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Anatomy of a Meltdown:
The Risk Neutral Density for the S&P 500 in the Fall of 2008

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ABSTRACT

We examine how the risk neutral probability density (RND) for the S&P 500 behaved from minute to minute during the fall of 2008, compared to earlier periods. The RND extracted from real-time bid and ask quotes for index options provides an exceptionally detailed view of how investors' expectations about returns and risk responded under extreme market stress, as intraday volatility increased to a level five times higher than it had been two years earlier. Due to arbitrage, the mean of the RND remains closely tied to the market index, but its fluctuations are much different. While the S&P index shows moderate positive autocorrelation, there is consistently large negative autocorrelation in both the RND mean and standard deviation over short intervals. Moreover, we find a strong pattern in how the shape of the RND responds to changes in the level of the stock index: The middle portion of the RND is more volatile, amplifying moves in the index by as much as a factor of 1.5, or more in some cases. This phenomenon increased in size during the crisis and, surprisingly, was stronger for up moves than for down moves in the market.

Keywords: risk neutral density; implied probabilities; stock index options; 2008 financial crisis

JEL Classification: G13, G14, D84
I. Introduction

The financial crisis that struck with fury in the fall of 2008 began in the credit market and particularly the market for mortgage-backed collateralized debt obligations in the summer of 2007. It did not affect the stock market right away. In fact, U.S. stock prices hit their all-time high in October 2007, when the S&P 500 index reached 1576.09. Although it has since been determined that the economy entered a recession in December 2007, the S&P was still around 1300 at the end of August 2008.

Over the next couple of months, it would fall more than 500 points, and would trade below 800 by mid-November. The "meltdown" of fall 2008 ushered in a period of extreme price volatility, and general uncertainty, such as had not been seen in the U.S. since the Great Depression of the 1930s. Not only were expectations about the future of the U.S. and the world economy both highly uncertain and also highly volatile, the enormous financial losses sustained by investors sharply reduced their willingness, and their ability, to bear risk.

Risk attitudes and price expectations are both reflected in the prices of options and are encapsulated in the market's "risk neutral" probability distribution. The risk neutral density (RND) is the market's objective estimate of the probability distribution for the level of the stock index on option expiration date modified by investors' risk aversion when the objective probabilities are incorporated into market option prices. In this paper, we examine how the RND was affected during this extraordinary period.

Thirty years ago, Breeden and Litzenberger (1978) showed how the RND could be extracted from the prices of options with a continuum of strikes. There are a number of significant difficulties in adapting their theoretical result for use with option prices observed in the market, but Figlewski (2009) develops a methodology that performs well. We will apply it to a new and extraordinarily detailed dataset of real-time best bid and offer quotes in the consolidated national options market, which allows a very close look at the behavior of the RND, essentially in real-time.

This is the first study of the "instantaneous" RND for the U.S. stock market and we uncover a number of interesting results, not just about the "meltdown" period. In periods of both high and low volatility, we find the RND to be strongly left-skewed, in sharp contrast to the lognormal density assumed in the Black-Scholes model. Also, to avoid profitable arbitrage between markets, the RND mean should equal the forward level of the index, and we find that they are very close to each other, even at the shortest time intervals during periods of extreme market disruption. However, the RND is always much more volatile than the forward index and it exhibits very strong negative autocorrelation. We explore several hypotheses that might help explain this seeming anomaly, and are able to eliminate some that are based in one way or another on bad data.
We offer a potential explanation, based on the process of marketmaking in options, that is consistent with the data, although we are not able to test it rigorously in this paper. When an option trade is initiated by an outside investor, a marketmaker counterparty first takes the position into his/her inventory, and then moves the posted quotes on nearby contracts in order to attract orders that will allow the risk to be offset. The initial trade thus produces a deformation of the RND, but as the orders arrive in the market and the marketmaker rebalances his/her exposure, the RND reverts back toward its previous shape.

The next section offers a brief review of the literature on using risk neutral densities extracted from option prices to look at financial market events. Section III gives an overview of the procedure for constructing RNDs. Section IV describes the real-time S&P 500 index options data used in the analysis. In Section V, we present summary statistics that illustrate along several dimensions how sharply the behavior of the stock market changed in the fall of 2008, as reflected in the risk neutral density. Section VI looks more closely at how the minute-to-minute changes in the different quantiles of the RND are related to fluctuations in the level of the stock market (the forward index). Section VII offers a summary of our results. The Appendix provides a more detailed exposition of the RND-fitting methodology.

II. Literature

There is a wide and continuously evolving literature on the extraction and analysis of option-implied risk-neutral distributions. Much of the literature has focused on identifying the best methodologies for extracting the RND. Jackwerth (2004) and Figlewski (2009) give detailed reviews of the prior literature on extracting option-implied distributions.

Less has been done using the RND to explore the market’s probability beliefs about specific stock market events. One of the first was Bates (1991), who utilized S&P 500 futures options during the period leading up to the 1987 market crash to determine whether the market predicted the impending crash. He concluded that the possibility of a crash was anticipated in the options market as much as two months in advance. Subsequently, Bates (2000) found that after the 1987 crash, the RND from S&P 500 options consistently over-estimated left tail events. Rubinstein (1994) and Jackwerth and Rubinstein (1996) arrived at a similar conclusion in their analysis of S&P options, finding that the option-implied probability of a significant decline in the index was much higher in the post-crash period.

Lynch and Panigirtzoglou (2008) summarize results from the literature and present stylized facts and summary statistics for RNDs extracted from S&P index options and variety of other equity and non-equity options around the world, for the 1985-2001 data period. Their overall assessment is that RNDs respond to market events but are not very useful for forecasting them. Gemmill and Saflekos (2000) used RNDs extracted from FTSE options to study the market’s expectations ahead of British elections, while Liu et
al. (2007) obtained real-world distributions from option-implied RNDs and assessed their explanatory power for observed index levels relative to historical densities. The forecasting ability of index options was tested in the Spanish market by Alonso, Blanco, and Rubio (2005), and in the Japanese market by Shiratsuka (2001).

Much of the work in this area has focused on implied volatilities (IVs). Among papers that explicitly analyze IVs around financial crises, in addition to those already cited, Bhabra et al. (2001) examined whether index option IVs were able to predict the 1997 Korean financial crisis. Like Lynch and Panigirtzoglou (2008), their results suggested that the options market reacted to, rather than predicted the crisis. Malz (2000) provided evidence that option implied volatilities in a number of markets contain information about future large returns. Similarly, Fung (2007) found that option IV gave an early warning sign and performed favorably compared to other measures in predicting future volatility during the 1997 Hong Kong stock market crash.

It is important to note that the RND has a significant advantage over implied volatility because it is model-independent, while IV is nearly always extracted using a specific model. The ubiquitous "volatility smile" found for equity options is an artifact of using the Black-Scholes model in computing the implied volatilities. A different model would produce a different smile. The intrinsic dependence on the probability distribution assumed by a particular model makes it much harder to disentangle the effect of risk neutralization from investors' objective probability beliefs.

A large number of factors in addition to expected return volatility have been found to influence implied volatilities and higher moments of the RND. A strongly negative relationship between returns and implied volatility has been well-documented in myriad studies: implied volatility goes up when the market falls. Risk attitudes are also embedded in implied volatility. Whaley (2000) considers the S&P 500 implied volatility index (VIX) to be an "investor fear gauge" and other researchers have also treated it that way, including Malz (2000), Low (2004), Giot (2005) and Skiadopoulos (2004).

Other recent studies attempting to identify factors that affect the moments of the RND have focused on changing demand for options (Bollen and Whaley (2004)), information releases (Ederington and Lee (1996)), and transaction costs (Pena, Rubio, and Serna (1999)). Option-implied skewness has been shown to reflect investor sentiment (Han 2008, Rehman and Vilkov (2009)). Dennis and Mayhew (2002) also found risk-neutral skewness for individual equities to be driven by aggregate market volatility, firm systematic risk, firm size, and trading volume, while Taylor, Yadav, and Zhang (2009) documented a relationship between risk-neutral skew and firm size, systematic risk, market volatility, firm volatility, market liquidity and the firm’s leverage ratio.

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1. In 2003, the Chicago Board Options Exchange replaced its methodology for calculating the VIX implied volatility index using the Black-Scholes formula with a non-parametric procedure similar to what we use below. The purpose is to compute an estimate of the standard deviation of the risk neutral density for a constant maturity 30-day horizon.
III. Extracting the Risk Neutral Density from Option Prices

In the following, the symbols C, S, X, r, and T all have the standard meanings of option valuation: C = call price; S = time 0 price of the underlying asset; X = exercise price; r = riskless interest rate; T = option expiration date, which is also the time to expiration. P will be the price of a put option. We will also use \( f(x) = \text{risk neutral probability density function} \), also denoted RND, and \( F(x) = \int_{-\infty}^{x} f(z)dz \) = risk neutral distribution function.

