Managerial Incentives and Stock Price Manipulation

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Abstract

This paper presents a model of optimal executive compensation in a setting where managers are in a position to manipulate short-term stock prices, and managers’ propensity to manipulate is uncertain. Stock-based incentives elicit not only productive effort, but also costly information manipulation. We analyze the tradeoffs involved in conditioning pay on long- versus short-term performance and characterize a second-best optimal compensation scheme. The paper shows manipulation, and investors’ uncertainty about it, affects the equilibrium pay contract and the informational efficiency of asset prices. The paper derives a range of new cross-sectional comparative static results and sheds light on corporate governance regulations.

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1 Introduction

The dissemination of timely and accurate value-relevant information is a key responsibility of corporate management and the foundation of a well-functioning stock market. But the temptation to manipulate stock prices is a perennial issue of concern that has come to the forefront again in a continuing spate of accounting scandals, of which Enron and WorldCom are only the most notorious: the GAO (2002, 2006) reports that substantive restatements of company accounts have become increasingly commonplace, with financial reporting fraud and/or accounting errors both high and growing significantly between 1997 and 2005. This increase in misleading corporate disclosures has come in the wake of an unprecedented increase in performance-related pay, particularly in the form of option-based compensation. The use of stock and option based compensation packages for top executives has increased dramatically since the late 1980s (see, for example, Murphy, 1999 and Hall and Murphy, 2003). Over a period of less than a decade, the average sensitivity of pay to firm value rose tenfold from the 0.3% reported by Jensen and Murphy (1990) to the 3% reported by Hall and Lieberman (1998).

It is natural to think that the two trends may be related, and that stock-based compensation gives executives an incentive to influence the stock price that determines their total pay. A growing body of empirical evidence confirms that there is a link. Recent studies finding a positive relationship between stock based compensation and indicators of information manipulation (such as the degree of earnings management as measured by accruals, SEC accounting enforcement actions, accounting restatements, and shareholder class action litigation) include Bergstresser and Philippon (2006), Burns and Kedia (2006), Peng and Röell (2008), and Johnson Ryan and Tian (2008). This suggests that there is a downside to performance-based pay, raising the issue of how compensation contracts can be designed to balance the incentives for productive effort against the incentives for wasteful manipulation.

This paper presents a model of optimal executive compensation in a setting where managers are in a position to manipulate short-term stock prices. Stock-based incentives elicit not only productive effort, but also costly information manipulation. We analyze the tradeoffs involved in conditioning pay on long- versus short-term performance and characterize a second-best optimal compensation scheme. The model used has a number of salient features.

First, a key innovative feature of our framework is the idea that investors are uncertain about the degree to which reports on the firm’s performance are inflated. If all managers had the same propensity to exaggerate their reported performance, a rational investor would be able to back out the degree of manipulation and a correct assessment would be reflected in the share price as, for example, in Stein’s (1989) signal-jamming model. In such a setting, manipulation can be costly and wasteful, but it does not have any systemic impact on the efficiency of stock market price formation. In reality, investors are often faced with a significant amount of ex ante uncertainty about the degree to which the
information released by managers is over-optimistic: it depends on the manager’s ethical compass and his ability to project a possibly unjustified aura of success, on the susceptibility of the firm’s business opportunities to hype, or on the manager’s unique ability to sell the merits of an innovative project to the investing public. The success of business enterprises can be hard to evaluate, particularly in high-tech growth industries, industries with intangible assets (such as patents), and more generally industries where current earnings are a poor guide to the future (for example, the case of IT companies that develop large but infrequent customer-specific software projects, for which the timing of revenue recognition is a major issue). Not all management teams are equally adept at managing investors’ perceptions, and indeed a few are downright dishonest (Enron is an extreme example of a strikingly huge chasm between the financial reports and the true state of the company). Faced with such uncertainty about the degree to which reports are inflated, investors cannot correctly gauge the true state of the firm. This uncertainty imposes an additional source of risk on both the investors and the managers, which results in informationally inefficient stock prices and less effective contracting, even in settings where the direct costs of manipulation are not so large or merely involve a transfer of relatively small amount of money from the shareholders to the managers. The introduction of manipulation uncertainty into a model of optimal contracting results in three new implications that has not been discussed in the previous literature.

First, we show that, paradoxically, when manipulation uncertainty is high, pay is actually more sensitive to the short-term stock price even though the equilibrium effort level induced is lower. The reason is that when there is more uncertainty associated with manipulation propensity, the stock prices are less responsive to managers’ reports since they are less reliable. Then a more high powered contract is needed to incentivize effort. Our model therefore has implications for the optimal contract across different firms or industries – those that exhibit a more uncertain propensity to manipulate are more likely to use higher powered incentive contracts. Furthermore, such firms have less informative stock prices in the short run and higher ex ante return uncertainty in the long run.

Secondly, our model illustrates that the strength of incentives from stock-based compensation depends on the product of the price sensitivity of pay and the sensitivity of stock prices to reported firm performance. The empirically often used stock price sensitivity (or elasticity) of pay may be a misleading measure of the strength of incentives. A higher sensitivity of pay to stock price could actually correspond to lower effort if the stock market is less responsive to performance reports due to a higher likelihood of manipulation.

Thirdly, the model show that regulation or policies that help to reduce manipulation uncertainty can potentially increase firm value by improving contracting efficiency and making it possible to induce more effort. On the contrary, regulations that simply make manipulation more costly on average may not have any real impact.

A second area of focus of our analysis is the optimal mix of long- and short-
term incentives, extending Peng and Röell (2008b), which analyzes only the optimal short term contract. It is important to allow for long term incentive pay components in the contract, because long term incentives align the manager’s objectives with long term shareholder value, thereby mitigating the waste of valuable managerial time and resources on manipulation associated with short term incentives. The disadvantage of long term incentive pay is the extra risk from longer term exogenous shocks to firm value that are incorporated into the risk averse manager’s pay. In general, when long term incentives are allowed, at least part of the incentive pay is shifted from the short term to the long term, resulting in higher equilibrium effort choice and enhanced firm value. On the other hand, pay will be more closely linked to the short-term stock price if waiting for the long term outcome entails a substantial increase in the riskiness of the pay package, due to shocks that are outside the current manager’s control. Practical examples of pay that is de facto sensitive to long term performance include stock and option pay plans with long vesting periods; the structuring of private equity contracts so that any excessive carried interest distributions to the general partners are “clawed back” at the end of the fund’s life; and more generally, the clawback of unwarranted pay in the wake of misconduct imposed in Section 304 of the Sarbanes-Oxley Act of 2002. For example, in December 2007 the ex-CEO of United Health settled an options backdating case by agreeing to reimburse the company for $600 mn worth of previously awarded bonus pay and options.

A third key feature of the analysis is that we formulate a multiplicative model of optimal executive pay contracts, in which the manager’s preferences are constant relative risk averse and Cobb-Douglas in wealth (consumption) and leisure, effort is modeled as a time cost and impacts firm value proportionately rather than additively, and firm value is lognormally distributed. Meanwhile, compensation schemes are limited to a loglinear form, rather than the usually posited linear form, in order to keep the analysis tractable and generate closed-form solutions. The traditional basic agency model of executive compensation has normally distributed firm value, on which effort has an additive impact, the manager is constant absolute risk averse and the cost of effort is independent of his wealth, and pay is linear in firm value. These features may be poor descriptors of reality and thus hard to bring to the data: a concern that was, for example, the central focus of Baker and Hall (2004). Therefore, quite independently of the issues relating to manipulation, it is useful to add a new model to the arsenal of examples of agency models of executive compensation that can be analyzed with relative ease. Our specification is worth exploring in its own right because it has features that seem more realistic than the usual normal-exponential model: in particular, the lognormal distribution for firm value; the notion that the money value that executives ascribe to their time is likely to be increasing in their wealth, while the fruits of their efforts are likely to be more valuable for larger firms; and constant relative risk aversion. Different from the traditional models, we show that the optimal incentives may depend on firm size since managers’ effort affects firm value proportionately to firm size while the disutility of effort is proportional to managerial wealth.
Interestingly, the model makes direct predictions about elasticities rather than sensitivities of pay to firm value, thus providing testable comparative static insights that are more closely aligned with the empirical literature. In his survey of empirical work on executive compensation, Murphy argues that in comparison with the sensitivity approach, the elasticity approach generally produces a better empirical "fit" in cross-sectional analyses of the relationship between pay and firm value and has the desirable property that it can be better compared across firms of different size. It is always useful to check the robustness of model predictions to seemingly quite minor changes in modeling strategy when analyzing executive compensation contracts and drawing conclusions regarding their optimality.

