Paper 1: THE PRINCIPAL PRINCIPLE: Optimal Modification of Distressed Home Loans (Why Lenders should Forgive, not Foresake Mortgages)

Paper 2: STRATEGIC LOAN MODIFICATION: An Options based response to strategic default (joint work with Ray Meadows)

Papers at http://algo.scu.edu/~sanjivdas/research.htm

Sanjiv R. Das
Santa Clara University

@Fordham, October 1, 2010.
The Housing Crisis

There are approximately 110 million households in the U.S. Of the 110 million, about 75.5 million are homeowners. Of those 75.5 million homeowners, approximately 68%, or 51.6 million have mortgages.¹ DB estimates that, as of the end of Q1 2009, 14 million U.S. homeowners had negative equity, or approximately 27% of all homeowners with mortgages. Applying DB’s most recent MSA-level home price projections,² we estimate that 25 million homeowners will have negative equity before home prices stabilize, or 48% of all mortgagors.³

Figure 1: Homeowners with mortgage balances > property value

Source: Economy.com, First American CoreLogic, Deutsche Bank
Current Statistics

✓ Q3 2009: 1/136 received foreclosure filing (RealtyTrac).
✓ 1 in 380 households received a foreclosure filing in August 2010 (RealtyTrac).
✓ Sep 2010: pre-sale foreclosure inventory 2.038 million homes (LPS).
✓ Sep 2010: 4.9 million mortgages 30-days overdue, 2.374 million 90-days past due (LPS).
✓ Sep 2010: 11 million borrowers (23% of households with a mortgage) have negative equity (CoreLogic).
CitiMortgage CEO Sanjiv Das helps people keep their homes

By Stephanie Armour, USA TODAY
April 27, 2009

There is very little in Sanjiv Das' uncluttered office.

Just snapshots of his wife and his 20-year-old daughter, Natasha. No pictures of his favorite sports: golf and cricket.

Das has moved around the world, run credit card acquisitions for American Express in India, and handled the mortgage business for Citibank in Sydney. But none of it compares to what he's doing now.
A tale of two Das: Citi CEO, academic and mortgages

NEW YORK (Reuters) - Sanjiv Ranjan Das, a professor at California's Santa Clara University, last fall attacked the problem of "underwater" mortgages often cited as an Achilles' heel to the U.S. housing market.

HOUSING MARKET
He had a special rant: Sanjiv Das, the top executive at CitiMortgage, the nation's fourth-largest home loan underwriter and servicer of $723 billion in mortgages.
Mr. Das,

I apologize for sending this email via your Santa Clara University email address. I only contact information I could locate for you. I understand that you are the
CitiMortgage, Inc. Our office is having difficulty obtaining information from CitiFinancial in regards to consumer complaints filed with our office related to consumer complaints, the responses often fail to address the consumer's questions and to follow up with the authors of the responses, they either fail to return our calls or otherwise informed on the details of the file and thus cannot provide us with even basic information.

Would it be possible to set up a conference call to further discuss these issues?

Thank you,

Lisa S. Wolf
Deputy Attorney General
Office of the Attorney General
Consumer Protection Division
Homeowner Protection Unit
302 W. Washington St. 5th Floor
Indianapolis, IN 46204

Mayor Eric J. Brewer - CitiMortgage's foreclosure rescue plan - I'd like to implement in my city

The following message was sent from the e-mail form in the SCU Online Phonebook:

A message from: Mayor Eric J. Brewer
Mr. Das
I am the Mayor of the City of East Cleveland, Ohio, one of the cities hit hardest by foreclosure in this area. Today I read an article that you have initiated a foreclosure rescue plan. I held a foreclosure prevention summit last year with 200 attendees to help people keep from losing their homes. I'd like to work with CitiMortgage to get the word out in my area. My cell is 216-310-1110. I don't answer restricted calls as a mayor. I'd appreciate the opportunity to work with CitiMortgage in helping my residents and more. Please call.

Mayor Eric J. Brewer
City of East Cleveland
14340 Euclid Avenue
East Cleveland, OH 44112
Office: 216-681-2208
Executive Asst: Ms. Kim Woodson
Website: eastcleveland.org
Rate reductions are value-destroying.
Maturity extensions also destroy value.
Principal reductions are optimal.
Capitalization of payments into back-ended principal will also destroy loan value.
Shared-appreciation mortgages improve ability to pay, mitigate moral hazard.
Optimal modifications may be computed in closed-form in a reduced-form model.
The best way to modify an underwater loan is to reduce the principal balance, lowering the monthly payment and restoring equity. But for the most part, lenders have refused to reduce principal because it would force them to take an immediate loss on the loan. Lenders also have vehemently and successfully resisted Congressional efforts to change the law so that bankruptcy courts could reduce the mortgage balances for bankrupt borrowers.

New York Times, January 4, 2010
The administration decided not to press lenders to grant principal reductions in the flawed belief that simply making payments more affordable would be enough to forestall foreclosures. It hasn’t. The administration also didn’t fight for the bankruptcy fix when it was before Congress last year despite President Obama’s campaign promise to do so.

The economy is hard pressed to function, let alone thrive, when house prices are falling. As home equity erodes, consumer spending falls and foreclosures increase. Lenders lose the ability and willingness to extend credit and employers are disinclined to hire. True economic recovery is all but impossible.

To avert the worst, the White House should alter its loan-modification effort to emphasize principal reduction. Job creation should also be a priority so that rising unemployment does not cause more defaults.
Treasury Sets Guidance to Simplify "Short Sales"

By REUTERS
Published: November 30, 2009

Filed at 6:58 p.m. ET

NEW YORK (Reuters) - The U.S. Treasury on Monday set long-awaited guidance on a plan for mortgage companies to speed "short sales" of homes and other loan modification alternatives to stem a rising tide of foreclosures.

