* \left(F_i^0\right)/\partial B > 0 \).

That is, each bank’s best response is increasing in its own manager’s \( B \). In a symmetric Nash equilibrium therefore, \( \partial F^* / \partial \theta^* > 0 \). To establish the sign of \( \partial F^* / \partial \beta \), note that the sign of \( \partial F_i^* \left(F_i^0\right)/\partial \beta \) depends solely on the sign of \( \partial A / \partial \beta \) where \( A = \delta \left[H^+ - L\right] - \delta \left[L - H^-\right] \). Now,
\[
\partial A / \partial \beta = -\left[H^+ - L\right] \left[\left(1 - \gamma\right) \left(1 - \delta\right)\right] - \left[1 - \delta\right] \left[1 - \gamma\right] \left[L - H^-\right] < 0.
\]
Since \( \partial F_i^* \left(F_i^0\right)/\partial A > 0 \), we have \( \partial F_i^* \left(F_i^0\right)/\partial \beta < 0 \). In a symmetric Nash equilibrium therefore,
\[
\partial F^* / \partial \beta < 0.
\]
Moreover, it can also be verified that \( \partial F_i^\rho \left(F_i^0\right)/\partial \rho < 0 \) and \( \partial F^\rho / \partial \rho < 0 \).

**Proof of Proposition 4:** The \( F_i \) that maximizes \( V_i + V_j \) is the one that maximizes:
\[
-\left\{F_j^3 D(F_j)/2x_{\text{max}}\right\} + \left[1 - \rho\right] \theta^* F_i + \text{constant},
\]
\[
-\left\{F_j^3 D(F_j)/2x_{\text{max}}\right\} + \left[1 - \rho\right] \theta^* F_j + \text{constant,}
\]  
(A-15)
where
The first-order condition for the regulator’s optimal choice of $F_i$, given $F_j$, is:

$$-\left\{F_i D(F_i) / x_{max}\right\} + [1 - \rho] \theta_i + \left\{F_j^2 / 2 x_{max}\right\} \left\{\lambda^2 \left[\partial A_i / \partial F_i\right]\right\} = 0 \quad \text{(A-16)}$$

where

$$A_i = \delta_i(F_i)[H^+ - L] - \delta_i(F_i)[L - H^-]$$

Rearranging (A-16) yields:

$$F_i = \frac{[1 - \rho] \theta_i x_{max} + \left\{F_j^2 / 2 x_{max}\right\} \left\{\lambda^2 \left[\partial A_i / \partial F_i\right]\right\}}{D(F_i)} \quad \text{(A-18)}$$

Comparing (A-18) and (23), we see that $F_i < F_i^*$ IFF $\partial A_i / \partial F_i < 0$. Now, using $H^+ - L = L - H^-$, we can write $A_i$ as:

$$A_i = \left[H^+ - L\right] \left[\delta_i(F_i) - \delta_i(F_i)\right]$$

$$= [1 - \delta_i][1 - \gamma][1 - \beta - \beta \{1 - \gamma\}] + [1 - \beta] \gamma$$

Thus,

$$\partial A_i / \partial F_i = -[1 - \gamma][1 - \beta - \beta \{1 - \gamma\}] \frac{\partial \delta_i}{\partial F_i}$$

$$= -[1 - \gamma][1 - \beta - \beta \{1 - \gamma\}] \frac{[1 - \lambda]}{x_{max}}$$

$$< 0 \text{ since } 1 - \beta > \beta \{1 - \gamma\} \text{ due to } \beta \leq 0.5.$$ 

Thus, $F_i < F_i^*$.

To verify that $F_i$ is a unique global maximum, let us examine the second-order condition:

$$-D(F_i) / x_{max},$$
which is clearly negative.

To obtain (26), substitute $F_j = F^*$, where $F^*$ is taken from (23), into (A-16), so that, the social optimum can be written as:

$$
\hat{F}_i = \frac{1 - D \left[ F_j \right]}{a + bF_j}
$$

(A-19)

where

$$
D = \frac{\lambda^2 [1 - \lambda] [1 - \gamma] [1 - \beta (2 - \gamma)]}{2 [1 - \rho] \theta [x_m]^2}
$$

Using symmetry, we write $\hat{F}_i = \hat{F}_j = \hat{F}$, we can express (A-19) as a quadratic equation in $\hat{F}$ to obtain (26).

Note that $D > 0$ whenever $\beta < [2 - \gamma]^{-1}$, which holds because $\beta \leq 0.5$.  