A final innovative feature of the model that deserves mention is the formulation of the cost of manipulation as a time cost. In the model, managers spend time attempting to improve the stock market’s valuation of the company, when they could be spending the time on efforts directed at enhancing the firm’s true value. In the real world, the time constraint is one of the most important constraints faced by managers. And they do complain of the significant amount of time and attention they are forced to devote to public relations and reassuring the stock market (in Europe, prominent business leaders have pointed out that the threat of a takeover, now that corporate control is more contestable than it used to be, is having the unfortunate side effect of distracting management from running the underlying business). This time cost comes out clearly in the London Stock Exchange’s *A Practical Guide toListing*¹:

>*Both the flotation process itself and the continuing obligations – particularly the vital investor relations activities . . . - use up significant amounts of management time which might otherwise be directed to running the business . . . It is vital that you maintain your company’s profile, and stimulate interest in its shares on a continuing basis. Many listed companies, even relatively small ones, employ specialist financial public relations and investor relations advisors on a retainer basis to keep the business on the financial pages and in the minds of investors . . . However, you cannot leave press or investor relations to your advisers. Top executives will commonly devote at least a couple of days a month to developing and nurturing such contacts . . . This commitment will increase sharply around regular announcements , . . , at the launch of a new product or strategy, or at times when the business or its profile have been hit by adverse events. This must be regarded as time well-spend . . . As a publicly-quoted company, it is a core element of running your business properly and responsibly.” (pp. 11, 47-48)

We model the cost of manipulation entirely as a demand on managerial time, complementing the approaches taken in the literature in a plausible way. Stein (1989) and Bolton *et al.* (2005, 2006) view the main cost as a distortion of real

¹Available at http://www.londonstockexchange.com.
investment decisions towards projects that give palpable results in the short run. Other papers such as Goldman and Slezak (2006) regard it as a money cost: expenditure on accountants, etc. Kedia and Philippon (2008) argue that overinvestment and excessive employment are associated with earnings management. Our approach complements these other views in an interesting and plausible way.

In the context of the literature on manipulation, our focus on manipulation uncertainty is in the spirit of Fischer and Verrecchia (2000), who show how it affects the relationship between stock prices and earnings reports, although they do not attempt to model the optimal incentive contract. Our model shares a common purpose with that of Goldman and Slezak (2006) in that we focus on characterizing a second-best optimal compensation scheme considering information manipulation by the managers. Unlike our model and that of Fischer and Verrecchia, they assume that all managers share the same inclination to manipulate so that in equilibrium, managers’ reports are all equally biased, generating a "signal jamming" equilibrium in which investors can perfectly infer the true level of performance. Their model also differs substantially from this paper in the four aforementioned aspects. While in our setup the principle maximizes the long-term firm value, Bolton, Scheinkman and Xiong (2005 and 2006) explore the conflict between the current and future shareholders: the current shareholders are shortermistic and gives an incentive contract that encourages driving up short-term share prices. Lastly, this paper extends Peng and Röell (2008b) by investigating the interplay of short-term and long-term incentives in the optimal pay contract.

The rest of the paper is organized as follows: In Section 2 we describe the model setup and solve for equilibrium stock price, manager’s problem and expected firm value, for any given pay contract. Section 3 presents the benchmark model of optimal contracts in the absence of manipulation. We analyze short term optimal pay contracts with manipulation in Section 4. We characterise the more general contract that trades off short- and long- term incentives and discuss the empirical predictions of the model and policy implications in Section 5. Section 6 concludes. The analytical proofs of the propositions are presented in the Appendix.

2 The model

In this section we describe and motivate the basic model and solve it for the equilibrium stock price, expected managerial utility and pay, and expected firm value, for any given managerial pay contract. These results will be drawn upon in subsequent sections to characterize executive pay contracts in a variety of settings.
2.1 Model assumptions

We analyze a multiperiod model with decisions taken at dates 0 and 1 and final payoffs established at date 2. At time 0, the manager and the firm’s shareholders sign a contract *ex ante*, before the cost of manipulation is observed. The shareholders are risk neutral: their objective is to maximize expected firm value net of managerial compensation. The manager is assumed to be risk averse. Between dates 0 and 1, the manager finds out the cost of manipulation. He then chooses his level of productive effort as well as his degree of manipulation; the latter is modeled as a factor by which he inflates a report regarding firm value at date 1. The true underlying value of the firm at date 1 is determined by an exogenous shock and the manager’s level of effort. The performance report released by the manager at this point conflates the true value of the firm at date 1 and his degree of manipulation. The stock price at time 1 is based on the manager’s report. At date 2, the true long-term value of the firm, which may incorporate a further exogenous shock, is revealed. For simplicity, we set the interest rate to zero.

The manager is constant relative risk averse, with preferences that are Cobb-Douglas in leisure and money:

$$U = \frac{1}{\phi} \left[ (L - C_E E - C_M M)^\Psi W \right]^\phi \quad \text{where } \Psi > 0 \text{ and } \phi < 1$$  \quad (1)

where $L$ is his time endowment, $E \geq 0$ is effort devoted increasing the firm’s true value, $M \geq 0$ is effort devoted to upward manipulation of the firm’s perceived value, and $W$ is the manager’s wealth, derived from his employment at the firm.\(^2\)

The manager’s personal time cost for the two types of effort is parameterized by the constants $C_E$ and $C_M$ respectively. For convenience we assume that $C_E$ is a fixed, known cost of effort that is identical for all managers, but that the manipulation cost parameter $C_M$ is random, with the following log normal distribution:

$$\ln C_M = c_M \sim N (\sigma_M, \Omega)$$  \quad (2)

The true value of the company is subject to random shocks, $\epsilon_1$ and $\epsilon_2$, at dates 1 and 2 respectively. It is assumed that the firm’s value in the short run, $V_1$, and in the long run, $V_2$, is multiplicative in the manager’s *bona fide* effort $E$:

\(^2\)An alternative model of manipulation cost regards it as a psychological cost for the manager, that is:

$$U = \frac{1}{\phi} \left[ (L - C_E E)^\Psi (R - C_M M)^\phi W \right]^\phi .$$

where $R$ is an endowment of "self-respect". This formulation yields very similar results to the version used in this paper.
where multiplicative value shocks $\epsilon_1$ and $\epsilon_2$ are independent mean-one lognormally distributed, $\epsilon_t \equiv \ln \epsilon_t \sim N\left(-\frac{1}{2}\Sigma_t, \Sigma_t\right)$ for $t = 1, 2$.

At time 1, the manager sends a report $S$ about the firm’s true value that portrays the firm’s true value $V_1$ as observed by him at time 1, factored up by a manipulation multiple $M$:

$$S = MV_1 = MEV_0 \epsilon_1$$

Participants in the stock market observe the report, and determine the market price as

$$P_1 = E[V|S]$$

where for convenience we define $P_1$ as the gross-of-pay stock price, that is, the gross expected value of the firm, before executive pay is deducted: the actual stock market capitalization would be equal to this expected gross value of the firm, minus expected executive pay.

We conjecture that $P_1$ takes the following log-linear price form:

$$P_1 = \pi V_0^{1-\gamma} \cdot S^\gamma$$

for some values of the parameters $\pi$ and $\gamma$ that are to be determined in equilibrium.

We restrict attention to a three-parameter pay contract where the manager’s compensation package (parameterized by $\{\omega, \mu, \eta\}$) is constant-elastic in the firm’s long- and short-term value, that is, the pay contract takes the log-linear form:

$$W = \omega P_1^\mu V_2^\eta$$

This multiplicative functional form is chosen as a convenient approximation that yields closed-form solutions.

Note that the manager’s pay does not depend directly on his report $S$: such a report is presumed to be unverifiable and too complex to summarize into a form that a pay contract can be based upon. It may include predictions about market share, product quality, earnings, the business climate, the competence and health of the management team, etc. It is left to the impersonal judgment of stock market participants to distil this information into a summary judgment about the firm’s underlying value, captured by $P_1$.

A rather unusual aspect of the multiplicative specification is that performance at dates 1 and 2 does not enter additively into pay, but in mutually reinforcing fashion. This is not unrealistic. The pay contract can be thought of as one in which short-run performance as represented by the stock price at time 1 determines the size of a package of fixed pay, stock and options to be
awarded to the manager; if the firm outperforms in the long run, this package will increase further in value. The chosen functional form, though restrictive, is tractable and gives direct insights into the optimal elasticity of pay to firm performance.