The value of a call option is the expected value of its payoff on the expiration date T, discounted back to the present. Under risk neutrality, the expectation is taken with respect to the risk neutral probabilities and discounting is at the risk free interest rate.

\[
C = e^{-rT} \int_{X}^{\infty} (S_T - X)f(S_T)dS_T
\]

Taking the partial derivative in (1) with respect to the strike price \( X \) and solving for the risk neutral distribution \( F(X) \) yields

\[
F(X) = e^{rT} \frac{\partial C}{\partial X} + 1
\]

Taking the derivative with respect to \( X \) a second time gives the Risk Neutral Density function

\[
f(X) = e^{rT} \frac{\partial^2 C}{\partial X^2}
\]

In practice, we approximate the solution to (3) using finite differences. In the market, option prices for a given maturity \( T \) are available at discrete exercise prices that can be far apart. To generate smooth densities, we interpolate to obtain option values on a denser set of equally spaced strikes.

To estimate the probability in the left tail of the risk neutral distribution up to \( X_2 \), we approximate \( \frac{\partial C}{\partial X} \) at \( X_2 \) and compute \( F(X_2) \equiv e^{rT} \frac{C_3 - C_1}{X_3 - X_1} + 1 \). The probability in the right tail from \( X_{N-1} \) to infinity is approximated by,

\[
1 - F(X_{N-1}) \equiv 1 - \left( e^{rT} \frac{C_N - C_{N-2}}{X_N - X_{N-2}} + 1 \right) = -e^{rT} \frac{C_N - C_{N-2}}{X_N - X_{N-2}}.
\]

The approximate density \( f(X_n) \) is given by
Equations (1) - (4) show how the portion of the RND lying between \( X_2 \) and \( X_{N-1} \) can be extracted from a set of call option prices. A similar derivation yields equivalent expressions to (2) and (3) for puts:

\[
F(X) = e^{rT} \frac{\partial P}{\partial X}
\]

and

\[
f(X) = e^{rT} \frac{\partial^2 P}{\partial X^2}
\]

The Risk Neutral Density aggregates the individual risk neutralized subjective probability beliefs within the investor population. The resulting density is not a simple transformation of the true (but unobservable) distribution of realized returns on the underlying asset, nor does it need to obey any particular probability law. Obtaining a well-behaved RND from market option prices is a nontrivial exercise. There are several key problems that need to be dealt with, and numerous alternative approaches have been explored in the literature.

Many investigators (e.g., Gemmill and Saflekos (2000), Eriksson, et al (2009)) impose a known distribution on the data, either explicitly or implicitly, which constrains the RND in the region that can be directly extracted from market option prices and fixes its tail shape by assumption. For example, assuming Black-Scholes implied volatilities are constant outside the range spanned by the data (e.g., Bliss and Panigirtzoglou(2004)) forces the tails to be lognormal. Imposing a specific distribution can easily produce anomalous densities with systematic deviations from the observable portion of the RND. Density approximations, using a technique such as Gram-Charlier, can even have negative portions (see Eriksson, et al (2009)). Using the empirical portion of the RND but assuming a specific density for the tails can lead to discontinuities or sharp changes in shape at the point where the new tail is added on.\(^2\)

We adopt a more general approach by extracting the empirical RND from the data and extending it with tails drawn from Generalized Extreme Value (GEV) distributions. The GEV parameters are chosen to match the shape of the RND estimated from the market data over the portions of the left and right tail regions for which it is available. Figlewski (2009) reviews the methodological issues in fitting a well-behaved RND to real world

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\(^2\) The observable portion of the RND determines both the total probability in the tail and the density at the point the new tail must begin. The problem is that for the lognormal (or whatever density is chosen), matching both the total tail probability and the density at the connection point generally produces a sharp spike or kink, or even a discontinuous jump in the fitted RND at that point.
price data and develops a consistent approach that works well. We summarize the steps in the Appendix; the interested reader can refer to that article for full details.

IV. Data

The intraday options data are the national best bid and offer (NBBO) extracted from the Option Price Reporting Authority (OPRA) data feed for all equity and equity index options. OPRA gathers pricing data from all exchanges, physical and electronic, and distributes to the public firm bid and offer quotes, trade prices and related information in real-time. The NBBO represents the inside spread in the consolidated national market for options. Exchanges typically designate one or more "primary" or "lead" marketmakers, who are required to quote continuous two-sided markets in reasonable size for the options they cover, and trades can always be executed against these posted bids and offers.\(^3\)

The quoted NBBO bid and ask prices are a much more useful reflection of current option pricing than trades are. Because each underlying stock or index has puts and calls with many different exercise prices and expiration dates, option trading for even an extremely active index like the S&P 500 is relatively sparse, especially for contracts that are away from the money. However the NBBO is available at all points in time and is continuously updated for all contracts that are currently being traded. Whether a particular option is trading actively or not, marketmakers must constantly adjust their quotes on OPRA or risk being "picked off" as the underlying index fluctuates.

The stock market opens at 9:30 A.M. New York time. Options trading begins shortly after that, but it can take several minutes before all contracts have opened and the market has settled into its normal mode of operation. To avoid introducing potentially anomalous prices at the beginning of the day from contracts that have not yet begun trading freely, we start the options "day" for our analysis at 10:00 A.M. Note that S&P options trading takes place in Chicago, which is in a different time zone from New York. To avoid ambiguity, all times of day cited in this paper will refer to New York time.

We extract the NBBO's for all S&P 500 options of the chosen maturity from the OPRA feed and record them in a pricing tableau. The full set of current bids and offers for all strikes is maintained and updated whenever a new quote is posted. Every quote is assumed to remain a current firm price until it is updated. Our data set for analysis consists of snapshots of this real-time price tableau taken once every minute, leading to about 366 observations of the RND per day.\(^4\) The current index level is also reported in the OPRA feed, which provides a price series for the underlying that is synchronous with the options data.

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3 In the present case, S&P 500 index options are only traded on the Chicago Board Options Exchange, due to a licensing agreement.
4 The market closes at 4:00 P.M. but there are often prices that come in for a few minutes after that. We start the day at 10:00 A.M. (observation 31) and stop at 4:05 (observation 396) at the latest, giving us up to 366 trading minutes per day.
To examine risk neutral densities from the fall of 2008 and compare them against those from the fall of 2006 and 2007, we use options with December expirations. These are European options, so no uncertainty is introduced by early exercise. An extracted RND is therefore the market's continuously updated risk-neutralized estimate of the probability density for the opening index level on the third Friday of December. This means that the RND standard deviation shrinks as the forecast horizon telescopes downward over time. Where relevant, we adjust for this effect by converting values to a common time horizon, either annual or daily.

Bid and ask implied volatilities needed for fitting the RND are computed using Merton's continuous dividend version of the Black-Scholes model. The riskless rate and dividend yield data required for this and for computing forward values for the index come from OptionMetrics. We interpolate U.S. dollar LIBOR to match option maturity and convert it into a continuously compounded rate. The projected dividends on the index are also converted to a continuous annual rate.

We use the RND as a tool to examine the behavior of the S&P index options market during the financial crisis of Fall 2008. Given the sheer volume of data, we do not try to include every day in the sample. Our full sample consists of 39 days over a 3 year period. To provide a base for comparison, we examine the RND on 7 days in September and October 2006 and 7 from September-October 2007. The Fall 2008 sample is in two parts. We have RNDs for every day in the period from September 8 through September 30, when the crisis first broke open, and then for the next 8 Wednesdays, from October 1 through November 19, 2008. Each RND is fitted at index levels from 0 to 2000 in increments of 0.50, making 4001 values in each of over 15,000 minutes.

Figure 2 provides a graphic illustration of the differences in the RNDs across the three years included in the sample. To make the comparison easier, we have chosen three dates in early October, each with exactly 71 days to expiration. Under the same conditions these curves should have the same shape, with means equal to the forward index values for their respective expiration dates. Clearly that is not the case here. In October 2006, index volatilities and many other market risk measures like credit default swap spreads, were close to their all-time record lows. On October 4, the VIX index closed at 11.86 and a one step ahead forecast of annualized volatility from the GARCH model we will be using below was only 8.32%. October 10, 2007 featured substantially higher volatility (16.67 on the VIX and 12.61% from the GARCH model), but the key fact about the rightmost curve in Figure 2 is that within the next week the S&P 500 closed at 1565.15, its highest level in history. But by the time the meltdown was in full swing one year later on October 8, 2008, the S&P was down to 984.08, the VIX was 57.53 and the GARCH volatility forecast was 65.88%.

5 Following the convention adopted by OptionMetrics, we treat this as if the contracts actually expired at the close on the Thursday before expiration Friday. For example, on Wednesday of expiration week, we would treat the contracts as having one day to expiration.

6 The calculation of the index dividend yield is based on an assumption that put-call parity holds for these options, as described in OptionMetrics (2008, p.30).