The Home Affordable Foreclosure Alternatives Program provides financial incentives and simplifies the procedures for completing short sales, a growing practice in which a lender agrees to accept the sale price of a home to pay off a mortgage even if the price falls short of the amount owed, according to an announcement on the Treasury's website.

Short sales are favored by real estate agents and community groups over foreclosure because they can preserve the borrower’s credit rating and leave the property in better condition than when a homeowner is evicted. While primary lenders typically realize steep losses, their recovery is typically far better than under foreclosure.

"If there was a short sale program that didn't recognize the second lien holder position, it could have pretty damaging consequences for the industry," Sanjiv Das, chief executive officer of CitiMortgage, said in an interview last week.
How Long Will Negative Equity Last?

BY: CARRIE BAY  

03/29/2010

There has been a lot of recent talk about mortgages in negative equity – underwater homes – and the impact on the housing market. In response, First American CoreLogic asked the question: When will these homes start to float?

The company estimates that the typical underwater homeowner will not begin to surface until late 2015 to early 2016. It’s an even longer stretch for some of the most depressed markets, where First American CoreLogic says the typical borrower in negative equity may not experience positive equity until 2020 or later.

Even in markets with low shares of negative equity, the recovery time will still be long because the few borrowers that are upside down are deeply in negative equity and these are typically not high appreciation markets, the company has concluded.

Although house price appreciation will, over time, offset negative equity, First American CoreLogic says amortization (the paying down of loan balances) will be a more significant remedy to negative equity. The company’s data shows that over average loan balance will decrease by an annual rate of 3.3 percent; meanwhile home prices are expected to increase at a 3 percent annual rate over the next decade.

To forecast when the typical U.S. homeowner will achieve neutral and positive equity, First American CoreLogic looked at 10 key markets, plotting equity trends over the next decade, and assuming a nominal annual appreciation rate of 3 percent.

Of the markets studied, the Washington D.C. area is expected to reach positive equity by 2015.

Atlanta, Georgia; Dallas, Texas; and Riverside-San Bernardino, California are projected to rise to the surface in 2016. Boston, Massachusetts should find a balance in 2017.

Cape Coral-Fort Myers, Florida; Pittsburgh, Pennsylvania; Las Vegas; and Lancaster, Pennsylvania are forecast to reach positive territory by 2020.

Detroit, though, is not projected to recover even by 2020, because of its depressed economy.

The latest numbers from First American CoreLogic show that more than 11.3 million, or 24 percent, of all residential properties with mortgages were underwater at the end of the fourth quarter of 2009.

Among the new housing initiatives announced by the administration Friday was assistance for borrowers with negative equity. In order to deter these homeowners from strategically defaulting, the Treasury will begin requiring servicers to consider principal write-downs as part of their Home Affordable Modification Program (HAMP) evaluations for borrowers whose loan balance is more than 115 percent of the property’s current value. The plan also includes a Federal Housing Administration (FHA) refinancing program for negative equity mortgages.
Figure 1: Negative Equity By CBSA
(When Marker Increases Past Zero = Positive Equity)
FDIC to test principal-reduction program for underwater borrowers

By Renae Merle
Friday, Feb 26, 2010

The Federal Deposit Insurance Corp. is developing a program to test whether cutting the mortgage balances of distressed borrowers who owe significantly more than their homes are worth is an effective method for saving homeowners from foreclosure.

The program would be aimed at a growing population of homeowners who are underwater on their loans, estimated at more than 20 percent of borrowers, or 11 million homeowners. Economists consider these borrowers among the most vulnerable to foreclosure, and some industry officials worry that more of them will simply walk away from their mortgages, or "strategically default," rather than spend a decade or more trying to regain positive equity.

Under the FDIC program, borrowers would be eligible for a reduction in their mortgage balances if they kept up their payments on the mortgage over a long period. The performance of those borrowers would be compared with borrowers given more traditional mortgage relief packages, such as those that cut the interest rate on loans.
HAMP UPDATE

Announcing HAMP Principal Reduction Alternative

Yesterday, June 3, 2010, Supplemental Directive 10-05: Modification of Loans with Principal Reduction Alternative, was issued offering mortgage relief to eligible homeowners whose homes are worth significantly less than the remaining amounts owed under their first lien mortgage loans. The Principal Reduction Alternative (PRA) guidance applies to non-GSE loans eligible for the Home Affordable Modification Program (HAMP) only.

Principal Reduction Alternative (PRA)
With this new guidance, servicers are required to evaluate all HAMP-eligible loans with a mark-to-market loan-to-value (MTMLTV) greater than 115% to determine if a principal reduction is beneficial. If the evaluation shows the net present value (NPV) for a HAMP modification using PRA is positive, servicers are encouraged to offer the principal reduction to the borrower. An updated NPV model reflecting principal reduction will be available to use for this evaluation. Additional details are as follows:

- **Effective Date** -- The PRA Effective Date (i.e., the date the principal reduction evaluation is required) will be either October 1, 2010, or the date of the HAMP NPV Model 4.0 release (whichever is later). However, servicers may immediately offer PRA for HAMP-eligible modifications as long as the reduction follows all PRA requirements.
- **Application** -- PRA is earned over a three-year period and is initially treated as a PRA Forbearance. Each year (for three years) that the borrower is in good standing on the anniversary of their trial period effective date, one-third of the original PRA forbearance amount will be reduced. This reduced amount will be applied to their unpaid principal balance.
- **Second Lien** -- Servicers participating in the Second Lien Program (2MP) will be required to provide a principal reduction on the borrower's second mortgage in proportion to any principal reduction offered on the borrower's first mortgage.
- **Investor Incentive** -- Investors will receive an incentive based on loan delinquency, LTV ratio, and the amount of the principal reduction. Note: Guidance on principal reduction and related investor incentives will be forthcoming for loans in active HAMP Trial Period Plans or that were permanently modified prior to June 3, 2010 (i.e., the SD 10-05 effective date).
# Loan Modification

As indicated in the summary table below, the FDIC’s Loan Modification Program is primarily based on two principals:

1) Determining a payment the borrower can afford by **multiplying the borrower’s gross monthly income times the appropriate housing-to-income (HTI) ratio, less taxes and insurance to achieve a minimum payment reduction of 10 percent, and**

2) Protecting investors’ interests by requiring that the cost of the modification is less than the estimated cost of foreclosure (the **Net Present Value (NPV) floor**).