2.2 The manager’s problem

After signing the compensation contract at date 0, the manager discovers \( C_M \), that is, how costly it is to manipulate the performance signal. He then chooses the level of effort \( E \) that he will exert and the manipulation factor \( M \) by which he will scale up his report of the firm’s value at time 1:

\[
\max_{E, M} \mathbb{E}_1 \left[ \left( \bar{L} - C_E E - C_M M \right)^\Phi \hat{W} \right] ^\phi \tag{8}
\]

Substituting out \( P_1 \) and \( V_2 \) in equation (7) using equations (3) and (6), the manager’s compensation can be expressed as:

\[
\hat{W} = \omega \hat{P}_1 V_2 = \omega \pi^\mu V_0^{\mu + \eta} M^{\gamma \mu + \eta} \varepsilon_1^{\mu + \eta} \tag{9}
\]

The manager’s optimal choice of effort, manipulation and leisure is then given by:

\[
E = \frac{\bar{L}}{C_E \Psi + 2 \gamma \mu + \eta} \gamma \mu + \eta \tag{10}
\]

\[
M = \frac{\bar{L}}{C_M \Psi + 2 \gamma \mu + \eta} \exp \left\{ k_M - c_M \right\}
\]

\[
L = \bar{L} - C_E E - C_M M = \frac{\Psi}{\Psi + 2 \gamma \mu + \eta} \bar{L} \tag{11}
\]

2.3 The short run stock price

At time 1, the stock market observes only the report sent by the manager, not the true value of the firm. Market participants do observe the pay contract signed at time 0 and understand the manager’s incentives to inflate the performance report. Thus they correctly back out the optimal level of effort exerted by the manager and form estimates of the firm’s value and the level of manipulation based on the ex-ante distribution of the manipulation cost \( C_M \) and the shock to the firm’s value \( \varepsilon_1 \). The rational-expectations short term stock price (gross of expected compensation) at time 1 is the conditional expectation of the firm value given the manager’s report \( S \), set down in the following proposition.
Proposition 1  The short term gross-of-pay stock price is given by

\[ P_1 = \pi V_0^{1-\gamma} S^\gamma \]

where

\[ \gamma = \frac{\Sigma_1}{\Omega + \Sigma_1} \]  \hspace{1cm} (12)

and

\[ \pi = \left( \frac{\mathcal{T}}{C_E \Psi + 2\gamma \mu + \eta} \right)^{1-\gamma} \left( \frac{\mathcal{T}}{\exp \tau_M \Psi + 2\gamma \mu + \eta} \right)^{-\gamma} \]

i.e.

\[ \ln \pi = (1-\gamma) (k_E - c_E) - \gamma (k_M - \tau_M) \]  \hspace{1cm} (13)

and \( k_E, k_M \) are defined in equation (A1).

Proof: see appendix.

The logarithm of the short term stock price is a weighted average of the logarithms of the initial firm value at time 0 and manager’s report. Accounting for the possibility that the report \( S \) may be manipulated, the market attaches less weight \( \gamma \) to the report, the more noisy it is as a result of uncertainty about the manager’s manipulation cost (\( \Omega \)). On the other hand, holding the report’s noisiness constant, the greater the fundamental uncertainty about the firm (\( \Sigma_1 \)), the more important it is to update the estimate of firm value and therefore the greater the weight on the report. In short, the sensitivity \( \gamma \) of the short-term stock price to the manager’s report decreases with \( \Omega \) but increases with \( \Sigma_1 \).

2.4 Manager’s expected utility and the net firm value

Given any compensation contract \( \{\mu, \omega, \eta\} \), the manager expects to choose optimal levels of effort and manipulation at the interim date 1, and the stock market to respond rationally to his report. Standing at date 0, before the realization of the manipulation cost or any random shocks to the firm’s value, his ex ante expected utility is as given in the following proposition.

Proposition 2  The manager’s ex ante log expected utility, before he is aware of his own propensity to manipulate, is given by:

\[
\ln E_0 [U] = -\ln \phi + \phi \Psi \ln \frac{\Psi}{\Psi + 2\gamma \mu + \eta} \mathcal{T} + \phi \left\{ \ln \omega + (\mu + \eta) \ln \left( \frac{\gamma \mu + \eta}{\Psi + 2\gamma \mu + \eta} \frac{V_0 \mathcal{T}}{C_E} \right) \right\} - \frac{\phi}{2} \left( (\gamma \mu + \eta) \Sigma_1 + \eta \Sigma_2 \right) + \frac{\phi^2}{2} \left[ (\gamma \mu^2 + 2\gamma \mu \eta + \eta^2) \Sigma_1 + \eta^2 \Sigma_2 \right]
\]

where \( \gamma \) is given in Proposition 1 and \( \omega, \mu \) and \( \eta \) are the terms of the compensation contract that is signed.

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For notational convenience we will sometimes work with money-equivalent transformed expected utility $E^{eq}[U]$, defined as the amount of money needed to give the agent utility $E[U]$ if he has leisure equal to $L$, that is, he does not spend any time working or manipulating $E^{eq}[U] = 1/L E[U]$. Therefore, the money-equivalent transformed expected utility can be expressed as:

$$E^{eq}[U] = \frac{1}{\phi} \left( \frac{\phi E[U]}{L} \right)^{1/\phi}. \quad (15)$$

Using Proposition 2, the logarithm of money-equivalent expected is given by:

$$\ln E^{eq}_0[U] = \Psi \ln \frac{\Psi}{\Psi + \alpha (\gamma \mu + \eta) + \beta \gamma \mu} + \ln \omega$$

$$+ (\mu + \eta) \ln \left\{ V_0 \left[ \frac{L}{C E} \frac{\alpha (\gamma \mu + \eta)}{\Psi + \alpha (\gamma \mu + \eta) + \beta \gamma \mu} \right] \right\}^{\alpha}$$

$$- \frac{1}{2} (\gamma \mu + \eta) \Sigma_1 + \eta \Sigma_2 + \frac{\phi}{2} \left[ \left( \gamma \mu^2 + 2 \gamma \mu \eta + \eta^2 \right) \Sigma_1 + \eta^2 \Sigma_2 \right]. \quad (16)$$

Using the proof of Proposition 2, inserting $\phi = 1$ and $L = L$, we have the ex ante expected wealth of the manager:

$$E_0[W] = \omega \left( \frac{V_0 L}{C E} \right)^{\mu+\eta} \left( \frac{\gamma \mu + \eta}{\Psi + 2 \gamma \mu + \eta} \right)^{\mu+\eta}$$

$$\cdot \exp \left\{ \frac{1}{2} \left[ (\gamma \mu + \gamma \mu^2 + 2 \gamma \mu \eta - \eta + \eta^2) \Sigma_1 + (\eta + \eta^2) \Sigma_2 \right] \right\}. \quad (17)$$

The expected firm value is the gross expected value minus the expected payment to the manager.

**Proposition 3** The ex ante company expected firm value, net of executive compensation, is:

$$E_0[V_2 - W] = \frac{\gamma \mu + \eta}{\Psi + 2 \gamma \mu + \eta} \left( \frac{V_0 L}{C E} \right)^{\mu+\eta} - \omega \left( \frac{\gamma \mu + \eta}{\Psi + 2 \gamma \mu + \eta} \frac{V_0 L}{C E} \right)^{\mu+\eta}$$

$$\cdot \exp \left\{ \frac{1}{2} \left[ (\gamma \mu + \gamma \mu^2 + 2 \gamma \mu \eta - \eta + \eta^2) \Sigma_1 + (\eta + \eta^2) \Sigma_2 \right] \right\}. \quad (18)$$
3 Optimal contracts in the absence of manipulation

In this section, we analyse two benchmark settings. First, we consider the first-best level of effort and pay in the absence of agency problems. We next characterize the optimal second-best contract in a setting without manipulation, that is, the setting of the classical principal-agent problem. The results will serve as a basis for comparison with the optimal contracts in the presence of manipulation that will be analyzed subsequently.

3.1 The first best

In the first best optimum, Pareto-efficient effort and compensation are chosen to maximize the expected wealth of the firm’s owners, subject to the agent’s participation constraint of money-equivalent utility being at least $W$, the equivalent reservation wage. Of course no wasteful manipulation takes place ($M^* = 0$), and effort and compensation solve the program:

$$\max_{\{E, W\}} E \cdot V_0 - W$$
subject to \((L - C_E E)^\Psi W \geq L^\Psi W\)
and \(E \geq 0\).