7 Most of the dates chosen were Wednesdays, although data problems forced us to use other weekdays in a few weeks.
Table 1 presents some summary data for the data sample. The full sample contains 12,274 observations, drawn from 7 different months in 2006-2008. As Figure 2 illustrates, there was considerable variability in the location and shape of the risk neutral density prior to the 2008 crisis. The highest and lowest levels of the expiration day forward for the S&P 500 index in each subsample show how extreme the changes in some of these periods were. We compute RND standard deviations in terms of the index value on expiration day. As described above, this shrinks over time as maturity approaches. To allow comparisons against volatility estimates from a GARCH model and from the VIX volatility index, that are expressed as annualized percents, we annualize the RND standard deviation as a percent of the forward index level at each point in time.

The GARCH model estimates come from the standard GJR-GARCH specification of Glosten, Jagannathan and Runkle (1993).

\[
\begin{align*}
    r_t &= \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma_t^2) \\
    \sigma_t^2 &= C + \alpha \sigma_{t-1}^2 + \beta \varepsilon_{t-1}^2 + \gamma \varepsilon_{t-1}^2 \mathbb{I}_{\{\varepsilon_{t-1} < 0\}}
\end{align*}
\]

where \( r_t \) is the logarithmic return on date \( t \) and \( \mathbb{I}_{\{\varepsilon_{t-1} < 0\}} \) is an indicator function taking the value 1 if the return was negative in period \( t-1 \) and 0 otherwise. This specification allows the asymmetric response of volatility to positive and negative returns that has been found to be strongly supported in the data. The numbers reported here are for one day ahead forecasts, with the model parameters refitted for each date using S&P daily index returns over the previous 2000 days. The rightmost columns report the high and low closing values of the VIX volatility index over the days in each subsample. The VIX is actually computed in a somewhat similar way to our method of extracting the RND, so it is expected to be closely related to the RND standard deviation we calculate. However, the VIX is designed to focus on (synthetic) options with exactly 30 days to maturity, which is shorter than the time horizons for all but the last two of our sample dates.\(^8\)

We have annualized the RND and GARCH volatilities using the customary "square root of \( T \)" rule, as the CBOE already does in calculating the VIX. This does express the three different estimates on a comparable time scale, but it can lead to misleading comparisons. If returns are independent and variance follows a martingale, the volatility scales upward by the square root of the length of the horizon. But this will give an incorrect estimate of long-term volatility when variance is mean-reverting, as it is in most common models with stochastic volatility, including GARCH. Indeed, we consider this to be the best explanation of the pattern of volatilities shown in Figure 3.

Figure 3 plots the values of these three volatility measures for the 39 dates in our sample. In 2006-07, the RND standard deviation was the highest; the VIX was a little lower and

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\(^8\) For full details of how the VIX is now calculated, see Chicago Board Options Exchange (2003).
the GARCH volatility was the lowest of the three volatility estimates. This pattern is to be expected because the VIX and the RND standard deviation are risk neutral values while the GARCH model estimates the empirical volatility. Since investors dislike volatility, the volatility risk premium should push the risk-neutralized value above the empirical estimate. But in the fall of 2008, extremely volatile returns caused the GARCH forecasts to rise sharply, reaching values between 4 and 5 percent per day. The pattern of 2006-07 was reversed, with the GARCH volatility now regularly higher than the VIX and the VIX higher than the RND volatility. In a time of unusually high volatility, this pattern is consistent with mean-reversion in the variance process: The one day ahead forecast (GARCH) is above the predicted average variance over the next 30 days (VIX) which is, in turn, above the average expected through option expiration (RND). In the fall of 2008, the force of expected mean-reversion outweighed the volatility risk premium.9

V. The Properties of the Risk Neutral Density Before and During the Meltdown

September 2008 was when the crisis hit in force. Our data for this period begin on September 8, the day after Fannie Mae and Freddie Mac were taken over by the Federal government. The following Monday, September 15, Lehman Brothers declared bankruptcy and Merrill Lynch unexpectedly sold itself to Bank of America. Two weeks later, on Monday, Sept. 29, the U.S. House of Representatives shocked the financial markets by voting down the first comprehensive TARP bailout plan. When the news hit the market shortly after 1:30 P.M. that day, the S&P 500 index responded by falling more than 4.8% in two hours.

By October, the crisis had spread to many other markets throughout the world. The stock index continued to fall and trading was marked by extreme volatility, such as had not been seen since the 1930s. On 55% of the trading days in October and November 2008, the index moved more than 3% up or down (corresponding to annualized volatility in excess of 47%). Interestingly, while it is well-known that the market tends to move faster and further on the downside, in this extraordinary period sharp moves to the upside were just as common. On the two days with the largest price changes in October, the market rose more than 10%.

The Shape of the RND and its Connection to the S&P Forward Index Level

Table 2 presents a variety of summary statistics regarding the behavior of the S&P 500 index and the RND during several periods, both before and during the financial

9 One might have expected GARCH model estimates to lag behind actual market volatility when it increased so rapidly in the fall of 2008, but recent work by Brownlees, Engle and Kelly (2009) shows that this intuition does not hold: GARCH did as well in tracking market volatility during the Meltdown as in earlier calmer periods.
meltdown. The first two columns cover the 14 days in 2006 and 2007 that we use for comparison. The next three columns summarize the results for September, October, and November 2008, respectively.

Analysis of the RND at one-minute intervals provides information about the shape and behavior of the "instantaneous" risk neutral density and allows us to address several important questions, including the following. (1) Arbitrage between index options and the underlying stocks should tie the RND to the current level of the stock index. How closely are they connected at the shortest time intervals? Was the relationship between prices in the two markets weakened by the financial crisis? (2) Under the assumptions of the Black-Scholes model, the RND is lognormal. How do the moments of the observed density compare to those from a lognormal density? Did the shape of the RND change during the crisis period? (3) There is extensive evidence that the empirical returns distribution for the S&P index has fatter tails than a lognormal. Our procedure of matching GEV tails to the directly observed portion of the RND entails estimating a "tail-shape" parameter \( \xi \) (see equation (A2)). Are the estimated tails of the instantaneous RND fat, Gaussian or thin on average? Are the left and right tails the same or different? Did the tail shapes change in the fall of 2008?

The first two lines in Table 2 show the average level of the S&P cash index and its forward value over all of the 1-minute intervals in each subperiod. The forward is defined as

(8) \[ F_t = S_t e^{(r_t-d_t)(T-t)} \]

where \( F_t \) is the forward level of the index, \( S_t \) is the current spot index, \( r_t \) is the riskless interest rate for the period from date \( t \) to expiration at date \( T \) and \( d_t \) is the annual dividend yield on the index.

In equilibrium, the possibility of arbitrage between index options and the underlying stock index portfolio should enforce equality between the expected value for the index level at option expiration under the risk neutral distribution and the forward level of the index in the spot market. The next section of the table provides information about that relationship. In all subperiods, the RND mean is lower than the forward on average, but the difference is tiny, only a few basis points. The third line in this section shows that the two values are rarely very far apart. The root mean squared difference between them is on the order of only 5-10 basis points. The strength and stability of the arbitrage relationship between these markets is supported by the fact that neither measure of pricing discrepancy appears to have been much altered in the meltdown.

Turning to the higher moments of the RND, we report two measures of the average dispersion of the RND around its mean. The first line in this subsection shows that the average RND standard deviation rose sharply from 2006 to 2008, peaking in October 2008. However, simple comparison of these numbers is misleading, because the RND standard deviation is a function of both the level of the index and the time to option expiration.
The next line expresses the RND standard deviations in a more comparable way. To adjust for different levels of the index across the subperiods, the standard deviation is reported as a percent of the RND mean. Then, because the RND collapses around the forward index level as expiration approaches, the raw figures on percent standard deviation are "annualized" by multiplying them by \( \sqrt{\frac{365}{\text{days to option maturity}}} \). Expressed in this way, we see that RND standard deviation was 4 to 5 times bigger by the end of the sample than in 2006, and November 2008 was more extreme than October.

The final two lines in this section show the risk neutral skewness and kurtosis. Strongly inconsistent with lognormality, but as expected from results in the literature, the RND is highly left-skewed. Also, the kurtosis values are all above 3.0, indicating fatter tails than the normal, although it is difficult to interpret kurtosis for an asymmetrical distribution. Interestingly, both of these higher moments of the RND went down in size during the crisis, meaning the risk neutral density looked considerably more like a normal distribution at the end of our sample than at the beginning. Note, however, that the RND calculated here is defined over prices rather than returns. Under the Black-Scholes assumption of lognormality, the density in price space should be lognormal, i.e., positively skewed. Finding it close to normal here does not indicate that the RND became consistent with Black-Scholes.

Lastly, the table reports average values of the estimated tail shape parameters for the two GEV tails appended to the density. In the early periods, and as we have found with other daily and intraday datasets, the tails of the RND are quite different from Gaussian and also different from each other. The left tail generally has a positive estimated tail parameter \( \xi \), meaning a fatter tail than the normal, and the right tail has a negative \( \xi \), implying a finite tail that does not extend out to positive infinity. These properties are both counterintuitive. Since the index can not go below 0, the left tail of the distribution must be bounded from below, so it can not extend to negative infinity. There is no such bound on the right tail, however. These average tail parameters show the effect of the market's risk neutralization of the expected empirical density. While the right tail stayed about the same, the shape of the left RND tail changed considerably during the crisis of 2008, becoming truncated to an even greater extent than the right tail. It may be that once the market had fallen so far, investors became more aware of the lower limit for the level of a stock index.