<table>
<thead>
<tr>
<th>FDIC Loan Modification Program</th>
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<tbody>
<tr>
<td><strong>Strategy</strong></td>
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<tr>
<td>Offer proactive workout solutions designed to address borrowers who have the willingness but limited capacity to pay.</td>
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<tr>
<td>Provide borrowers the opportunity to</td>
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Modification Process

Step 4: Solve for “Affordable Payment” through a three step waterfall process

1) **Interest Rate Reduction**: Cap the life-of-loan interest rate at the Freddie Mac Weekly Survey rate as of the week of the modification offer, then reduce the interest rate incrementally to as low as 3 percent to achieve the “affordable” payment per the adjusted unpaid principal balance (UPB) and remaining amortization term. An interest rate floor of 3 percent will enable the borrower to maintain approximately a 38 percent HTI ratio throughout the life of the loan, assuming modest borrower earnings growth commensurate with the inflation rate. The reduced rate remains in effect for 5 years. After this period, the interest rate increases by not more than one percent annually until the Freddie Mac Weekly Survey rate is achieved. If the “affordable” modified PITI payment amount has not been achieved, proceed to the next step.

2) **Extend Amortization Term**: For loans with an original term of 30 years, re-amortize the adjusted UPB at the reduced interest rate (3 percent floor) over an extended amortization term of 40 years from the original first payment date. For securitized loans, the amortization will be extended to 40 years from the original first payment date, but the maturity date will not change, resulting in a balloon payment. For loans with an original term of less than 30 years, extend the amortization period for only 10 years if the modified PITI payment amount has not been achieved, proceed to the next step.

3) **Partial Principal Forbearance**: Reduce the adjusted UPB for amortization purposes and amortize over a 40 year period at the reduced interest rate (3 percent floor). This process splits the debt into an interest-bearing, amortizing portion and a zero percent, zero payment portion of the loan. The repayment of the “postponed” principal will be due when the loan is paid in full. For loans within securitizations, this principal forbearance should be passed as a write-off of principal to the trust, with any future collections at time of pay-off submitted to the trust as a recovery.
July 20, 2009

BACK TO BUSINESS

Subprime Brokers Resurface as Dubious Loan Fixers

By PETER S. GOODMAN

LOS ANGELES — From the ninth floor of a downtown office building on Wilshire Boulevard, Jack Soussana delivered staggering numbers of mortgages to homeowners during the real estate boom, amassing a fortune.

Yet the dangers assailing Mr. Soussana’s clients have yielded fresh business for him: Late last year, he and his team — ensconced in the same office where they used to broker mortgages — began working for a loan modification company. For fees reaching $3,495, with most of the money collected upfront, they promised to negotiate with lenders to lower payments on the now-delinquent mortgages they and their counterparts had sprinkled liberally across Southern California.

“We just changed the script and changed the product we were selling,” said Mr. Soussana, who ran the Los Angeles sales office of Federal Loan Modification Law Center. The new script: You got a raw deal, and “Now, we’re able to help you out because we understand your lender.”
Deadweights costs of foreclosure. Foote, Gerardi, Goette and Willen (2009) estimate: $180 BN or 1% of GDP.

This is most likely a huge overestimate!
Choice: Allow foreclosure or modify the loan? Which one loses less loan value?
Ability to pay

&

Willingness to pay
Guiso, Sapienza and Zingales (2009) find that 26% of defaults are strategic in nature.

Cohen-Cole and Morse (2009) find that in the presence of negative equity, borrowers pay credit cards first, and prefer foreclosure.
More homeowners are opting for 'strategic defaults'

Underwater on their mortgages and angry at banks, more borrowers are choosing to hand over the keys, even if they can afford the payments.

By Alana Semuels

March 17, 2010

Wynn Bloch has always dutifully paid her bills and socked away money for retirement. But in December she defaulted on the mortgage on her Palm Desert home, even though she could afford the payments.

Bloch paid $385,000 for the two-bedroom in 2006, when prices were still surging. Comparable homes are now selling in the low-$200,000s. At 66, the retired psychologist doubted she'd see her investment rebound in her lifetime. Plus, she said she was duped into an expensive loan.

The way she sees it, big banks that helped fuel the mess all got bailouts while small fry like her are left holding the bag. No more.

"There was not a chance that house was ever going to be worth anywhere near what my mortgage was," said Bloch, who is now renting a few miles away after defaulting on the $310,000 loan. "I haven't cheated or stolen."
23% of owner-occupied single-family homes have negative equity (national average). States: California (33%), Arizona (37%), Nevada (40%)

Greater problems are likely to arise with cash-out refinancings [Mian & Sufi (2009); Khandani, Lo & Merton (2009)].
New Evidence on the Foreclosure Crisis

Zero money down, not subprime loans, led to the mortgage meltdown.

By STAN LIEBOWITZ

What is really behind the mushrooming rate of mortgage foreclosures since 2007? The evidence from a huge national database containing millions of individual loans strongly suggests that the single most important factor is whether the homeowner has negative equity in a house— that is, the balance of the mortgage is greater than the value of the house. This means that most government policies being discussed to remedy woes in the housing market are misdirected.

Many policy makers and ordinary people blame the rise of foreclosures squarely on subprime mortgage lenders who presumably misled borrowers into taking out complex loans at low initial interest rates. Those hapless individuals were then supposedly unable to make the higher monthly payments when their mortgage rates reset upwards.