The solution is:

$$E^* = \frac{L}{C_E} \left(1 - \left(\frac{\Psi W C_E}{V_0 L}\right)^{\frac{1}{\Psi + 1}}\right)$$
(20)

$$W^* = \frac{W}{\Psi W C_E} \left(1 - \frac{V_0 L}{\Psi W C_E}\right)^{\frac{1}{\Psi + 1}}$$
$$L^* = \frac{L}{\Psi W C_E} \left(1 - \frac{1}{\Psi W C_E}\right)^{\frac{1}{\Psi + 1}}$$
(21)

as long as:

$$W \leq \frac{V_0 L}{\Psi C_E}$$
(22)

Otherwise effort is zero ($E^* = 0, L^* = 0$) and presumably shareholders would not wish to enter into an employment relationship with the manager if $W > 0$.

The first-best expected firm value net of executive compensation is:

$$\frac{V_0 L}{C_E} \left(1 - \frac{\Psi + 1}{\Psi} \left(\frac{\Psi W C_E}{V_0 L}\right)^{\frac{1}{\Psi + 1}}\right)$$
(23)
which is nonnegative as long as
\[ W \leq \frac{V_0 L}{\Psi C_E} \left( \frac{\Psi}{\Psi + 1} \right)^\frac{1}{\Psi + 1} \]  
and clearly this condition must be met if the firm’s shareholders are to be willing to participate.

Note that there is a downward-sloping efficient locus of \( \{E^*, W^*\} \) combinations mapped out by solutions with different levels of managerial utility:

\[ E^* = \frac{T}{C_E} - \frac{W^* \Psi}{V_0} \]

In the executive’s preferences, money and leisure are complementary. While \( E^* \) decreases with \( W \), \( W^* \) increases with \( W \). As the executive’s reservation utility increases, he both obtains a higher wage and puts in less effort in a first-best contract. The preference formulation suggests that "fat cats" put in less effort than "lean and hungry" executives.

Consider next what happens in a situation where executives have an initial endowment of money, say a personal wealth endowment of \( cW > 0 \), and suppose that they only need to be paid enough to compensate them for the effort they put into the company and no more. Then our analysis above applies, but the firm only needs to pay the executive \( (W^* - \hat{W}) \) rather than \( W^* \). The net firm value is then:

\[ V_0 L C_E \left( 1 - \frac{\Psi + 1}{\Psi} \cdot \left( \frac{\Psi \hat{W}}{V_0 L} \right)^{\frac{1}{\Psi + 1}} \right) + \hat{W} \]

It can be shown that the net firm value is decreasing in \( \hat{W} \) over the range where (22) is valid. The firm prefers to hire a lean and hungry executive rather than a rich one. Both are endowed with an equal amount of time. But the poor one is more willing to exert effort in return for extra money. Naturally, this conclusion rests on the assumption that they are equally skilled.

3.2 Second-best contract in the absence of manipulation

We now characterize the second-best optimal contract in a classical agency model setting without opportunities for manipulation, in which pay is constrained to depend only on the long-term value of the firm \( (M = 0 \text{ and } \mu = 0) \). As is well known in this setting, there is a tradeoff between riskbearing and incentives which leads to a below-first-best level of effort.

Applying the usual agency model that maximizes expected net firm value as expressed in equation (18) subject to the constraint that the manager’s ex ante money-equivalent wealth in equation (16) be greater than or equal to the reservation wealth, we have the following:

\[ \max_{\omega, \eta} \left\{ \omega \left( \frac{\eta}{\Psi + \eta} \right) \left( \frac{V_0 L}{C_E} \right)^\eta \exp \left\{ \frac{1}{2} \left( -\eta + \eta^2 \right) \Sigma \right\} \right\} 

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subject to:

$$\omega \left( \frac{\Psi}{\Psi + \eta} \right)^{\psi} \left( \frac{\eta}{\Psi + \eta} \right)^{\mu} \left( \frac{V_0 L}{C_E} \right)^{\eta} \exp \left\{ \frac{1}{2} \left( -\eta + \phi \eta^2 \right) \Sigma \right\} \geq W \quad (28)$$

where $$\Sigma \equiv \Sigma_1 + \Sigma_2$$ is the total exogenous volatility over the long-run horizon.

Substituting out for $\omega$ on the assumption that the participation constraint is binding we can simplify this to:

$$\max_{\eta} \frac{\eta}{\Psi + \eta} \left( \frac{V_0 L}{C_E} \right) - W \cdot \left( \frac{\Psi + \eta}{\Psi} \right)^{\psi} \exp \left\{ \frac{1}{2} (1 - \phi) \eta^2 \Sigma \right\} \quad (29)$$

**Proposition 4** When managerial pay depends only on the long-term value of the firm, there is no opportunity for manipulation. The optimal contract is characterized by an elasticity of pay to the long term firm value, $\eta$, given by the following first-order condition:

$$\frac{V_0 L}{WC_E} = \left( \frac{\Psi + \eta}{\Psi} \right)^{\psi+1} \left[ \Psi + (\Psi + \eta) \eta (1 - \phi) \Sigma \right] \cdot \exp \left\{ \frac{1}{2} (1 - \phi) \eta^2 \Sigma \right\} \quad (30)$$

where $$\Sigma \equiv \Sigma_1 + \Sigma_2$$.

As long as the parameters satisfy equation (22), that is, as long as first-best effort $E^*$ would be strictly greater than zero, the constrained second-best value of the pay parameter $\eta$ will be greater than zero (even so, it needs to be checked that net expected shareholder value is nonnegative).

If the agent were risk neutral so that $\phi = 1$ (holding constant $\Psi$ so that the tradeoff between money and leisure remains unchanged), this would elicit the first best effort level: $\eta$ would be the solution to:

$$\frac{V_0 L}{WC_E} = \left( \frac{\Psi + \eta}{\Psi} \right)^{\psi+1} \Psi \quad (31)$$

which implies, from equation (10), that:

$$E = \frac{L}{C_E} \frac{\eta}{\Psi + \eta} = \frac{L}{C_E} \left( 1 - \left( \frac{C_E W}{V_0 L} \right)^{\psi+1} \right)$$

and this corresponds to the first-best effort level derived in equation (20). But if the agent is risk averse ($1 > \phi$), as in usual in agency problems, there is a tradeoff between incentives and riskbearing which means that incentives for effort fall short of the first best level. Specifically, since the right hand side of the first order condition (30) is increasing in both $(1 - \phi)$ and in $\eta$, $\eta$ will have to take a lower value than that needed to elicit the first-best effort level. As a result, the optimal level of effort will be lower than the first best level.
3.3 Discussion

It is worth noting that in our model it is the elasticity of managers’ wealth to firm value (percent-for-percent) rather than the sensitivity (dollar-for-dollar) that determines effort directly, because the model assumes Cobb-Douglas managerial preferences and multiplicative impact of effort on firm value, in contrast to the traditional model specifications in the literature that are additive in both respects. That is,

\[ E = \frac{T}{C_E} \frac{\eta}{\Psi + \eta}. \]

Thus if this model captures reality, empirical measurement of the power of incentives should focus on elasticities not sensitivities, and it is inappropriate to conclude, as Jensen and Murphy (1990) do, that a $3.25 change in CEO wealth for every $1000 change in firm wealth is a sign of weak incentives. For if the manager is not independently wealthy and his average pay is low relative to the value of the firm, such a low slope for the pay-performance relationship may well entail strong incentives in terms of the elasticity. In our model, a manager who has no wealth of his own and is compensated entirely through the appreciation of his stock will put in an equal amount of effort, no matter what proportion of the total stock he holds: the elasticity of his pay to firm value is one, regardless. His stake needs only be large enough to induce him to accept the job. What matters in inducing effort is the elasticity of the compensation scheme - the more elastic it is, the more effort is induced. In practice, since most managers have positive wealth of their own beyond their pay, the elasticity of their total wealth to firm value is likely to be below one, unless there is option pay and/or bonus pay is suitably convex. Dittmann and Maug (2007) calibrate a standard principal-agent model and argue that optimal contracts should not include options: if anything, there should be negative option holdings. Thus they find that the pay-performance relationship should take a concave form that has an elasticity well below one for realistic parameter values. It would be interesting to revisit these issues in the setting of our model, for equation (30) suggests that if the manager is not too risk averse (the CRRA \(1 - \phi\) not too high), volatility is low (median \(\Sigma\) is 0.19 in Dittmann and Maug’s data), the manager’s taste for leisure is not too strong (\(\Psi\) low enough) and the company’s base value is a large multiple of the manager’s reservation wage, then the optimal elasticity \(\eta\) could substantially exceed one.