Proof of Proposition 5: The proof of capital structure irrelevance is obvious (note that (27) is independent of $F$). Comparing the objective functions in (20) and (27), we see that if we ignore $c$, then (20) can be made equivalent to (27) by setting $F = 0$. But the $F$ that maximizes ex ante liquidity in (20) is strictly positive. Hence, for $c > 0$ small enough, the optimized value of $V(F)$ in (20) exceeds the $V_{LOLR}$ in (27).

Proof of Lemma 3: In any state, the bank’s shareholders will be willing to buy out the creditors at a price $L$ (which makes the creditors indifferent between liquidating and not liquidating) if their net payoff after buying out the creditors and relying on compensation-based incentives exceeds the price, $L$, of buying out the creditors. That is, if

$$
[1 - \theta^*] qE(H|\text{liquidation threat}) > L
$$

(A-20)

where $E(H|\text{liquidation threat})$ is the expected value of $H$ conditional on creditors threatening liquidation, and we have assumed that the $x$ realized at $t=1$ is used to pay creditors.

Since $\partial \theta^*/\partial B > 0$, it is clear that (A-20) will not hold for $B$ sufficiently large.

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**Proof of Proposition 6:** Since the LOLR credibly precommits to bail out any bank that has suffered a systematic asset-value impairment shock, no bank can fail due to this shock. Moreover, since the bailout is secret, no other bank’s creditors learn that the asset-value impairment shock has occurred if they did not receive the signal. Finally, since no bank is liquidated due to the asset-value-impairment shock and no liquidations based on $x$ are prevented, the bank’s leverage-choice problem is the same as in (10), except that $H$ is replaced by $H^\ast$. Hence, (28) is the same as (12) except that $H$ is replaced by $H^\ast$. ■
**Figure 1: Sequence of Events**

<table>
<thead>
<tr>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Bank needs liquidity $I$ for project.</td>
<td>• Project throws off cash flow of $x$. Observable only to each bank’s managers, shareholders and creditors.</td>
<td>• Project yields terminal cash flow $Hx$ with probability $q$ and $0$ with probability $1 - q$ if good, and $Hx$ with probability $p$ and $0$ with probability $1 - p$ if it is the private-benefit project.</td>
</tr>
<tr>
<td>• Bank chooses equity or debt financing.</td>
<td>• Manager makes choice of good or private-benefit project after observing $x$.</td>
<td>• Creditors may expend $c$ to discover the manager’s project choice. They can enforce a different project choice if they so desire.</td>
</tr>
<tr>
<td>• Debt face value is $F$.</td>
<td>• Creditors may expend $c$ to discover the manager’s project choice. They can enforce a different project choice if they so desire.</td>
<td>• Creditors decide whether to liquidate the bank or continue after observing $x$ and systematic liquidity shock.</td>
</tr>
<tr>
<td>• All cash flows are pledgeable.</td>
<td>• Creditors may expend $c$ to discover the manager’s project choice. They can enforce a different project choice if they so desire.</td>
<td>• Creditors decide whether to liquidate the bank or continue after observing $x$ and systematic liquidity shock.</td>
</tr>
<tr>
<td>• Shareholders choose whether to give the manager a share $\theta$ of the interim cash flow ($x$) and of the terminal payoff and in which interim cash flow ($x$) states.</td>
<td>• Creditors decide whether to liquidate the bank or continue after observing $x$ and systematic liquidity shock.</td>
<td>• If the bank is not liquidated and $x &gt; F$, the manager receives $\theta x$.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• The LOLR may intervene to provide liquidity if creditors threaten to liquidate.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• All payments are made to managers, shareholders and (if any outstanding, to) creditors.</td>
</tr>
</tbody>
</table>
Figure 2
The Continuation/Liquidation Decision of Creditors

- Creditors fully paid off
- No creditor investigation
- Compensation incentives provided by equity ensure choice of good project.
- Continuation

- Liquidation by creditors
- Bank’s project choice is irrelevant

- Market discipline of debt results in choice of good project by the bank
- Continuation

$X_{\text{max}}$

$F - \frac{c}{\Delta}$

$F/[1+L]$

$\lambda F$

$F/[1+H]$

$X_{\text{min}}$
Figure 3

The Difference Between the Privately Optimal Leverage and the Socially Optimal Leverage

A. As a Function of the Private Benefit $B$

$F^o - \hat{F}$

B. As a function of $\gamma$, the Visibility of the Systematic Shock to Creditors

$F^o - \hat{F}$
C. As a Function of $\beta$, the Probability of the Systematic Shock

\[ F^o - \hat{F} \]

D. As a Function of the Manager’s Discount Factor, $\rho$.

\[ F^o - \hat{F} \]
REFERENCES


Hayek, Friedrich, *The Road to Serfdom*, University of Chicago Press, 1944.