But the focus on subprimes ignores the widely available industry facts (reported by the Mortgage Bankers Association) that 51% of all foreclosed homes had prime loans, not subprime, and that the foreclosure rate for prime loans grew by 488% compared to a growth rate of 200% for subprime foreclosures. (These percentages are based on the period since the steep ascent in foreclosures began—the third quarter of 2006 -- during which more than 4.3 million homes went into foreclosure.)
What about upward resets in mortgage interest rates? I found that interest rate resets did not measurably increase foreclosures until the reset was greater than four percentage points. Only 8% of foreclosures had an interest rate increase of that much. Thus the overall impact of upward interest rate resets is much smaller than the impact from equity.

To be sure, many other variables -- such as FICO scores (a measure of creditworthiness), income levels, unemployment rates and whether the house was purchased for speculation -- are related to foreclosures. But liar loans and loans with initial teaser rates had virtually no impact on foreclosures, in spite of the dubious nature of these financial instruments.

Instead, the important factor is whether or not the homeowner currently has or ever had an important financial stake in the house. Yet merely because an individual has a home with negative equity does not imply that he or she cannot make mortgage payments so much as it implies that the borrower is more willing to walk away from the loan.

The difference in policy implications is enormous: A significant reduction in foreclosures will happen when and only when housing prices stop falling and unemployment stops rising (see chart nearby).

Although the government is throwing money -- almost $2 trillion and counting -- at the mortgage markets with the intent of stabilizing house prices, its methods are poorly targeted. While Federal Reserve actions have succeeded in reducing mortgage interest rates, low interest rates induce refinancings more than they do home purchases.

Other government policies are likely to be even less effective in reducing foreclosures. The Obama administration's "Making Homes Affordable" plan focuses on having the government help lower obligation ratios (the share of income devoted to house payments) down to 31% from levels somewhat above 38%. But my analysis finds that mortgages having such obligation ratios at closing did not later experience high foreclosure rates. This suggests that reducing these ratios is not likely to significantly improve the foreclosure problem.
Game theoretic problem: Lender determines modification that maximizes value of loan given that borrower will act strategically in his best interest.
Three Cases

- F: foreclosure present value
- A: Ability to pay
- W: Willingness to pay

- Case 1: F > A  Foreclose
- Case 2: W > A > F  Set to A
- Case 3: A > W > F  Set to W
Model

Loan balance: \( L(t) \)

Home value:

\[
\frac{dV(t)}{V(t)} = (r - \delta)V(t) \, dt + \sigma_1 \, dZ_1(t) + \sigma_2 \, dZ_1(t)
\]

Equity: \( E(t) = V(t) - L(t), \quad \text{LTV} = L(t)/V(t) \)

Kelly (2006) shows that negative equity is a strong determinant of default.

Interest rates:

\[
df(t, T) = \alpha(t, T) \, dt + \beta(t, T) \, dZ_1(t), \quad \forall T.
\]
Quadrinomial Tree

Extended CRR Tree

\[
V(t + 1) = \begin{cases} 
V(t) \exp \left( +\sigma_1 \sqrt{h} + \sigma_2 \sqrt{h} \right), & \text{w/prob } q/2 \\
V(t) \exp \left( +\sigma_1 \sqrt{h} - \sigma_2 \sqrt{h} \right), & \text{w/prob } (1 - q)/2 \\
V(t) \exp \left( -\sigma_1 \sqrt{h} + \sigma_2 \sqrt{h} \right), & \text{w/prob } q/2 \\
V(t) \exp \left( -\sigma_1 \sqrt{h} - \sigma_2 \sqrt{h} \right), & \text{w/prob } (1 - q)/2
\end{cases}
\]

Heath-Jarrow-Morton (HJM) Tree

\[
\forall T, \quad f(t + 1, T) = \begin{cases} 
f(t, T) + \alpha(t, T) h + \beta(t, T) \sqrt{h}, & \text{w/prob } \frac{1}{2} \\
f(t, T) + \alpha(t, T) h - \beta(t, T) \sqrt{h}, & \text{w/prob } \frac{1}{2}
\end{cases}
\]

\[
\sum_{t=1}^{T} \alpha(t, T) = \frac{1}{h^2} \ln \left[ \cosh \left( \sum_{t=1}^{T} \beta(t, T) h^{3/2} \right) \right], \quad \forall T
\]
Risk-neutral Probabilities

\[ V(t) e^{(r(t) - \delta)h} = E[V(t + 1)] \]
\[ = V(t) \exp \left( +\sigma_1 \sqrt{h} + \sigma_2 \sqrt{h} \right) \times q/2 \]
\[ + V(t) \exp \left( +\sigma_1 \sqrt{h} - \sigma_2 \sqrt{h} \right) \times (1 - q)/2 \]
\[ + V(t) \exp \left( -\sigma_1 \sqrt{h} + \sigma_2 \sqrt{h} \right) \times q/2 \]
\[ + V(t) \exp \left( -\sigma_1 \sqrt{h} - \sigma_2 \sqrt{h} \right) \times (1 - q)/2 \]

Solving this equation for \( q \) gives:

\[ q = \frac{2 \ e^{(r(t) - \delta)h} - (u_2 + u_4)}{u_1 + u_3 - u_2 - u_4} \]

where

\[ u_1 = \exp \left( +\sigma_1 \sqrt{h} + \sigma_2 \sqrt{h} \right) \]
\[ u_2 = \exp \left( +\sigma_1 \sqrt{h} - \sigma_2 \sqrt{h} \right) \]
\[ u_3 = \exp \left( -\sigma_1 \sqrt{h} + \sigma_2 \sqrt{h} \right) \]
\[ u_4 = \exp \left( -\sigma_1 \sqrt{h} - \sigma_2 \sqrt{h} \right) \]
Modeling the Mortgage

\[ L_0 = A \int_0^T e^{-r_L t} \, dt = A \left( \frac{1 - e^{-r_L T}}{r_L} \right) \]

\[ A = \frac{r_L L_0}{1 - e^{-r_L T}} = \frac{r_L L_t}{1 - e^{-r_L (T-t)}} \]

\[ A = m \times S = m \times \frac{i L_0}{1 - (1+i)^{-N}} \]

\[ L(t+1) = L(t) \left[ (1+i) - \frac{i}{1 - (1+i)^{-(N-t)}} \right] \]
Mortgage Value