The elasticity approach aligns well with the empirical literature. Murphy’s (1999) survey of empirical work on executive compensation notes that models phrased directly in terms of elasticities fit better:

"The primary advantage of the elasticity approach is that it produces a better "fit" in the sense that rates of return explain more of the cross-sectional variation of \(\Delta \ln(CEO\ Pay)\) than changes in shareholder value explain of \(\Delta(CEO\ Pay)\)."
Our model focuses on elasticities directly. In particular, while the optimal pay sensitivity $\eta$ is independent of firm size in traditional models, firm size plays an important role in the paper. Equation (30) predicts that the larger the firm relative to the manager’s reservation wage, the more performance sensitive the pay contract. The intuition is that the gain in firm value per unit of managerial time devoted to effort is proportional to $V_0$, while the disutility of time spent on effort is proportional to the manager’s wealth. So, all else equal, a more elastic pay contract is optimal for larger firms. Similarly, more able managers with a low cost of effort $C_E$ will be more strongly incentivised. In contrast, “fat cat” managers with independent wealth, that is, with a high reservation wage $W$, will choose more leisure and a less elastic contract, because leisure is a normal good.

Our model shares its multiplicative structure and some basic comparative statics with that of Edmans, Gabaix and Landier (2008). Their model differs in featuring a two-state distribution of firm value, a binary effort choice, and risk neutral preferences. In that model, the pay elasticity needs to be high enough to induce the single first-best effort level and therefore is constant across firms. In contrast, our model endogenises the second-best optimal effort level for risk averse management, and thus provides a rich set of insights into the determinants of optimal pay-performance sensitivities. It can also potentially be used to address the issue of how incentives and pay vary with firm size. This topic is addressed by Edmans et al., who make assumptions about the population distribution of firm size and managerial ability, and endogenise the distribution of reservation wages at the top end using the fact that more able managers are matched to larger firms in equilibrium. We will not pursue these issues here, and instead turn to the main topic of our paper: how manipulation affects the optimal incentive contract.

4 Optimal short-term pay contracts with manipulation

We now turn to the setting analysed in Peng and Röell (2008b) where pay can be linked to the short-term stock price, which is based on a manipulable report by the manager, but where it is impossible to tie pay to long-term performance so that $\eta \equiv 0$, and characterise the constrained second-best optimum. Inducing effort by making pay depend on short term performance has the inevitable side effect of encouraging the manager to manipulate the stock price. Writing down the optimization problem for this case, maximizes net firm value as expressed in equation (18) subject to the constraint that the manager’s money-equivalent wealth in equation (16) be greater than or equal to the reservation wealth, we
have:

$$\max_{\omega, \mu} \frac{\gamma \mu}{\Psi + 2\gamma \mu} \left( \frac{V_0 L}{C_E} \right) - \omega \left( \frac{\gamma \mu}{\Psi + 2\gamma \mu} \right)^\mu \left( \frac{V_0 L}{C_E} \right)^\mu \exp \left\{ -\frac{\gamma \mu}{2} (1 - \mu) \Sigma_1 \right\}$$

subject to:

$$\omega \left( \frac{\Psi}{\Psi + 2\gamma \mu} \right)^\Psi \left( \frac{\gamma \mu}{\Psi + 2\gamma \mu} \right)^\mu \left( \frac{V_0 L}{C_E} \right)^\mu \exp \left\{ -\frac{\gamma \mu}{2} (1 - \phi \mu) \Sigma_1 \right\} \geq W(32)$$

where $\gamma$ is determined by equation (12). Substituting out again for $\omega$ on the assumption that the manager’s participation constraint binds, we have:

$$\max_{\mu} \frac{\gamma \mu}{\Psi + 2\gamma \mu} \left( \frac{V_0 L}{C_E} \right) - \mathbf{W} \left( \frac{\Psi + 2\gamma \mu}{\Psi} \right)^\Psi \cdot \exp \left\{ \frac{\gamma \mu^2}{2} (1 - \phi) \Sigma_1 \right\}$$

(33)

**Proposition 5** When managerial pay depends only on the short-term value of the firm, the optimal contract is characterized by an elasticity of pay to the short term firm value, $\mu$, given by the following first-order condition:

$$\frac{V_0 L}{\mathbf{W} C_E} = \left( \frac{\Psi + 2\gamma \mu}{\Psi} \right)^{\Psi + 1} \cdot |2\Psi + (\Psi + 2\gamma \mu) \mu (1 - \phi) \Sigma_1| \cdot \exp \left\{ \frac{\gamma \mu^2}{2} (1 - \phi) \Sigma_1 \right\}$$

(34)

To compare the predictions of this proposition with a situation where manipulation is impossible, we can use as our benchmark the case of long-term stock price based pay described in Proposition (4), under the additional assumption that $\Sigma_2 = 0$, so that $\Sigma \equiv \Sigma_1$. There are two distinct reasons why the optimum level of effort is lower in the presence of manipulation. First, the incentive compensation encourages effort, but brings with it the undesirable side effect of encouraging manipulation, which carries with it a resource cost in terms of available managerial time. In the setting of a Cobb-Douglas utility function, the time resources needed to produce a unit of productive effort are essentially doubled since an equal amount of time is channeled into manipulation ($C_E E = C_M M$).

The time wasted on manipulation is a source of agency cost, making it more wasteful to incentivize effort and reducing the optimal effort level.

Second, the uncertainty about the manipulation cost further reduces the equilibrium level of effort. This uncertainty makes it hard to distinguish true performance from managerial hype, and therefore contaminates the value of the short-term stock price as a measure of managerial effort. Managers who do not exaggerate their performance as much as others are unable to persuade the market that they are simply modest, not lazy. This imposes an additional risk on the risk-averse managers *ex ante*, before they learn their type.

To verify that effort is lower than in the absence of manipulation cost uncertainty, consider a setting where the cost of manipulation is certain, that is,

Note that a similar intuition holds in a more general setting where the ratio of time spend on effort versus manipulation is $\frac{\beta}{\alpha}$ rather than 1 by allowing for nonlinear time costs, $C_E E^\alpha$ and $C_M M^\beta$ for $\alpha, \beta \geq 1$. 

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\( \Omega = 0 \) and therefore \( \gamma = 1 \). This corresponds to the "signal jamming" setting analysed by Goldman and Slezak (2006). The first order condition (34) would simplify to:

\[
\frac{V_0 L}{WC_E} = \left( \frac{\Psi + 2\mu}{\Psi} \right)^{\Psi+1} \cdot \left[ 2\Psi + (\Psi + 2\mu) \{ \mu (1 - \phi) \Sigma_1 \} \right] \cdot \exp \left\{ \frac{\mu^2}{2} (1 - \phi) \Sigma_1 \right\}.
\]

(35)

Now when \( \Omega > 0 \), the stock price becomes less sensitive to the signal, and \( \gamma < 1 \). Comparing (35) with (34), it is readily verified that \( \mu \) in (34) is greater than \( \mu \) in (35), but that \( \gamma \mu \) in (34) is less than \( \mu \) in (35). Since by equation (10) the equilibrium level of effort is determined by the product \( \gamma \mu \), the effort level is now lower.

More surprisingly, the elasticity of pay with stock price, \( \mu \), is higher when uncertainty about manipulation cost is high. One driver of the result is the fact that the volatility of the period 1 stock price is decreasing in \( \Omega \), thus reducing the risk imposed on the manager by stock-price-driven incentive pay. In particular, the variance of the period 1 log-return is given by \( \gamma \Sigma_1 \), where \( \gamma \equiv \frac{\Sigma_1}{\Sigma_1 + \Omega} \). Intuitively, the stock price is becoming less responsive to managerial reports regarding firm value; this is at least partially offset by an increase in elasticity of pay to the stock price. This suggests that for companies or industries in which the uncertainty about managerial manipulation is high, \( \mu > 1 \), it is harder to disentangle the true firm value from hype in the manager’s disclosures, the optimal contract should actually be more elastic with respect to price. Thus the model predicts that options, viewed as a means of adding convexity (\( \mu > 1 \)) to the pay scheme, might be more prevalent for such companies.

The current model setup provides a good description of a situation arising when a firm newly enters into a line of business characterised by a new and different value of \( \Omega \); the stock price will start out being relatively unresponsive if \( \Omega \) has just increased. In a long-run steady state, where a firm has settled into a particular line of business with associated underlying per-period fundamental risk \( \Sigma_1 \), and manipulation uncertainty \( \Omega \), the volatility from period to period is necessarily equal to the fundamental variance \( \Sigma_1 \). In computations not shown in this paper, we check the impact of manipulation uncertainty \( \Omega \) on the incentive parameter \( \mu \) in a steady-state setting where the stock market discovers the full impact of the previous management team’s efforts between dates 0 and 1, thus adding a shock of variance \( (1 - \gamma) \Sigma_1 \) to the period’s return but otherwise leaving the model unaffected. The impact of \( \Omega \) on the incentive parameter \( \mu \) is still positive, as long as \( \Omega \) is not too large relative to \( \Sigma_1 \).