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10 We have already mentioned the potential problems in using the "square root of T rule" to annualize volatility estimates that apply to shorter periods when volatility is time-varying. We justify the procedure here in two ways. First, some adjustment is necessary if these standard deviations are to be compared at all, and the square root of T approach is both well-established in the literature and much better than doing nothing. Second, and more persuasive, results from unpublished research we have conducted on daily S&P 500 RNDs strongly support the hypothesis that on average the RND standard deviation does, in fact, go down over time at a rate proportional to the square root of the remaining time to expiration.

11 Another possible explanation for the change in tail shape has to do with the way puts were priced at the very lowest part of the range of available strikes during the meltdown period. Bid prices for those deep out of the money puts were unusually high, so the density in that range has to be high, as well. But the total probability in the RND's left tail is determined by option prices in its observable portion. Having a very
Short Run Dynamics of the RND and the S&P Forward Index

The results presented in Table 2 on the shape of the risk neutral density and its arbitrage-based connection to the underlying stock market showed that the RND mean and the forward level of the current index in the cash market were virtually identical on average and the differences between them were small. Table 3 focuses on the intraday variability of the forward index level and the RND. This illustrates the change in their behavior during the meltdown period from another dimension, and reveals that despite the close connection between the levels of the index forward and the instantaneous RND shown in Table 2, their short run dynamics are surprisingly different.

The first line in Table 3 reports the average daily trading range for the forward index. The range contains a substantial amount of information about price variability. Also, it incorporates the effect of short term return correlation, which is not measured in volatility estimated from one-minute returns, so it can give a somewhat different picture of intraday price risk. Remarkably, in September and October 2006, on average the forward traded over a range well under 1% during the day. This rose to a more typical range of about 1 1/4 percent in September-October 2007. But in the fall of 2008, the daily range widened out to the point that the index fluctuated over a range of about 5% on an average day.

Return volatility is the standard measure of price fluctuation for an option's underlying asset. Estimating realized volatility from intraday data requires correcting for the effects of market microstructure noise and non-diffusive price jumps. We do not attempt to deal with those issues here, and simply report the volatility computed as the standard deviation of log returns over 1-minute intervals in basis points. The differences in intraday volatility of the S&P index forward across the subperiods are striking: minute-to-minute volatility was more than seven times higher in October 2008 than in the fall of 2006.

The third line in the table reports the autocorrelation in the one-minute returns on the index forward. An informationally efficient market should exhibit no serial correlation, but given the way the index is constructed it will always contain some stale prices, so it is common to find a small amount of positive serial correlation in returns observed at short intervals. Here, values of 0.062 in the fall of 2006 and 0.032 in the fall of 2007 for measured one-minute autocorrelation seem quite reasonable. Interestingly, autocorrelation did not rise in the financial crisis, despite the market disruption.

Turning our attention to the mean of the RND, we find that it behaves very differently from the S&P forward. It is much more volatile minute-to-minute than the forward.

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12 See Garman and Klass (1980) for example.
13 See, for example Andersen, et al (2003).
14 We do not put much faith in the point estimate of -0.009 for November 2008, since there were only three days in the sample for that month.
index, even during the earlier periods. In fact, the RND was more than twice as volatile as the forward index during 2006 and 2007. During the crisis period, the RND mean was still substantially more volatile than the forward index, but the ratio dropped considerably. The explanation for the difference in measured volatility is that, in sharp contrast to the forward index, the RND mean is highly negatively autocorrelated.

The final section of Table 3 examines the 1-minute percent changes of the RND standard deviation. We find even stronger negative autocorrelation at short intervals than for the RND mean. The volatility of RND volatility seems extraordinarily high, and unlike the volatility of the RND mean, it does not go down during the fall of 2008.

**Possible Explanations for Negative Autocorrelation in RND Moments**

What accounts for the striking negative serial dependence in the mean and standard deviation of the RND? Although we will not be able to settle this question here, we can suggest a few hypotheses and are able to offer some evidence against a couple of the more obvious candidates.

The first hypothesis is that negative serial correlation is commonly observed in very short horizon returns computed from transactions data, but it is a market microstructure artifact of the marketmaking process. Marketmakers quote bid and ask prices that evolve fairly slowly and smoothly, but transactions typically bounce between trades at the bid and the ask. The measured volatility of transactions prices at very short intervals is therefore much higher than at longer intervals and there is substantial negative autocorrelation. But that explanation can not apply here because there is no "bid-ask bounce" in our data, which come from the slowly moving bid and ask quotes, not from trade prices.

A related hypothesis is that our method for constructing the RND includes use of some stale prices. Perhaps the fact that quotes can not all be updated simultaneously induces artificial autocorrelation in our data, because whenever we take a snapshot of the market, some of the quotes will be fresher than others.\(^{15}\) Our empirical evidence rules this explanation out also. Delay in updating posted quotes when new information arrives should produce positive, not negative, measured serial correlation. Innovations arrive randomly but their impact on price quotes gets spread out over two or more time periods, causing positive correlation in the changes.

The same counterargument applies to the hypothesis that it is investors who are sluggish in adjusting their expectations in response to the arrival of new information. Again, a

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\(^{15}\) Some staleness in the data is unavoidable, due to the fact that quotes must be updated one at a time. But we expect the problem to have very little effect on our RND estimation. The options market moves so fast that these updates are entered automatically by computer (under a trader's control, of course), with the result that the lags involved are in milliseconds. Marketmakers have to update their quotes as rapidly as possible, to protect themselves against the non-marketmaker traders using computers that look for and attempt to profit from short-lived pricing anomalies in the posted quotes.
market that was informationally inefficient in this way would exhibit positive rather than negative autocorrelation.

Negative autocorrelation will arise from a data error, such as a price quote that is reported for the wrong option. This will cause a distortion in the RND, which is then reversed whenever a new quote that is not erroneous is posted for the option involved. Perhaps the explanation is that there are numerous such errors in our data.

These hypotheses all turn on there being price quotes in the data sample that differ from investors’ true valuations for those options given current information. Autocorrelation arises when the bad quotes are corrected. One distinguishing feature of "bad data" explanations is that measured autocorrelation should diminish rapidly when changes over longer intervals are considered. As soon as the erroneous data is overwritten, or the posted quotes are updated that correctly incorporate the new information, there should be no further serial correlation.

To explore this point further, Table 4 reports autocorrelations for changes in the mean and the standard deviation of the risk neutral density measured over different time intervals, from 1 minute to 30 minutes. We computed higher order autocorrelations with up to 25 lags, but the results beyond the first few lags showed nothing noteworthy so only the first 5 are displayed in the table. We only consider non-overlapping intervals within the same trading day, and continue to eliminate observations prior to 10:00 A.M. This greatly reduces sample size for the longer intervals. For example, while there are 14,196 observations of first order 1-minute changes, there are only 12 non-overlapping 30-minute intervals between 10:00 A.M. and 4:00 P.M, which translates to a total of 273 data points in our sample for measuring the 5th order autocorrelation of 30-minute changes.

Table 4 shows strong and highly significant first order negative correlations at the one-minute interval of -0.286 for the RND mean and -0.476 for the standard deviation. First order autocorrelation is large and highly significant out to 15-minute intervals for the mean, and over all of the intervals considered for the standard deviation. While the 20- and 25-minute values are not significant for the mean, at 20 minutes the second order autocorrelation is a significant -0.116. Except for odd entries here and there, there is little evidence of strong autocorrelation beyond one lag.

The amount of autocorrelation in the RND mean diminishes from -0.286 at the 1-minute interval to around -0.10 for 5-minutes, but it remains at about that level after that, except for the 25-minute estimate (and, arguably, the 20-minute value, if one ignores significantly negative correlation at the 2nd lag). It certainly does not appear consistent with the positive serial correlation for the S&P index forward reported in Table 3. This behavior is even stronger for the RND standard deviation, for which all of the first order autocorrelations are highly significant and highly negative, with the smallest in size being about -0.4.
The RND mean and standard deviation are computed over the whole range of prices, including those in the tails where we have no data from the market. A possibility that needs to be checked is whether the negative autocorrelation could somehow be an artifact of the tail-fitting procedure. This issue can be addressed easily by examining autocorrelation for quantiles of the RND rather than its moments. The middle portion of the curve is extracted directly from option market prices; it is not affected by the tail-fitting procedure.

Table 5 reports autocorrelations for lags 1 to 3 for the 25th, 50th (the median), and 75th percentiles of the RND. The results tell the same story as Table 4, with nearly all first order autocorrelations being strongly negative and the size of the coefficients decaying quite slowly even as the interval expands from 1 to as long as 30 minutes. In particular, the 75th percentile show significant negative autocorrelation at every differencing interval.