- Lender’s loan value: $B^L(t)$
- Borrower’s loan value: $B^b(t)$
- Prepayment option triggered if $B^b(t) > L(t)+S$
- Put option to default: $P(t) = \max[P(t), V(t) - B^b(t) - K_R]$ 
- On default, $B^L(t) = \phi \; V(t)$, $B^b(t) = V(t)$
“Iso-Service” Surface

Choose

Loan balance = $300,000
Home value = $250,000
Remaining maturity = 25 years
A = $1,933 per month

\[ A \leq A_{\text{max}} \]

\[ A_{\text{max}} = \frac{\text{Home value}}{\text{Remaining maturity}} \times \frac{12}{A} = \frac{250,000}{25} \times \frac{12}{1,933} = $20,000 \text{ per year} \]

\[ (1,667 \text{ per month}) \]

\[ L_0 = \frac{A_{\text{max}}}{m} \left[ \frac{1 - (1 + r_L/m)^{-N}}{r_L/m} \right] \]
Iso-service loans
Values of Iso-Service Loans
Deadweight Foreclosure Costs

Figure 3: How loan principal and deadweight costs determine loan value. In this graph we plot the value of the loan from our model, for a range of loan principal balances, varying from $200,000 to $300,000, given a current home value of $250,000. The parameters used for this graph are: home value volatility parameters $\sigma_1 = 0.02$ and $\sigma_2 = 0.03$, service flow level $\delta = 0.01$, interest rate volatility per annum $\beta = 0.0050$ (i.e., 50 bps), time step $h = 1/4$, loan rate $r_L = 0.06$, relocation costs $K_R = 0$, foreclosure recovery rate $\phi = \{0.7, 0.9\}$, loan maturity $T = 25$ years, and a flat forward rate curve at 5%.
Default Put Exercise Region

L=225,000

L=250,000
Non-zero Relocation Costs
$K_R = 20,000$
Modifying Maturity

The graph illustrates the relationship between the loan value (in thousands) and the annual payment for different maturity periods: T=25 yrs, T=30 yrs, and T=35 yrs. The x-axis represents the annual payment, while the y-axis represents the loan value. The graph shows how the loan value decreases as the annual payment increases for each maturity period.
Figure 7: Loanvalues for with varying drift (net service flow incorporating mean-reversion). We set loan balance to $245,000. Loan maturity is 25 years. The parameters used for this graph are: home value volatility parameters $\sigma_1 = 0.02$ and $\sigma_2 = \{0.03, 0.10\}$, service flow level $\delta = \{-0.01, 0.01, 0.07\}$ for the first five years and $\delta = 0.07$ thereafter, interest rate volatility per annum $\beta = 0.0050$ (i.e., 50 bps), time step $h = 1/4$, loan rate $r_L = 0.06$, relocation costs $K_R = 0$, foreclosure recovery rate $\phi = 0.7$, and a flat forward rate curve at 5%. The figure shows the loan value for three levels of $\delta$ in the first five years of the loan, and for two levels of volatility $\sigma_2$. 
# Empirical Snapshot

<table>
<thead>
<tr>
<th>Source: OCC/Treasury</th>
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## Number of Home Retention Actions

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<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>Loan Modifications</td>
<td>68,001</td>
<td>127,940</td>
<td>114,142</td>
<td>119,220</td>
<td>185,156</td>
<td>55.3%</td>
<td>172.3%</td>
</tr>
<tr>
<td>Payment Plans</td>
<td>134,624</td>
<td>126,114</td>
<td>154,649</td>
<td>177,314</td>
<td>152,036</td>
<td>-14.3%</td>
<td>12.9%</td>
</tr>
<tr>
<td>Total</td>
<td>202,625</td>
<td>254,054</td>
<td>268,791</td>
<td>296,534</td>
<td>337,192</td>
<td>13.7%</td>
<td>66.4%</td>
</tr>
</tbody>
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## Modified Loans 30 or More Days Delinquent

(30+ Re-Default Rate for 2008 Modifications)

- First Quarter: 40.4%
- Second Quarter: 50.4%
- Third Quarter: 58.8%
- Fourth Quarter: 63.3%

Source: OCC/Treasury
# Types of Modifications

<table>
<thead>
<tr>
<th>Type</th>
<th>Total Number of Changes in Each Category</th>
<th>Percentage of 185,156 Modifications Made in the First Quarter of 2009</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capitalization</td>
<td>129,929</td>
<td>70.2%</td>
</tr>
<tr>
<td>Rate Reduction</td>
<td>117,067</td>
<td>63.2%</td>
</tr>
<tr>
<td>Rate Freeze</td>
<td>23,346</td>
<td>12.6%</td>
</tr>
<tr>
<td>Term Extension</td>
<td>46,488</td>
<td>25.1%</td>
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<tr>
<td>Principal Reduction</td>
<td>3,398</td>
<td>1.8%</td>
</tr>
<tr>
<td>Principal Deferral</td>
<td>1,979</td>
<td>1.1%</td>
</tr>
<tr>
<td>Unknown(^{15})</td>
<td>12,962</td>
<td>7.0%</td>
</tr>
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</table>

[15] Total unknown modifications include changes that were not classified in any of the above categories.
Other Findings

- Foote, Gerardi, Goette and Willen (2009) find that a 10% fall in house prices raises the probability of delinquency by more than 50%.

- Liebowitz (2009) analyzes a database from McDash Analytics of 30 million loans and shows that it is not the features of the loan (subprime, Alt-A, etc.) that explain foreclosure, but the extent of negative equity. While only 12% of the loans had negative equity, they accounted for 47% of all foreclosures.