\(^4\)Thinking of each period as the lifespan of a management team in a relay setting, the pay contract is signed before uncertainty about the success of the previous team’s efforts is resolved; and this uncertainty adds volatility of \( (1 - \gamma) \Sigma_1 \) to the stock price. Manipulation uncertainty does not affect the total volatility of the stock price per period; rather, it delays the incorporation of information regarding a management team’s performance into the stock price to beyond the date at which their pay is determined.
Regarding the remaining determinants of the optimal pay contract, an increase in the fundamental volatility of the firm ($\Sigma_1$) reduces $\mu$, the elasticity of pay to the short term stock price. It also reduces $\gamma \mu$ and therefore managerial effort due to the usual tradeoff between incentives and riskbearing, (as can be verified by noting that the RHS of condition (34) is increasing in the two composite variables $\gamma \mu$ and $\Sigma_1 + \Omega$). Increased risk aversion $(1 - \phi)$ has a similar impact, decreasing $\mu$ and thus reducing the level of effort. Equation (34) also implies that for firms whose value is large relative to the manager’s reservation wage ($\frac{V_0 T \Omega}{WC_E}$ high), the optimal pay contract should be more elastic. This prediction is similar to the case of optimal contract in the absence of manipulation as analyzed in Section 3. However, in the presence of manipulation possibilities, the strong incentives mean that the managers in these firms also manipulate more.

5 The optimal tradeoff between short- and long-term pay

So far, we have considered constrained second-best optimal contracts with manipulation in which only short-term performance are permitted in the compensation scheme. We now consider what happens when both can be included. We solve for the unconstrained optimization problem of maximizing net firm value as expressed in equation (18) subject to the constraint that the manager’s money-equivalent wealth in equation (16) be greater than or equal to the reservation wealth. Again eliminating the pay scaling constant $\omega$ using the participation constraint of the manager, the optimal incentive contract is the set of parameters $\{\mu, \eta\}$ that solves the optimization problem:

$$\max_{\mu, \eta} \frac{\gamma \mu + \eta}{\Psi + 2\gamma \mu + \eta} \left(\frac{V_0 L}{C_E}\right)$$

$$-\frac{\Psi}{\Psi - \eta} \left(\frac{\Psi + 2\gamma \mu + \eta}{\Psi}\right)^{\psi+1} \cdot \exp \left\{\frac{1 - \phi}{2} \left[\left(\gamma \mu^2 + 2\gamma \mu \eta + \eta^2\right)\Sigma_1 + \eta^2 \Sigma_2\right]\right\}$$

The first order conditions are:

FOC w.r.t. $\mu$ :

$$\frac{V_0 L}{WC_E} = \Psi^{-1} \left(\frac{\Psi + 2\gamma \mu + \eta}{\Psi}\right)^{\psi+1} \cdot \exp \left\{\frac{1 - \phi}{2} \left[\left(\gamma \mu^2 + 2\gamma \mu \eta + \eta^2\right)\Sigma_1 + \eta^2 \Sigma_2\right]\right\}$$

(or $\mu = 0$ and $\leq$ replaces $=$ in the FOC).
We first establish some properties of the optimal pay contract in the presence of long-term incentives.

**Proposition 6** The second-best optimal contract has the following properties:

(i) the maximised net firm value is (weakly) decreasing in risk aversion \((1 - \phi)\), in short-term volatility \(\Sigma_1\) (holding constant either \(\Omega\) or \(\gamma\), regardless), in long-term volatility \(\Sigma_2\), and in the manipulation uncertainty \(\Omega\)

(ii) the long-term incentive parameter, \(\eta\), is strictly greater than zero but the short-term parameter, \(\mu\), is only weakly greater than zero;

(iii) long- and short-term incentives are substitutes in the sense that in the unconstrained optimal contract, the short-term incentive parameter \(\mu\) is lower than it would be if long-term incentives were constrained to be absent \((\eta = 0)\);

(iv) effort is below the first-best optimal level.

Proof. See appendix.

In general, the optimal contract includes long- as well as short-term incentives. Long-term incentives enable the firm to align the manager’s objectives with long-term firm value, mitigating the undesirable side effect associated with short-term incentives, namely the waste of valuable time on manipulating the short-term stock price. Long-term incentives, on the other hand, have the disadvantage of bringing extra risk (captured by \(\Sigma_2\)) from longer-term exogenous shocks to firm value into the manager’s pay. Interestingly, in the model long-run incentives may displace the short-run incentives altogether for some parameter configurations; but it is always advantageous to include at least some long-term element in pay.

The comparative statics relating the properties of the optimal pay contract to the underlying parameters is quite complex because it is not just the overall intensity of incentives that varies. The relative power of long- and short-run incentives varies as well. We will use numerical analysis to illustrate these effects. We present the results using a set of benchmark parameter values and varying some of these parameters individually to gain insight on their effects. Our results are robust to a wide range of reasonable parameter values.

Without loss of generality, we normalize \(C_E = \overline{L} = \overline{W} = 1\) and \(V_0=100\). These 4 parameters are basically just one as what matters for the optimal incentive is the ratio \(\frac{V_0L}{WC_E}\). We set short run return variance \(\Sigma_1\) to be 1 and
long run return variance $\Sigma_2$ to be 5. The manipulation uncertainty $\Omega$ is set to 1. The value of $\phi$ is set to $-1$, which corresponds to a constant relative risk aversion coefficient $(1-\phi)$ of 2 that has been established as a reasonable value for the relative risk aversion coefficient in various work. The coefficient on leisure in the manager’s utility function $\Psi$ is set to 1. The budget share of leisure is $\Psi/(\Psi + 1)$. A low value for this ratio indicates a workaholic manager and a high value indicates a slacker.

A key focus of our model is uncertainty about the manager’s propensity to manipulate, captured by randomness in his manipulation cost $C_M$. In Figure 1, we demonstrate some of the key results when varying the manipulation uncertainty parameter, $\Omega$. Panel A considers the optimal contract where both short term and long term incentive pay are allowed and Panel B considers a restricted contract with only short term incentive pay. The left graph shows the elasticities of pay to stock price, $\mu$ (and $\gamma \mu$) and $\eta$. In the middle graph, the areas labeled $E$, $M$ and $L$ represent the proportion of time the manager spends on the effort, manipulation and leisure. The right graph show the ex-ante expected gross and net firm value.

The top panel of Figure 1 shows that an increase in the uncertainty (\Omega) regarding the manager’s propensity to manipulate naturally leads to a shift away from short-term incentives ($\gamma \mu$ falls) towards long-term incentives ($\eta$ rises). Even so, in the case pictured, the elasticity of pay to the short-term stock price ($\mu$) rises so that the pay contract becomes more elastic to both short- and long-run value. However, effort falls despite the increase in the elasticity of pay to both short- and long-term value, because the performance report at date 1 is less informative and so the stock price is less responsive to the report ($\gamma$ falls). Manipulation declines even more precipitately, because unlike effort it is not subject to an offsetting push from the rising long-term incentives. When long-term incentives are not available, the case depicted in the bottom panel of Figure 1, the impact of manipulation cost uncertainty on short-term incentives is qualitatively similar but much stronger. In particular, firm value is much more sensitive to $\Omega$, decreasing dramatically by about 45% as $\Omega$ increases (only 5% in the unconstrained contract); Similarly, the stock price elasticity of pay $\mu$ increases more markedly, to nearly 3.5 while it remains near its initial value below 0.5 in the unconstrained setting.

The role of manipulation uncertainty on the elasticity (convexity) of optimal incentive pay provides a potential explanation that reconciles Dittman and Maug’s (2007) finding with optimal contracting. They calibrate a standard principle-agent model with constant relative risk aversion and lognormal stock prices with executive compensation data and argue that the observed contracts with a significant amount of options pay can not be rationalized by the standard models. Our model shows that, once the manipulation uncertainty is taking to account, the optimal contract needs to be more convex relative to the case when manipulation is absent. Even for reasonable levels of risk aversion as shown in Figure 1, options can be used to achieve the optimal convexity if the manipulation uncertainty is high, especially for firms that prefers more short term based pay due to reasons such as high long run fundamental uncertainty.
Figure 2 describes how the optimal contract, effort-manipulation-leisure choice, and firm value vary with the managers’s coefficient of relative risk aversion \((1 - \phi)\). In this case, \(\gamma\) is a constant that equals to 0.5. Contrast to the results we have on the restricted contract where only short term incentive pay is allowed, the incentives under the general contract display interesting and non-monotonic patterns when risk aversion varies: when the risk aversion is low enough, only long-term incentives are used: pay is not linked to the short-term stock price. However, as risk aversion increases, short-term incentives become an attractive way to manage risk and the elasticity of pay to long-term value is reduced; and as a side effect, managers devote part of their time to manipulation. As risk aversion increases further, both long- and short-term elasticities fall due to the costs of bearing risk, and both effort and manipulation decline. The figure thus illustrates that short-term incentives are not monotonic in risk aversion: from zero, they first increase as they replace long-term incentives, and then they decline alongside the long-term incentives as risk aversion increases further.