The unexpected short run behavior of the RND documented in Tables 4 and 5 is inconsistent with explanations that rest on bad data or sluggish investors. An alternative hypothesis might be offered, rooted in behavioral irrationality. The pattern we are seeing is what one would expect if investors in the index options market were basically skittish and subject to waves of irrational exuberance and pessimism, but the price effects of their speculative excesses were restrained by arbitrage with the more stable market for the underlying. Incoming information would have a bigger impact in the derivatives market than the cash market, producing price changes that would tend to overshoot the change in the cash market and then partially reverse as the system returned to equilibrium.

Believers in efficient markets will be very uncomfortable with such an explanation based on investor irrationality. We are not able to rule it out at this point, but we are also quite loath to accept it. Instead, our final hypothesis is much more consistent with theories of rational asset pricing, as well as with our understanding of price formation in real world markets, than those we have considered so far. Under this hypothesis, strong negative serial correlation in the RND mean and other moments is a natural result of rational marketmaking in the derivatives market.

To see the argument easily, consider the following scenario. Starting from a position of market equilibrium in which marketmakers have hedged their current option holdings to their target levels of risk exposure, imagine that an outside investor then sells a large number of S&P calls with some particular exercise price, say 1000. The order is filled by marketmakers who take the contracts into their inventory temporarily. This alters their aggregate risk exposure and they will immediately begin trying to rebalance their positions. Ask quotes on 1000-strike calls will be lowered in order to attract buyers for the excess contracts, and bids will be reduced to avoid buying any more. Bids and offers for other options with exercise prices close to 1000 will be adjusted commensurately. This quote revision process will show up as a change in the risk neutral density.

The marketmakers' inventory imbalance will be redressed over time, as the new price quotes attract trades from outsider investors that allow them to lay off their unwanted
risk. As this happens, the quotes for the affected options will tend to revert towards their previous levels relative to the rest of the options market, which will produce negative autocorrelation in the RND.

How long it takes to offset the initial impact of a trade will depend on a number of things, including its size, the marketmakers' perceptions of how likely it is that the trade came from an information trader rather than a noise trader, how frequently that particular contract and similar ones trade in the market, how eager marketmakers are to avoid unbalanced risk exposure, and what other hedging vehicles they may have available. In a recent paper, Gârleanu, Pedersen, and Poteshman (2009) have proposed a model of options marketmaking in this spirit, and they provide convincing empirical evidence to support it.

The hypothesis that the strong negative correlation in the mean and standard deviation of the risk neutral density reflects the process of marketmaking in the index options market is the one we favor to explain the results in Tables 3 - 5. We are unable to explore this explanation rigorously in the current paper, but it will be the object of future investigation.

VI. The Response of the Risk Neutral Density to Fluctuations in the Stock Market

Under Black-Scholes assumptions, the RND is lognormal and its mean is equal to the forward price of the underlying. As the index moves, the RND shifts back and forth along the price axis, but its shape does not change: If the level of the forward index goes down 1 dollar, every quantile of the RND shifts 1 dollar to the left. By contrast, if some quantile does not move by the same amount as the index does, the shape of the RND will change.

The RND is a deformation, induced by risk preferences, of the market's aggregate subjective estimate of the true probability distribution for the S&P index on option expiration day. While the true density may reasonably be assumed to be lognormal or a standard fat-tailed alternative, there is no reason to expect the risk neutral density to obey any common probability law. To analyze how the shape of the RND changes over time in response to fluctuations in the underlying spot market, with greater precision than is possible simply by looking at its moments as in Table 2, in this section we break the RND down and look at the behavior of its different quantiles.

We regress the minute to minute changes of each quantile against the forward index, with the following regression,

\[ \Delta Q_{jt} = a_j + b_j \Delta F_t \]
where \( Q_{jt} \) refers to the \( j \)th quantile of the RND at observation \( t \) and \( \Delta F_t \) is the contemporaneous change in the forward index level. As above, we exclude the first half hour of trading and begin the option trading "day" at 10:00 A.M.

To provide a point of comparison in estimating equation (9) over the 1-minute intervals of our sample, the first three lines in Table 6 present the results from running this regression for 15 percentiles of the RND, from 1% to 99%, on a different dataset, that was studied in Figlewski (2009). Those RNDs were constructed on a daily basis from the bids and asks at the market close, as reported by OptionMetrics. The sample covers options maturing on the quarterly March-June-September-December cycle, with maturities ranging from about 90 down to 14 days. The sample period ran from January 4, 1996 through February 20, 2008, yielding a total of 2761 observations.

Every \( b_j \) coefficient is highly significant and they are quite different from 1.0, exhibiting a strong and nearly monotonic pattern across percentiles. The left tail quantiles move more than the forward and the coefficients become larger as one looks further out in the tail, reaching a maximum of 1.412 at the 2% level. By contrast, from the median (50%) up, the RND quantiles do not move as much as the forward and the sensitivity goes down uniformly for higher quantiles.

Turning to the current data sample, the next three lines in Table 6 report the estimated values, standard errors, and t-statistics of the \( b_j \) coefficients for the full sample of 14,664 1-minute changes. The pattern across quantiles is very striking and quite different from what was seen for the daily data. All of the estimates are positive and highly statistically significant, but they are all different from 1.0 and they vary widely in size.

In the middle of the distribution, from around the 20th to the 70th percentiles, the RND moves further than the index forward, as much as 46% more for the 40th percentile. However, the RND remains tied to the forward price, so its quantiles can not continue to move further than the forward for long. Instead, the negative autocorrelation seen above is plainly operating for these quantiles. A move in the stock market is amplified in the change of the RND, some of which is then reversed over time.

The remote tails at both ends are distinctly less sensitive to the change in the index than are the middle quantiles. A good reason to expect this result is that both tails are extracted from deep out of the money contracts, puts on the left and calls on the right. These options have very small deltas and wide bid-ask spreads, so their fair values are quite insensitive to changes in the forward index that might occur over a period as short as a minute.

The next three sets of lines show how the pattern of RND quantile sensitivity varied across our subsamples. The same general pattern is seen in all three, but as the crisis unfolded, it became much stronger in the middle quantiles, from 30% to 60% (where most of the options trading occurs), and distinctly less sensitive in the wings, even reversing sign in the furthest left tail in Oct-Nov 2008. But these are synthetic tails that have been appended to the market-determined middle portion of the RND, so we are
somewhat less confident about drawing strong conclusions from their minute-to-minute fluctuations.

Many studies have found that the stock market responds more to negative than to positive returns. Table 7 examines this proposition with our quantile regressions. The top half of the table displays the coefficients and their standard errors for minutes when the forward index fell, and the bottom half for minutes when it rose.

For the full sample, the expected result that negative returns have more impact than positive returns does appear to hold in the wings of the density, but it is not obviously true for the middle portion. The estimates of $b_j$ were larger for negative returns than for positive returns only up to about the 30th percentile, but those for positive returns were greater than for negative returns from the 50th to the 95th percentiles.

Comparing across subperiods, we see that the low volatility subsample from 2006 - 2007 exhibited both large and very small coefficients at different quantiles, with sharp differences in the left tail between positive and negative return minutes. In September 2008, the left portion of the RND, except for the remote tail, became more responsive to negative returns than it had been in the earlier subperiod, and less sensitive to positive returns, while the reverse was true for the right tail.

In the final "full meltdown" period of October - November 2008, the sensitivity of the middle portion of the RND became even stronger, with values above 1.6 for some of the quantiles. The largest coefficients were found for positive price changes. Could it be that under normal circumstances the market is expected to go up on average, so a down day is more of a shock relative to expectations than an up day, but by the time the financial crisis was in full force, the mood in the stock market had become so grim, and expectations for further price drops so widespread, that it was a greater surprise to see the market go up?

VII. Concluding Comments

The risk neutral probability density that can be extracted from market option prices contains a large amount of information about investors' price expectations and risk preferences. The challenge is to extract it. Applying a procedure developed in Figlewski (2009) to a new intraday data set on S&P 500 index options, we were able to construct well-behaved estimates of the RND and analyze its fluctuations at 1-minute intervals. This allows an extremely detailed picture of the sharp changes in the behavior of the U.S. stock market during the period of financial crisis, September - November 2008.

Most obvious was the extraordinary increase in risk measures in conjunction with the fall in stock prices. For example, between October 2006 and October 2008 the average daily trading range for the S&P index expanded from under 0.8 percent of the index level to about 5%. Intraday volatility rose by a factor of more than 7, from 2.2 to 17.2 basis points per minute. Along with the increase in its standard deviation, the shape of the
RND also changed. In contrast to the lognormal density assumed by the Black-Scholes model, the empirical RND is always negatively skewed. But the degree of skewness diminished in the financial crisis, as did the excess kurtosis. The estimated left tail changed from "fat" to "thin" during this period.

Our analysis also revealed an important contrast between the intraday dynamics of the forward value of the stock index and the mean of the RND. Arbitrage ties their levels together very closely, with the root mean squared difference between them being under 11 b.p. even during the worst of the market crisis. But fluctuations are much more volatile for the RND mean than the forward. This is possible because both the mean and the standard deviation of the RND exhibit very high negative autocorrelation while autocorrelation of the forward price is mildly positive.