- No difference in prime and subprime.

- Liebowitz: raising rates does not induce foreclosures.

- Foreclosures have contagion effects on neighborhood; Harding, Rosenblatt and Yao (2008).
Playing Mortgage Market Proves Tricky

By RUTH SIMON

As mortgage delinquencies have climbed, hundreds of investors have sought to profit by buying troubled loans from banks and other institutions, restructuring the mortgages to keep borrowers in their homes and quickly moving loans that can't be saved to foreclosure.

But as hedge-fund manager Ralph DellaCamera has learned, it has been a difficult strategy to pull off. Banks have been reluctant to sell loans at the price investors will pay for them. Meanwhile, the time it takes for investors to get their cash back has lengthened.

Some mortgage investors have made principal reduction a part of their strategy, in part because it gives borrowers who owe more than their houses are worth an incentive to keep making payments. It is also easier to ultimately refinance or sell the mortgage if the borrower has equity.

Henry, a respiratory therapist in Charlotte, N.C. Mr. Henry hadn't missed a single payment on his $149,000 interest-only mortgage, though falling prices meant he owed more than the house was worth.

When he opened a letter from iServe offering to reduce his mortgage balance, "my first reaction was to shred it," Mr. Henry recalls. Instead, he wound up with a $53,000 principal reduction that brought his mortgage in line with his home's value and made refinancing possible. Mr. Henry's monthly payments fell by about $380, though he now pays interest and principal.
Logit: Explaining Re-default

<table>
<thead>
<tr>
<th>2007</th>
<th>&lt; 450</th>
<th>450-620</th>
<th>620-820</th>
<th>820-1120</th>
<th>&gt; 1120</th>
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<td></td>
<td>Est.</td>
<td>$\chi^2$</td>
<td>Est.</td>
<td>$\chi^2$</td>
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<tr>
<td>$\Delta$ Rate</td>
<td>-0.11310</td>
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<td>-0.28570</td>
<td>4.212</td>
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<td>$\Delta$ Term</td>
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<td>-0.00133</td>
<td>0.220</td>
<td>0.0114</td>
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<tr>
<td>$\Delta$ Principal</td>
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<td>5.323</td>
<td>-0.00007</td>
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<td>0.00007</td>
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<td>0.02530</td>
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<td>PPC</td>
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<td>0.421</td>
<td>-0.01340</td>
<td>1.888</td>
<td>-0.00324</td>
</tr>
</tbody>
</table>

| Re-Default | 43 | 59 | 66 | 89 | 93 |
| No Default | 230 | 330 | 382 | 459 | 545 |
| Wald Stat | 18.9061 | 0.0043 | 18.8779 | 0.0044 | 15.1021 | 0.0195 | 23.4206 | 0.0007 | 20.8749 | 0.0019 |

<table>
<thead>
<tr>
<th>2008</th>
<th>&lt; 450</th>
<th>450-620</th>
<th>620-820</th>
<th>820-1120</th>
<th>&gt; 1120</th>
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<tr>
<td></td>
<td>Est.</td>
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<td>Est.</td>
<td>$\chi^2$</td>
<td>Est.</td>
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<td>Intercept</td>
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<td>Debt Ratio</td>
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<td>12.026</td>
<td>-0.00572</td>
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<td>-0.01130</td>
</tr>
</tbody>
</table>

| Re-Default | 177 | 205 | 286 | 375 | 522 |
| No Default | 747 | 819 | 1000 | 1071 | 1564 |
| Wald Stat | 32.3832 | < 0.0001 | 42.1224 | < 0.0001 | 46.1375 | < 0.0001 | 83.5345 | < 0.0001 | 81.2047 | < 0.0001 |
Shared-Appreciation Mortgages

Exercise value = $B^b(t) - [V(t) - \theta \cdot C(V(t), K, t)] - K_R$

Figure 8: Loan values with appreciation sharing. The annual payment is $A = 19,000$. The parameters used for this graph are: home value volatility parameters $\sigma_1 = 0.02$ and $\sigma_2 = 0.03$, service flow level $\delta = 0.01$, interest rate volatility per annum $\beta = 0.0050$ (i.e., 50 bps), time step $h = 1/4$, loan rate $r_L = 0.06$, relocation costs $K_R = 0$, foreclosure recovery rate $\phi = 0.7$, loan maturity $T = 25$ years, and a flat forward rate curve at 5%. Appreciation share $\theta$ takes values in the set $\{0, 0.1, 0.5\}$, and the strike of the appreciation sharing agreement is $250,000$. 
Home values

\[ dH_t = \mu H_t \, dt + \sigma H_t \, dZ_t \]

Normalize initial home value to 1. The option to default is ITM when \((H > L)\).

There is a home value \(D\) at which the borrower will default. \(D\) is a “default level” or default exercise barrier.

\(D\) is a function of the lender share \(\theta\), we write it as \(D(L, \theta)\).

\(D\) increases in \(L\) and in \(\theta\).

Foreclosure recovery as a fraction of \(H\) is \(\phi\).
Default Barrier and Lender Share

\[ D = L \exp[-\gamma(1 - \theta)] \]

1. The greater the willingness to pay (\( \gamma \)), the lower is the default level of home value \( D \).
2. When \( \gamma = \infty \), the willingness to pay is infinite, the default level \( D = 0 \). The borrower never defaults unless the home value goes to zero.
3. When \( \gamma = 0 \), there is no willingness to pay and the default level is \( D = L \), i.e., the borrower defaults the moment the home value drops infinitesimally below LTV at the time zero.