The impact of the riskiness of the company’s business is illustrated in Figure 3. Focusing on the impact of the volatility of the period 1 exogenous shock \((\Sigma_1)\), the impact on short-term incentives is rather straightforward: they decline steadily as risk increases, because the cost of compensating the manager for bearing the risk attached to the incentives increases; and there comes a point beyond which it is optimal to provide no short-term incentives \((\mu^* = 0)\). The impact of increasing risk on the long-term incentives is strikingly nonmonotonic. Initially there is a slight decline as would be natural given that the risk associated with any given \(\eta\) increases. But beyond that there is a range where the long-term incentives as captured by \(\eta\) actually increase, presumably as a substitute for the drop in short-term incentives \((\gamma\mu)\): as \(\Sigma_1\) increases (holding constant \(\Sigma_2\)) the risk-reduction advantages of short-term compensation become less salient and the manipulation-cost-saving advantages of long-term compensation dominate. Once short-term incentives have been driven down to zero altogether, there is no further substitution effect and so increases in risk simply lead to decreases in long-term incentives.

Regarding the impact of long-term volatility \(\Sigma_2\), it is readily verified (see the proof of Proposition 6) that when the additional risk from long-term incentives is low enough, it is optimal not to use short-term incentives: \(\mu^* = 0\) when \(\Sigma_2\) is low enough. For the parameter ranges examined, as \(\Sigma_2\) increases, incentives are loaded onto short-term performance because the extra risk from delaying payments increases.

The impact of changes in the size of the firm relative to the manager’s reservation wage is illustrated in Figure 4. The expression \(\frac{V_0 T}{W}\) that appears in Proposition 6 is the ratio of the maximum value of the firm, attained if the manager devotes 100% of his time to effort, to his reservation wage. An increase in the size of the firm \(V_0\) means that effort becomes more productive; not surprisingly, the optimal amount of effort elicited increases with firm size. Different from the monotonic pattern between short term incentive and firm size relative
to manager’s reservation wage in the case where the long-term incentive is unavailable, there is an interesting nonmonotonicity in the short-term incentives once the long-term incentive is allowed. At the lower end of the range of firm sizes, short-term incentives increase and the amount of manipulation increases concomitantly. But the proportion of time spent on manipulation reaches a maximum at an intermediate firm size and then decreases as relatively more reliance is placed on long-term incentives. This contrasts the comparative statics of the simpler model with short-run pay only, where both effort and manipulation were monotonically increasing in the relative firm size. Intuitively, as firm size increases effort increases and the manager’s time becomes a relatively more scarce resource. There comes a point beyond which the waste of time spent on manipulation outweighs the risk advantages of short-term contracts, and thus at the higher end of the firm size scale incentives are shifted to the long run.

In terms of the information efficiency of the short term stock prices, $P_1$, we analyze the return variance for the period between time 1 and 2. Everything else equal, a more informative $P_1$ would deviate less from the long run fundamental value and the associated return variance would be lower. Given the following logarithmic return,

$$\ln \left( \frac{V_2}{P_1} \right) = \ln \left( \frac{E V_0 \epsilon_1 \epsilon_2}{\pi V_0^{1-\gamma} (M E V_0 \epsilon_1)^{\gamma}} \right)$$

$$= \ln \frac{E^{1-\gamma}}{\pi} + (1-\gamma) \ln \epsilon_1 + \ln \epsilon_2 - \gamma \ln M,$$  \hspace{1cm} (37)

the log-return variance is:

$$\text{var} \left[ \ln \left( \frac{V_2}{P_1} \right) \right] = (1-\gamma)^2 \Sigma_1 + \Sigma_2 + \gamma^2 \Omega$$

$$= \left( \frac{\Omega}{\Sigma_1 + \Omega} \right)^2 \Sigma_1 + \left( \frac{\Sigma_1}{\Sigma_1 + \Omega} \right)^2 \Omega + \Sigma_2$$

$$= \frac{\Omega \Sigma_1}{\Sigma_1 + \Omega} + \Sigma_2,$$  \hspace{1cm} (38)

The first part of the return variance shows how much $P_1$ is "noised up" because of the uncertainty on the manager’s propensity to manipulate and is increasing in the uncertainty $\Omega$. If the uncertainty is zero, return variance would simply be $\Sigma_2$ and $P_1$ would capture the true state of the firm at the time. Given that firms or industries with high manipulation uncertainty are also more likely to use higher powered incentives, Equation (38) predicts that extreme poor performances are more likely to be observed for firms with more intangible assets and in growth industries where there are more manipulation uncertainty and the usage of higher powered incentives are more common. These companies are more likely to become targets of shareholder class action lawsuits, as shown in Peng and Röell (2008a).
The finding that manipulation uncertainty affects the inefficiency of stock price formation also provides insights for empirical studies on executive compensation. These studies typically use the sensitivity of managerial pay with respect to stock price as a measure of incentive. However, as shown in our model, the short term incentive depends on $\gamma \mu$, the product of the sensitivity of pay to stock price and the sensitivity of stock price to the performance report disclosed by the managers. In the presence of high manipulation uncertainty, stock market is less responsive to manager’s reports and therefore a higher powered incentive is necessary to induce the optimal effort. In this case, however, the product $\gamma \mu$ is actually lower, which results in less optimal effort and lower long run performance. The result of higher pay sensitivity to stock price but lower performance is consistent with optimal contracting. However, simply using $\gamma$ to proxy for incentive, as done typically in empirical studies, may give rise to misleading conclusions on whether the observed performance is consistent with optimal contracts.

In terms of policy implications, our discussion on the uncertainty about the manipulation cost suggest that policies that lower $\Omega$ (e.g. tighter accounting standards or better internal controls through more board monitoring) would allow investors to make sharper inference about the degree of manipulation and result in an optimal contract that induces more effort. Surprisingly, such policies that increases $\gamma \mu$ would also result in more manipulation in equilibrium. Even so, net firm value is higher, stock prices are more informationally efficient and investors’ welfare is improved. In contrast, policies that only changes the average cost of manipulation may not have real impact. This is because the stock market can correctly undo the average level of manipulation so both the stock price’s sensitivity to the signal and the elasticity of pay are independent of $C_M$. Therefore, policies that merely make manipulation more costly may not reduce the deadweight cost caused by manipulation.

6 Conclusion

We analyze a model of optimal executive compensation in a setting where managers are in a position to manipulate short-term stock prices, and managers’ propensity to manipulate is uncertain. Stock-based incentives elicit not only productive effort, but also costly information manipulation. We derive a second-best optimal compensation scheme that analyze the tradeoffs involved in balancing effort and manipulation and in conditioning pay on long- versus short-term performance.

We show that manipulation, and investors’ uncertainty about it, affects the equilibrium pay contract and the informational efficiency of asset prices. When manipulation uncertainty is high, pay can actually be more closely linked to the stock price even though the efficiency of equilibrium stock price formation is now lower, while the equilibrium level of effort is lower and the stock price is less sensitive to the manager’s report. The cross-sectional implication of
this result is that firms or industries that have high uncertainty regarding the propensity to manipulate, such as growth firms or industries that involves more intangible assets, are at the outset more likely to use high powered incentive contracts, such as stock options. Policies that reduce manipulation uncertainty would allow investors to make better inferences about the degree of manipulation and result in more efficient contracts that enhance shareholder value. However, policies that only change the average cost of manipulation may not have any real impact.

For companies with high manipulation uncertainty, our model suggests that shifting incentive pay from short term to long term based pay can help mitigate the economic waste associated with short term incentives, resulting in higher effort and improved firm value. For example, it might be optimal for these firms to use stock or options with long vesting periods, or to use "clawback" clauses that allow firms to recover unwarranted pay if long run performance falters.

Our analysis also cautions against the widely used empirical methodology of measuring incentives by simply looking at the sensitivity of pay to the contemporaneous stock price return, since the speed at which underlying performance is incorporated into the stock price can vary between industries. Seemingly powerful incentives may elicit little effort if the full impact of the effort on the stock price is delayed beyond the pay horizon. Ignoring this can lead to misleading conclusions regarding the relation between incentives provided by the contract and performance.