We considered several possible hypotheses for the RND's seemingly anomalous serial dependence, and argued that the pattern of autocorrelation in our results does not support explanations based on bad data, whether from bid-ask bounce, stale prices, slow adjustment of investors' expectations to new data, or erroneous price quotes. It is also not an artifact of our procedure for completing the tails of the RND.

Strongly negative autocorrelation could be consistent with a market in which "skittish" options traders irrationally overreact to new information, so that the options market overshoots the new equilibrium at first, then corrects afterwards. But while we can't rule irrationality out, we strongly prefer a different hypothesis, that negative autocorrelation in the dynamics of the RND at very short time intervals may simply reflect the mechanics of the normal process of liquidity provision by options marketmakers as they respond to fluctuating investor demand.

Viewing this phenomenon from another perspective, we used quantile regressions to examine how the shape of the RND changes when the stock market moves, and found a striking pattern. In the region at and somewhat below the current index, where the strike prices for the most actively traded options lie, the RND quantile moves substantially more than the change in the forward index, while the tails of the RND move much less. This effect became noticeably stronger during the crisis, with the median of the density moving 1.6 times as much as the forward index on a minute-to-minute basis. Comparing intraday RNDs against those drawn from an earlier study daily data, we found some interesting differences in behavior, both between the middle and the tails of the density, and also between periods with price increases versus periods with price drops.

This is the first study to examine the risk neutral density for the U.S. stock market using real-time data drawn from the OPRA data feed. The RND is an extremely sensitive measure of investors' return expectations and risk tolerance, that has great potential for advancing our understanding of how prices are formed in our financial markets. Plainly, "more research is called for."
References


APPENDIX

This appendix sketches out the steps we use to extract a well-behaved estimate of the risk neutral density (RND) from a set of market options data. A full exposition and discussion of alternative approaches can be found in Figlewski (2009).

1. **Use bid and ask quotes, eliminating options that are too far in or out of the money:**

   The RND must be extracted from contemporaneous option prices. Transactions are sporadic even in active options markets, but marketmakers quote firm bids and offers continuously throughout the trading day, so it is much better to take option prices from those quotes than from data on trades. We use bid and ask quotes for S&P 500 index options obtained from the real-time data feed of the national best bid and offer. Options that are too far out of the money are eliminated by requiring a minimum bid price, $0.50 in this case.

2. **Construct a smooth curve by 4th degree spline interpolation in strike-implied volatility space:**

   Interpolating option prices directly does not work well. Instead, we convert the option prices into Black-Scholes implied volatilities (IVs), interpolate the resulting volatility smile in Strike-IV space, and then convert the IV curve back into a dense set of option prices.\(^{16}\) We employ a 4th degree "smoothing spline" to limit the effect of noise in market option prices, while producing a volatility smile smooth enough to give a well-behaved risk neutral density.\(^{17}\) The results are insensitive to the number of knot points used in the spline, so here we use a single knot placed on the at money exercise price.

3. **Fit the spline to lie within the bid-ask spread:**

   Fitting a spline to the midpoint of the bid-ask spread by least squares applies equal weight to a squared deviation regardless of whether the interpolated value would fall inside or outside the quoted spread. Option spreads can be wide, so we are more concerned about the approximation if the spline falls outside the quoted spread. We therefore increase the weighting of deviations outside the spread relative to those that remain within it. To do this efficiently, we use the cumulative normal distribution function, which produces weights between 0 to 1 as a function of a single parameter \(\sigma\).

   \[
   w(IV_i) = \begin{cases} 
   N[ IV_i - IV_{Ask}, \sigma] & \text{if } IV_{Midpoint} \leq IV_i \\
   N[ IV_{Bid} - IV_i, \sigma] & \text{if } IV_i \leq IV_{Midpoint} 
   \end{cases} 
   \]

   \(^{16}\) Note that this procedure does not assume the Black-Scholes model holds for these option prices (which would require IV to be the same at all strikes). It simply uses the Black-Scholes equation as a computational device. We want a good estimate of the risk neutral density throughout its range, but the translation from probabilities to option prices is highly nonlinear. Converting to IVs permits a more balanced fit across the whole range of strikes.

   \(^{17}\) An Nth degree spline consists of a set of curve segments joined together at their endpoints, called "knot points," such that all derivatives of the resulting curve are continuous up to the N-1st, but the Nth derivative is allowed to change at the knots. Since the RND is obtained as the second derivative of the option value with respect to the strike price, the spline interpolation must have continuous derivatives up to the 3rd to prevent discontinuities in the first derivative of the RND and sharp spikes at the knots.
4. Use out of the money calls, out of the money puts, and blend the two at the money: It is generally felt that better information about the market's risk neutral probability estimates is obtained from out of the money and at the money contracts than from in the money options. Also, stock index puts regularly trade on somewhat higher implied volatilities than calls at the same strike price, so switching from one to the other at a single strike price would create an artificial jump in the IV curve, and a badly behaved density function. We blend the put and call bid and ask IVs to produce a smooth transition in the region around the current stock price.

5. Convert the interpolated IVs back to option prices to extract the middle portion of the risk neutral density, then complete it by adding tails from a Generalized Extreme Value distribution: The procedure described above produces the portion of the RND between the second lowest and second highest strikes used in the calculations. To complete the density, it is necessary to extend it into the left and right tails. We do this by appending tails drawn from a Generalized Extreme Value (GEV) distribution. Similar to the way the Central Limit Theorem makes the Normal a natural model for the sample average from an unknown distribution, the Fisher-Tippett Theorem shows that the Generalized Extreme Value distribution is a natural candidate for modeling the tails of an unknown density. Equation (A2) gives the GEV distribution function.

\[
F(S_F) = \exp \left[ - \left( 1 + \frac{S_F - \mu}{\sigma} \right)^{-1/\xi} \right]
\]

18 Deep in the money options have wide bid-ask spreads, very little trading volume, and high prices that are almost entirely due to their intrinsic values (which give no information about probabilities). For example, the current methodology for constructing the well-known VIX volatility index uses only out of the money puts and calls. See Chicago Board Options Exchange (2003).

19 How far these two implied volatilities can deviate from one another is limited by arbitrage, which in turn depends on the transactions costs of putting on the trade. In our S&P 500 index option data, even though they are European options, put IVs can easily be 1 to 2 percentage points higher than call IVs at the money.

20 In the analysis presented here, we have chosen a range of 20 points on either side of the current forward index value $F_0$. Specifically, let $X_{\text{low}}$ be the lowest traded strike such that $(F_0 - 20) \leq X_{\text{low}}$ and $X_{\text{high}}$ be the highest traded strike such that $X_{\text{high}} \leq (F_0 + 20)$. For traded strikes between $X_{\text{low}}$ and $X_{\text{high}}$ we use a blended value between $IV_{\text{put}}(X)$ and $IV_{\text{call}}(X)$, computed as

\[
IV_{\text{blend}}(X) = w \cdot IV_{\text{put}}(X) + (1 - w) \cdot IV_{\text{call}}(X)
\]

where

\[
w = \frac{X_{\text{high}} - X}{X_{\text{high}} - X_{\text{low}}}
\]

This is done for the bid and ask IVs separately to preserve the bid-ask spread for use in the spline calculation. The average forward value of the index was 1241 in our data sample, so that 20 points was on average less than 2% of the current level. The width of the range over which to blend put and call IVs is arbitrary. A small amount of experimentation suggested that the specific choice has little impact on performance of the methodology for this data set.

21 The class of distribution functions that satisfy this condition is very broad, including all of those commonly used in finance. See Embrechts, et al (1997) or McNeil, et al (2005) for further detail.
The GEV distribution has three parameters, which we set so that the tail satisfies three constraints: (1) The total probability mass in the fitted tail must equal the missing tail probability, that can be computed from the empirical RND. (2) The density of the GEV tail must equal the empirical RND density at the connection point. (3) The GEV density also must match the empirical density at a second point further out in the tail. If enough options data are available, we connect the GEV tails to the empirical RND at the 5th and 95th percentiles and require the densities to be equal also at the 2nd and 98th percentiles. Figure 1 illustrates of how this procedure works.