1. The greater the lender's share (\( \theta \)), the higher is the default level of home value \( D \). The likelihood of default is therefore greater.
2. When the lender share \( \theta = 0 \), the default level is \( L \exp[-\gamma] \).
3. When \( \theta = 1 \), the default level is \( D = L \). The borrower defaults the moment there is negative equity.
Barrier Model Intuition

\[ D = L \exp[-\gamma(1-\theta)] \]

Region of default

Region of no default and gains to SAM

\[ H_0 = 1 \]

Default Payoff = φD

No default Payoff = L
A Barrier Option Decomposition

Non-default component

\[ L e^{-rT} \int_{D(L, \theta)}^{\infty} p(H_T | H_t > D, \forall t < T) \, dH_T \]

where \( p(H_T | H_t > D, \forall t < T) \) is the density of the terminal home value conditional on no interim default.

Default component

\[ \phi D \int_{0}^{T} e^{-rt} f(t; D) \, dt \]

where \( f(t; D) \) is the first-passage time density for \( H_t = D \).

Shared Appreciation component

\[ e^{-rT} \int_{K}^{\infty} (H_T - K) p(H_T | H_t > D, \forall t < T) \, dH_T \]

PDE

\[
\frac{\partial F}{\partial H} [\mu - \lambda \sigma] H + \frac{1}{2} \frac{\partial^2 F}{\partial H^2} \sigma^2 H^2 + \frac{\partial F}{\partial t} = r F
\]
The Closed-Form Solution

\[ \text{LOANVAL} \equiv V(H, L, K, r, T, \phi, \theta, \mu, \lambda, \sigma, \gamma) \]

\[ = Le^{-rT} \left[ N(d'_2) - \frac{(D/H)^{2(R/\sigma^2)^2} - 1}{N(d'_{2b})} + \phi D \left[ (D/H)^{b_1} \cdot N(a_1) + (D/H)^{b_2} \cdot N(a_2) \right] + \theta \left[ \text{CSAM}(H, K) - D^{2(R/\sigma^2)^2} \cdot \text{CSAM}(D^2/H, K) \right] \right] \]

\[ d'_2 = \frac{\ln(H/D) + (R - 0.5\sigma^2)T}{\sigma \sqrt{T}} \]

\[ d'_{2b} = \frac{\ln(D/H) + (R - 0.5\sigma^2)T}{\sigma \sqrt{T}} \]

\[ a_1 = \frac{\ln(D/H) + \sqrt{2r\sigma^2} + (R - 0.5\sigma^2)^2 \cdot T}{\sigma \sqrt{T}} \]

\[ a_2 = \frac{\ln(D/H) - \sqrt{2r\sigma^2} + (R - 0.5\sigma^2)^2 \cdot T}{\sigma \sqrt{T}} \]

\[ b_1 = \frac{(R - 0.5\sigma^2) + \sqrt{2r\sigma^2} + (R - 0.5\sigma^2)^2}{\sigma^2} \]

\[ b_2 = \frac{(R - 0.5\sigma^2) - \sqrt{2r\sigma^2} + (R - 0.5\sigma^2)^2}{\sigma^2} \]

\[ \text{CSAM}(x, y) = xe^{-(r-R)T} N(d'_1) - ye^{-rT} N(d'_1 - \sigma \sqrt{T}) \]

\[ d'_1 = \frac{\ln(x/y) + (R + 0.5\sigma^2)T}{\sigma \sqrt{T}} \]
SAM or not?

Fig. 2. Loan value as LTV is varied for loans with and without appreciation sharing. The parameters for the plot are as follows: willingness to pay coefficient $\gamma = 0.1$, home price volatility $\sigma = 0.04$, foreclosure fraction $\phi = 0.7$, risk-free rate $r = 0.02$, the house value growth rate $\mu = 0.04$, price of risk $\lambda = 0.25$, and the horizon of the model $T = 5$ years. The appreciation share fraction is $\theta = 0.50$ for the case when a SAM is applied, and $\theta = 0$ when there is no share appreciation.
Fig. 3. Loan value as LTV is varied for loans with SAMs and the foreclosure recovery rate is varied across \{\phi = 0.5, \phi = 0.7\}. Both cases are with appreciation sharing. The parameters for the plot are as follows: willingness to pay coefficient \( \gamma = 0.1 \), home price volatility \( \sigma = 0.04 \), risk-free rate \( r = 0.02 \), the house value growth rate \( \mu = 0.04 \), price of risk \( \lambda = 0.25 \), and the horizon of the model \( T = 5 \) years. The appreciation share fraction is \( \theta = 0.50 \).
Fig. 4. Loan value as LTV is varied for loans with SAMs and housing price volatility is varied across \(\{\sigma = 0.04, \sigma = 0.10\}\). Both cases are with appreciation sharing. The parameters for the plot are as follows: willingness to pay coefficient \(\gamma = 0.1\), foreclosure percentage \(\phi = 0.7\), risk-free rate \(r = 0.02\), the house value growth rate \(\mu = 0.04\), price of risk \(\lambda = 0.25\), and the horizon of the model \(T = 1\) year. The appreciation share fraction is \(\theta = 0.50\).
Fig. 5. Loan value as LTV is varied for loans with SAMs and housing value growth rate is varied across \( \mu = -0.04, \mu = +0.04 \). Both cases are with appreciation sharing. The parameters for the plot are as follows: willingness to pay coefficient \( \gamma = 0.1 \), foreclosure percentage \( \phi = 0.7 \), risk-free rate \( r = 0.02 \), housing price volatility \( \sigma = 0.04 \), price of risk \( \lambda = 0.25 \), and the horizon of the model \( T = 1 \) year. The appreciation share fraction is \( \theta = 0.50 \).
Fig. 6. Loan value as LTV is varied for loans with SAMs and willingness to pay is varied across \( \{ \gamma = 0.01, \gamma = 0.10, \gamma = 0.20 \} \). All cases are with appreciation sharing. The parameters for the plot are as follows: the house value growth rate \( \mu = 0.04 \), price of risk \( \lambda = 0.25 \), foreclosure percentage \( \phi = 0.7 \), risk-free rate \( r = 0.02 \), housing price volatility \( \sigma = 0.04 \), and the horizon of the model \( T = 1 \) year. The appreciation share fraction is \( \theta = 0.50 \).
Fig. 7. Loan value as LTV is varied for loans with SAMs and appreciation is varied across \{\theta = 0.70, \theta = 0.50\}. The parameters for the plot are as follows: willingness to pay coefficient $\gamma = 0.1$, foreclosure percentage $\phi = 0.7$, risk-free rate $r = 0.02$, housing price volatility $\sigma = 0.04$, and the horizon of the model $T = 5$ years. The growth rate in home values is $\mu = 0.04$, and the price of risk $\lambda = 0.25$. 
Two-Horizon Model

We accommodate varying growth rates over short and long horizons.

This captures mean-reversion in house price levels.

The model is embedded on a Cox-Ross-Rubinstein tree that accommodates American option features and risk premia for non-tradeability of housing assets.
Mean-reversion in home value

Fig. 8. Two-horizon model: Loan value as LTV is varied for loans with and without appreciation sharing. The parameters for the plot are as follows: willingness to pay coefficient $\gamma = 0.1$, home price volatility is $\sigma = 0.04$, foreclosure recovery fraction $\phi = 0.7$, risk-free rate $r = 0.03$, the house value growth rate in each period is $\mu_1 = -0.05$ and $\mu_2 = +0.04$, price of risk $\lambda = 0.25$, and the two horizons of the model are $\tau = 1$ and $T = 10$ years. The appreciation share fraction is $\theta = 0.50$ for the case when a SAM is applied, and $\theta = 0$ when there is no share appreciation. The coupon rate on the loan is 3.3%. The coupon rate on the loan has been set such that at the optimal LTV, the loan with a SAM prices up close to par.
Macroeconomic Sequencing

Fig. 9. Two-horizon model: Loan value as LTV is varied for loans when there are periods of positive and negative growth in home values. The parameters for the plot are as follows: willingness to pay coefficient $\gamma = 0.1$, home price volatility is $\sigma = 0.04$, foreclosure recovery fraction $\phi = 0.7$, risk-free rate $r = 0.03$, the house value growth rate in each period is $\mu_1 = -0.05$ and $\mu_2 = +0.05$ (reversed in the second case), price of risk $\lambda = 0.25$, and the two horizons of the model are $\tau = 2$ and $T = 4$ years. The appreciation share fraction is $\theta = 0.50$. The coupon rate on the loan is 4.4%. The coupon rate on the loan has been set such that at the optimal LTV, the loan for the first up then down scenario prices up close to par.
Parsimonious Model

INPUTS:
- $\sigma$: Home price volatility (Case-Shiller)
- $\mu$: Growth rate in home values (Forecast)
- $\Phi$: Foreclosure fraction of home value (Empirical)
- $\lambda$: Market price of risk (?)
- $\gamma$: Borrower’s willingness to pay (?)

OPTIMIZATION PARAMETERS:
- $\theta$: Share in appreciation on mortgage
- $L$: LTV after modification
- $T$: Maturity of the analysis (1-2 years)

$\beta(\mu^* - r) = \lambda \sigma$

$\frac{\partial F}{\partial H}[g - \beta(\mu^* - r)]H + \frac{1}{2} \frac{\partial^2 F}{\partial H^2} \sigma^2 H^2 + \frac{\partial F}{\partial t} = rF$

Garman 1986
Summary

✓ Using a dynamic game-theoretic optimization problem, we see that principal write-downs are indeed optimal and much preferred to interest-rate reductions, or maturity extensions of loans.

✓ We derive a mathematical decomposition of the loan value into options that admit closed-form solutions, that may be used to optimize loan modification in practice.
“Resizing” -- A Long Run Solution

Income shocks render homeowners incapable of making all home-related payments, debt service plus costs like insurance, taxes, etc.

In the long run, moving homeowners to homes they can actually afford with a margin of wealth and income risk is the only solution.

Example: $V=$200,000. $L=$150,000. Paid in principal=$80,000. 
Equity = $V-L = $50,000

Monthly service=1% of $L$, i.e., $s=1000$. 
Costs=0.25% of $V$, i.e., $c=500$. 
$s+c=2,000$

After income shock can only make $y=1500$ in burden. 
If foreclosed, bank bears deadweight loss of 30%, i.e., $60,000$. 
A short-sale is better, saving credit quality for the homeowner, and saving money for the bank as sale price is usually better than foreclosed price.
Two alternatives

Principal Reduction

We consider first the loan modification option using our example. To make the loan affordable, the bank will have to write down the loan balance to $L = 100,000$, i.e., a principal reduction of $50,000$. This is less than the deadweight cost of foreclosure of $60,000$. Now, we see that the total loan service is

$$s + c = 0.01L + 0.0025V = 0.01(100,000) + 0.0025(200,000) = 1,000 + 500 = 1500 \leq y$$

Resizing Homes

Next, we explore the resizing option. Given that the homeowner can support a monthly burden of $y = 1,500$, we solve for the maximum house size that he should move to. Denote the value of the new home as $V^N$. The homeowner invests the recovered equity from the old home of $50,000$ for a loan amount of $L^N = V^N - 50,000$. The housing burden for this home size is given by the following equation:

$$0.01L^N + 0.0025V^N = 0.01(V^N - 50000) + 0.0025V^N = 1500$$

Solving, we get $V^N = 160,000$. For this home value, i.e., a 20% reduction in home size, the monthly service payment is $s = 1,100$ and the costs are $c = 400$. 
The Benefits

Help homeowner move into a smaller home that he can afford and lower costs. Bank gives equity to homeowner instead of principal reduction.

Bank saves
1. deadweight loss,
2. sells REOs,
3. keeps customer.

Homeowner
1. retains home ownership,
2. credit rating,
3. negotiates equity subsidy from bank,
4. has less debt, and
5. lower monthly costs.