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22 This was possible with our S&P 500 option data for the right tail on nearly all dates, but after the market sold off sharply during the 2008 financial crisis, available option prices often did not extend as far into the left tail, especially given the increase in volatility, which widened the range of the distribution. Where possible, we used 5% and 95% as the connection points; otherwise, if \( \alpha_L \) (\( \alpha_R \)) denotes the furthest point into the left (right) tail that was available from the data at a given time, we set the tail connection point to \( \alpha_L + 0.03 \) (\( \alpha_R - 0.03 \)).
Figure 1: Risk Neutral Density and Fitted GEV Tail Functions

- Density
- S&P 500 Index

Key Points:
- 2% and 5% connection points
- 95% and 98% connection points

Legend:
- Empirical RND
- Left tail GEV function
- Right tail GEV function
- Connection points
Figure 2: S&P 500 Index Risk Neutral Density on 3 Dates (December Expiration)
Figure 3: Risk Neutral Volatility vs GARCH Volatility and the VIX Index
Table 1
Description of Data Sample

The table reports summary information about the data sample by subperiod. For intraday data, the trading “day” is assumed to begin at 10:00 A.M. The S&P 500 Forward Index is computed at one minute intervals intraday, by multiplying the current spot index by \( \exp((r-d)T) \), where \( r, d, \) and \( T \) are, respectively, the maturity-matched level of LIBOR, the S&P dividend yield, and the time to option expiration. The Risk Neutral Density (RND) is estimated from options with December maturity, and its standard deviation is multiplied by \( \sqrt{\frac{365}{\text{days to expiration}}} \) to convert it into an annualized volatility. GARCH volatility estimates are one day ahead forecasts from a GJR-GARCH model fitted on S&P index returns over the previous 2000 days. The VIX Index is the end of day value for the 30-day (new) VIX. It is reported as an annual volatility.

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<th># Obs</th>
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<th>RND Annualized Standard Deviation</th>
<th>GARCH Annualized Volatility</th>
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<td>42.11</td>
<td>75.10</td>
<td>66.99</td>
</tr>
</tbody>
</table>
Table 2  
Intraday Values for the S&P 500 Index and the Risk Neutral Density, by Subperiod

The table reports intraday average values for the level of S&P 500 index in the spot market and the moments and tail shape parameters of the Risk Neutral Density during five subperiods. The relevant horizon in each case is the expiration date for December S&P index options. The S&P 500 forward is the current spot index multiplied by \( \exp((r-d)T) \), where \( r \), \( d \), and \( T \) are, respectively, the maturity-matched level of LIBOR, the S&P dividend yield, and the time to December option expiration. The risk neutral mean and higher moments are computed from the densities fitted as described in the text. The average and root mean squared difference between the RND mean and the S&P index forward measures the extent of potential arbitrage opportunities between the S&P spot market and index options. RND Standard Deviation in index points depends on the volatility in percent, the level of the index, and the time to option maturity. To facilitate comparison across days with different index levels and time to expiration, the table also reports RND standard deviation as a percent of the RND mean, annualized by multiplying by \( \sqrt{365 / \# \text{ days to expiration}} \). Tail shape parameters are the \( \xi \) values for the GEV distributions appended to complete the left and right tails of the RND (see eq. (A2)).

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<thead>
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<tbody>
<tr>
<td><strong>LEVEL OF S&amp;P INDEX</strong></td>
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<tr>
<td>S&amp;P spot</td>
<td>1342.30</td>
<td>1512.00</td>
<td>1209.30</td>
<td>991.04</td>
<td>893.41</td>
</tr>
<tr>
<td>S&amp;P forward (Dec expiration)</td>
<td>1351.40</td>
<td>1521.10</td>
<td>1212.60</td>
<td>994.51</td>
<td>892.42</td>
</tr>
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<td><strong>THE RND MEAN vs S&amp;P FORWARD</strong></td>
<td></td>
<td></td>
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<tr>
<td>RND mean</td>
<td>1350.80</td>
<td>1520.70</td>
<td>1212.10</td>
<td>994.22</td>
<td>892.09</td>
</tr>
<tr>
<td>Average(RND mean / forward - 1) as %</td>
<td>-0.041</td>
<td>-0.030</td>
<td>-0.040</td>
<td>-0.029</td>
<td>-0.038</td>
</tr>
<tr>
<td>RMS( RND mean / forward - 1 ) as %</td>
<td>0.057</td>
<td>0.067</td>
<td>0.103</td>
<td>0.066</td>
<td>0.110</td>
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<td><strong>HIGHER MOMENTS OF THE RND</strong></td>
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<tr>
<td>RND standard deviation</td>
<td>78.30</td>
<td>132.13</td>
<td>164.15</td>
<td>191.06</td>
<td>158.52</td>
</tr>
<tr>
<td>RND std dev as % of mean (annualized)</td>
<td>13.04</td>
<td>20.80</td>
<td>27.33</td>
<td>46.61</td>
<td>57.53</td>
</tr>
<tr>
<td>Skewness</td>
<td>-2.247</td>
<td>-1.476</td>
<td>-0.732</td>
<td>-0.607</td>
<td>-0.744</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>16.83</td>
<td>7.05</td>
<td>3.81</td>
<td>3.26</td>
<td>3.56</td>
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<tr>
<td><strong>RND TAIL SHAPE</strong></td>
<td></td>
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</tr>
<tr>
<td>Left tail shape</td>
<td>0.214</td>
<td>-0.096</td>
<td>-0.151</td>
<td>-0.256</td>
<td>-0.321</td>
</tr>
<tr>
<td>Right tail shape</td>
<td>-0.204</td>
<td>-0.251</td>
<td>-0.180</td>
<td>-0.196</td>
<td>-0.175</td>
</tr>
</tbody>
</table>
Table 3  
**Intraday Variability of the S&P 500 Index and the Risk Neutral Density**

The table reports the intraday variability of the S&P forward index level and the mean and standard deviation of the risk neutral density measured at 1-minute intervals. The S&P 500 forward is the current spot index multiplied by \( \exp((r-d)T) \), where \( r \), \( d \), and \( T \) are, respectively, the maturity-matched level of LIBOR, the S&P dividend yield, and the time to December option expiration. Risk neutral moments are computed from the fitted densities. The high-low range is computed for each day as the highest minus the lowest value of the forward index after 10:00 A.M. To make the values more comparable across subperiods the range is reported as a percent of the 10:00 A.M. level. "Volatility" in each case is the standard deviation in basis points of 1-minute log returns. Autocorrelation is computed from 1-minute changes.

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td><strong>VARIABILITY of S&amp;P FORWARD</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>High-low range (% of 10:00 AM level)</td>
<td>0.77</td>
<td>0.95</td>
<td>2.94</td>
<td>4.95</td>
<td>5.24</td>
</tr>
<tr>
<td>Volatility of forward (b.p. per minute)</td>
<td>2.2</td>
<td>3.7</td>
<td>9.8</td>
<td>17.2</td>
<td>13.2</td>
</tr>
<tr>
<td>Autocorrelation</td>
<td>0.062</td>
<td>0.032</td>
<td>0.037</td>
<td>0.041</td>
<td>-0.009</td>
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<td><strong>VARIABILITY OF RND MEAN</strong></td>
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<td></td>
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<td></td>
</tr>
<tr>
<td>Volatility of RND mean (b.p. per minute)</td>
<td>5.0</td>
<td>9.7</td>
<td>17.3</td>
<td>18.8</td>
<td>19.5</td>
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<tr>
<td>Autocorrelation</td>
<td>-0.275</td>
<td>-0.240</td>
<td>-0.307</td>
<td>-0.048</td>
<td>-0.245</td>
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<td><strong>VARIABILITY OF RND STD DEVIATION</strong></td>
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<td></td>
</tr>
<tr>
<td>Volatility of RND std deviation (b.p. per minute)</td>
<td>351.5</td>
<td>398.26</td>
<td>244.11</td>
<td>120.09</td>
<td>184.37</td>
</tr>
<tr>
<td>Autocorrelation</td>
<td>-0.466</td>
<td>-0.452</td>
<td>-0.489</td>
<td>-0.416</td>
<td>-0.455</td>
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</tbody>
</table>
Table 4
Intraday Autocorrelation of the Risk Neutral Density Mean and Standard Deviation

The first through fifth order autocorrelation of the log changes in RND mean and standard deviation are reported for differencing intervals from 1 to 30 minutes. The first interval begins at 10:00 A.M. and all intervals are completed within the same day. * indicates a correlation is significantly different from zero at the 5% confidence level.

<table>
<thead>
<tr>
<th>Interval in minutes</th>
<th>RND Mean</th>
<th>RND Standard Deviation</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>-0.286 *</td>
<td>-0.008</td>
</tr>
<tr>
<td>2</td>
<td>-0.211 *</td>
<td>0.020</td>
</tr>
<tr>
<td>3</td>
<td>-0.172 *</td>
<td>0.026</td>
</tr>
<tr>
<td>4</td>
<td>-0.126 *</td>
<td>0.010</td>
</tr>
<tr>
<td>5</td>
<td>-0.105 *</td>
<td>0.008</td>
</tr>
<tr>
<td>10</td>
<td>-0.091 *</td>
<td>0.027</td>
</tr>
<tr>
<td>15</td>
<td>-0.102 *</td>
<td>0.029</td>
</tr>
<tr>
<td>20</td>
<td>0.013</td>
<td>-0.116 *</td>
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<tr>
<td>25</td>
<td>-0.006</td>
<td>-0.056</td>
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<tr>
<td>30</td>
<td>-0.098 *</td>
<td>0.010</td>
</tr>
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</table>
Table 5
Autocorrelation of the 25th, 50th, and 75th Percentiles of the Risk Neutral Density

The first through fifth order autocorrelation of the log changes in the 25th, 50th, and 75th percentiles of the RND are reported for differencing intervals from 1 to 30 minutes. The first interval begins at 10:00 A.M. and all intervals are completed within the same day. These percentiles are drawn from the middle portion of the estimated Risk Neutral Density and are unaffected by the GEV tail fitting procedure. * indicates a correlation is significantly different from zero at the 5% confidence level.

<table>
<thead>
<tr>
<th>Interval in minutes</th>
<th>25th Percentile</th>
<th>50th Percentile (Median)</th>
<th>75th Percentile</th>
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<tr>
<td></td>
<td>Lag 1</td>
<td>Lag 2</td>
<td>Lag 3</td>
</tr>
<tr>
<td>1</td>
<td>-0.349*</td>
<td>-0.018*</td>
<td>-0.006</td>
</tr>
<tr>
<td>2</td>
<td>-0.306*</td>
<td>0.001</td>
<td>-0.027*</td>
</tr>
<tr>
<td>3</td>
<td>-0.258*</td>
<td>-0.018</td>
<td>-0.009</td>
</tr>
<tr>
<td>4</td>
<td>-0.252*</td>
<td>-0.001</td>
<td>-0.002</td>
</tr>
<tr>
<td>5</td>
<td>-0.208*</td>
<td>0.009</td>
<td>-0.094*</td>
</tr>
<tr>
<td>10</td>
<td>-0.184*</td>
<td>0.057</td>
<td>0.023</td>
</tr>
<tr>
<td>15</td>
<td>-0.154*</td>
<td>0.044</td>
<td>-0.090*</td>
</tr>
<tr>
<td>20</td>
<td>-0.027</td>
<td>-0.136*</td>
<td>0.082</td>
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<tr>
<td>25</td>
<td>-0.060</td>
<td>0.005</td>
<td>0.003</td>
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<tr>
<td>30</td>
<td>-0.139*</td>
<td>0.014</td>
<td>0.095</td>
</tr>
</tbody>
</table>
Table 6
Regression of Change in Quantile on Change in Forward Index, 1-Minute Intervals

The table reports the slope coefficient, standard error and t-statistic from the following regression, run over the full sample and three subperiods:

\[ \Delta Q_{jt} = a_j + b_j \Delta F_t \]

where \( \Delta Q_{jt} \) refers to the change in the \( j \)th quantile over minute \( t \) and \( \Delta F_t \) is the contemporaneous change in the forward index level. The sample for each trading day starts at 10:00 A.M. For comparison, the first section reports regression results from Figlewski (2009) using daily option closing prices, Jan. 4, 1996 - February 20, 2008.

<table>
<thead>
<tr>
<th>Quantile</th>
<th>1%</th>
<th>2%</th>
<th>5%</th>
<th>10%</th>
<th>20%</th>
<th>30%</th>
<th>40%</th>
<th>50%</th>
<th>60%</th>
<th>70%</th>
<th>80%</th>
<th>90%</th>
<th>95%</th>
<th>98%</th>
<th>99%</th>
</tr>
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<tbody>
<tr>
<td><strong>Daily data 1996 - 2008 -- NOBS = 2761</strong></td>
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</tr>
<tr>
<td>coef</td>
<td>1.365</td>
<td>1.412</td>
<td>1.385</td>
<td>1.297</td>
<td>1.172</td>
<td>1.089</td>
<td>1.027</td>
<td>0.974</td>
<td>0.926</td>
<td>0.881</td>
<td>0.832</td>
<td>0.773</td>
<td>0.730</td>
<td>0.685</td>
<td>0.659</td>
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<tr>
<td>std err</td>
<td>0.023</td>
<td>0.019</td>
<td>0.014</td>
<td>0.007</td>
<td>0.004</td>
<td>0.004</td>
<td>0.004</td>
<td>0.004</td>
<td>0.003</td>
<td>0.003</td>
<td>0.004</td>
<td>0.006</td>
<td>0.008</td>
<td>0.011</td>
<td>0.014</td>
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<tr>
<td>t-stat</td>
<td>58.65</td>
<td>72.43</td>
<td>98.62</td>
<td>180.26</td>
<td>285.09</td>
<td>255.35</td>
<td>251.50</td>
<td>269.88</td>
<td>298.32</td>
<td>299.81</td>
<td>227.50</td>
<td>131.16</td>
<td>88.75</td>
<td>60.08</td>
<td>46.60</td>
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<tr>
<td><strong>Full Sample -- NOBS = 14,664</strong></td>
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<tr>
<td>coef</td>
<td>0.496</td>
<td>0.281</td>
<td>0.391</td>
<td>0.799</td>
<td>1.313</td>
<td>1.420</td>
<td>1.460</td>
<td>1.425</td>
<td>1.310</td>
<td>1.156</td>
<td>0.975</td>
<td>0.876</td>
<td>0.767</td>
<td>0.497</td>
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<td>std err</td>
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<td>0.040</td>
<td>0.024</td>
<td>0.020</td>
<td>0.017</td>
<td>0.016</td>
<td>0.015</td>
<td>0.014</td>
<td>0.014</td>
<td>0.014</td>
<td>0.014</td>
<td>0.019</td>
<td>0.026</td>
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<tr>
<td>t-stat</td>
<td>2.327</td>
<td>3.346</td>
<td>8.139</td>
<td>20.229</td>
<td>53.730</td>
<td>85.414</td>
<td>91.709</td>
<td>86.810</td>
<td>80.396</td>
<td>71.834</td>
<td>64.570</td>
<td>53.899</td>
<td>25.516</td>
<td>11.391</td>
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<tr>
<td>coef</td>
<td>0.066</td>
<td>0.608</td>
<td>0.769</td>
<td>1.004</td>
<td>1.386</td>
<td>1.422</td>
<td>1.415</td>
<td>1.340</td>
<td>1.231</td>
<td>1.105</td>
<td>0.958</td>
<td>0.873</td>
<td>0.774</td>
<td>0.503</td>
<td>0.276</td>
</tr>
<tr>
<td>std err</td>
<td>0.710</td>
<td>0.212</td>
<td>0.099</td>
<td>0.060</td>
<td>0.053</td>
<td>0.047</td>
<td>0.036</td>
<td>0.028</td>
<td>0.028</td>
<td>0.028</td>
<td>0.028</td>
<td>0.028</td>
<td>0.028</td>
<td>0.050</td>
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<td><strong>Sept. 2008 -- NOBS = 6392</strong></td>
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<tr>
<td>coef</td>
<td>0.931</td>
<td>0.557</td>
<td>0.615</td>
<td>0.944</td>
<td>1.257</td>
<td>1.312</td>
<td>1.330</td>
<td>1.298</td>
<td>1.218</td>
<td>1.111</td>
<td>0.984</td>
<td>0.912</td>
<td>0.831</td>
<td>0.621</td>
<td>0.443</td>
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<tr>
<td>std err</td>
<td>0.331</td>
<td>0.121</td>
<td>0.066</td>
<td>0.048</td>
<td>0.033</td>
<td>0.027</td>
<td>0.023</td>
<td>0.021</td>
<td>0.022</td>
<td>0.022</td>
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<td>0.018</td>
<td>0.018</td>
<td>0.028</td>
<td>0.041</td>
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<tr>
<td>coef</td>
<td>-0.036</td>
<td>-0.159</td>
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<td>0.563</td>
<td>1.378</td>
<td>1.570</td>
<td>1.648</td>
<td>1.615</td>
<td>1.453</td>
<td>1.226</td>
<td>0.966</td>
<td>0.826</td>
<td>0.676</td>
<td>0.324</td>
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<td>std err</td>
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<td>0.178</td>
<td>0.114</td>
<td>0.056</td>
<td>0.043</td>
<td>0.038</td>
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<td>0.028</td>
<td>0.028</td>
<td>0.029</td>
<td>0.037</td>
<td>0.037</td>
<td>0.047</td>
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<tr>
<td>t-stat</td>
<td>-0.115</td>
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<td>0.139</td>
<td>5.570</td>
<td>24.659</td>
<td>36.526</td>
<td>43.894</td>
<td>44.145</td>
<td>41.644</td>
<td>39.797</td>
<td>34.823</td>
<td>29.809</td>
<td>23.310</td>
<td>8.835</td>
<td>1.833</td>
</tr>
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</table>

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Table 7
Regressions of Change in Quantile on Change in Forward Index for Positive and Negative Returns Separately

See the notes for Table 6. Regressions of the change in each quantile on the change in the index forward are run separately for negative and positive changes in the index forward.

<table>
<thead>
<tr>
<th>Quantile</th>
<th>1%</th>
<th>2%</th>
<th>5%</th>
<th>10%</th>
<th>20%</th>
<th>30%</th>
<th>40%</th>
<th>50%</th>
<th>60%</th>
<th>70%</th>
<th>80%</th>
<th>90%</th>
<th>95%</th>
<th>98%</th>
<th>99%</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Negative Return</strong></td>
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</tr>
<tr>
<td>Full Sample -- NOBS = 7146</td>
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