Lastly, we provide a tractable and realistic multiplicative model of optimal executive pay contracts, in which the manager’s preferences are constant relative risk averse and Cobb-Douglas in wealth and leisure, effort is modeled as an input of managerial time and impacts firm value proportionally, and firm value is lognormally distributed. This formulation generates a number of empirically testable predictions about the determinants of optimal pay contracts. The model makes direct predictions about the elasticity of pay to firm value rather than "dollar-for-dollar" sensitivities of pay, providing a convenient reference point for empirical research.

7 Appendix

We use lower-case letters to denote the logarithms of the corresponding uppercase variables throughout. It is useful to introduce the short hand notation

\[
\begin{align*}
k_E &= \ln \left( \frac{\gamma \mu + \eta}{\Psi + 2\gamma \mu + \eta} \frac{T}{L} \right) \quad \text{and} \quad c_E = \ln C_E \quad \text{(A1)} \\
k_M &= \ln \left( \frac{\gamma \mu}{\Psi + 2\gamma \mu + \eta} \frac{T}{L} \right) \quad \text{and} \quad c_M = \ln C_M
\end{align*}
\]

so that:

\[
\begin{align*}
E &= \exp \{k_E - c_E\} \\
M &= \exp \{k_M - c_M\}
\end{align*}
\]
Proof of Proposition 1. Equation (4) is rewritten in logarithmic form as:
\[ s = v_0 + m + e + \bar{e}_1 \]  
where
\[ \left( \begin{array}{c} m \\ \varepsilon_1 \end{array} \right) \sim N \left( \left( \begin{array}{cc} k_M - \tau_M \\ - \frac{1}{2} \Sigma_1 \end{array} \right), \left( \begin{array}{cc} \Omega & 0 \\ 0 & \Sigma_1 \end{array} \right) \right) \]

Then we have
\[ E[v_1 | s] = v_0 + k_E - c_E - \frac{1}{2} \Sigma_1 + \frac{\Sigma_1}{\Omega + \Sigma_1} \left( s - \left( v_0 + k_E - c_E + k_M - \tau_M - \frac{1}{2} \Sigma_1 \right) \right) \]
\[ = v_0 \frac{\Omega}{\Omega + \Sigma_1} + s \frac{\Sigma_1}{\Omega + \Sigma_1} + (k_E - c_E - \frac{1}{2} \Sigma_1) \frac{\Omega}{\Omega + \Sigma_1} - (k_M - \tau_M) \frac{\Sigma_1}{\Omega + \Sigma_1} \]
and
\[ \text{var} [v_1 | s] = \frac{\Omega \Sigma_1}{\Omega + \Sigma_1} \]
From (6), the logarithm of the (gross-of-expected-pay) market price is given by:
\[ p_1 = \ln P_1 = \ln E[V_1 | s] = \ln E[\exp (v_1) | s] = E[v_1 | s] + \frac{1}{2} \text{var} [v_1 | s] \]
\[ = v_0 + \frac{\Omega}{\Omega + \Sigma_1} + s \frac{\Sigma_1}{\Omega + \Sigma_1} + (k_E - c_E - \frac{1}{2} \Sigma_1) \frac{\Omega}{\Omega + \Sigma_1} - (k_M - \tau_M) \frac{\Sigma_1}{\Omega + \Sigma_1} \]
\[ = v_0 (1 - \gamma) + s \gamma + \ln \pi \]
where:
\[ \gamma = \frac{\Sigma_1}{\Omega + \Sigma_1} \]
\[ \ln \pi = (1 - \gamma) (k_E - c_E) - \gamma (k_M - \tau_M) \]

Proof of Proposition 2. Rewriting (9) in logarithmic form and substituting out for \( M \) and \( E \) using equation (10) and \( \pi \) using equation (13):
\[ w = \ln \omega + \mu \ln \pi + (\mu + \eta) v_0 + (\gamma + \eta) m + (\gamma \mu + \eta) e + (\gamma \mu + \eta) \varepsilon_1 + \eta \varepsilon_2 \]
\[ = \ln \omega + (\mu + \eta) v_0 + \gamma \mu (\tau_M - c_M) + (\mu + \eta) (k_E - c_E) + (\gamma + \eta) \varepsilon_1 + \eta \varepsilon_2 \]
and thus
\[ \overline{w} = \ln \omega + (\mu + \eta) v_0 + (\mu + \eta) (k_E - c_E) + \gamma \mu (\tau_M - c_M) \]
\[ - \frac{1}{2} \left[ (\gamma \mu + \eta) \Sigma_1 + \eta \Sigma_2 \right] \]
and

\[ \text{var}(\bar{w}) = (\gamma \mu + \eta)^2 \Sigma_1 + \eta^2 \Sigma_2 \]  

(A11)

Based on the optimal effort and manipulation choice in equation (10), the manager’s leisure is:

\[
L = L - C_E E - C_M M = \frac{\Psi \bar{L}}{\Psi + 2\gamma \mu + \eta} 
\]

(A12)

The logarithm of the interim expected utility of the manager is then:

\[
\ln E_1 [U] = -\ln \phi + \phi \Psi \ln L + \phi \left( \bar{w} + \frac{1}{2} \phi^2 \text{var}(\bar{w}) \right) 
\]  

(A13)

To obtain the ex-ante value of this expression, note that

\[
E_0 \left( \frac{1}{C_M} \right)^{\phi \gamma \mu} = E_0 \exp \left\{ -\phi \gamma \mu c_M \right\} 
\]

(A14)

so that the logarithm of the ex ante expected utility of the agent is:

\[
\ln E_0 [U] = -\ln \phi + \phi \Psi \ln \frac{\Psi}{\Psi + 2\gamma \mu + \eta} \bar{L} + \phi \left\{ \ln \omega + (\mu + \eta) (v_0 + k_E - c_E) \right\} 
\]  

(A15)

\[
-\frac{\phi}{2} [(\gamma \mu + \eta) \Sigma_1 + \eta \Sigma_2] 
\]

\[
+ \frac{\phi^2}{2} \left[ (\gamma \mu + \eta)^2 \Sigma_1 + \eta^2 \Sigma_2 \right] 
\]

\[
+ \frac{\phi^2}{2} \gamma^2 \mu^2 \Omega 
\]
Using the definition of $\gamma$, substituting out for $\Omega$ using $\Omega = \Sigma_1 \frac{1-\gamma}{\gamma}$, we have:

$$\ln E_0 [U] = -\ln \phi + \phi \Psi \ln \frac{\Psi L}{\Psi + 2\gamma \mu + \eta}$$

$$+ \phi \left[ \ln \omega + (\mu + \eta) [\nu_0 + kE - cE] \right]$$

$$- \frac{\phi}{2} [(\gamma \mu + \eta) \Sigma_1 + \eta \Sigma_2]$$

$$+ \frac{\phi^2}{2} [(\gamma \mu^2 + 2\gamma \mu \eta + \eta^2) \Sigma_1 + \eta^2 \Sigma_2]$$

(A16)

In money-equivalent terms, the logarithm of expected utility is given by:

$$\ln E_0 [U]' = \ln \omega + \Psi \ln \frac{\Psi}{\Psi + 2\gamma \mu + \eta} + (\mu + \eta) \ln \left( \frac{\Psi L}{\Psi + 2\gamma \mu + \eta} \frac{V_0 L}{C_E} \right)$$

$$- \frac{1}{2} [(\gamma \mu + \eta) \Sigma_1 + \eta \Sigma_2]$$

$$+ \frac{\phi}{2} [(\gamma \mu^2 + 2\gamma \mu \eta + \eta^2) \Sigma_1 + \eta^2 \Sigma_2]$$

(A17)

**Proof of Proposition 3.** The interim expected value of the company net of executive compensation, for given value of $C_M$:

$$E \cdot V_0 - E [W]$$

$$= \frac{\gamma \mu + \eta}{\Psi + 2\gamma \mu + \eta} \frac{V_0 L}{C_E}$$

$$- \frac{\omega}{2} \left( \frac{\gamma \mu + \eta}{\Psi + 2\gamma \mu + \eta} \frac{V_0 L}{C_E} \right)^{\mu + \eta} \left( \frac{C_M}{C_M} \right)^{\gamma \mu}$$

$$\cdot \exp \left\{ \frac{1}{2} \left[ \{ - (\gamma \mu + \eta) + (\gamma \mu + \eta)^2 \} \Sigma_1 + \{ - \eta + \eta^2 \} \Sigma_2 \right] \right\}$$

Hence *ex ante* company expected value is:

$$E[E \cdot V_0 - W]$$

$$= \frac{\gamma \mu + \eta}{\Psi + 2\gamma \mu + \eta} \left( \frac{V_0 L}{C_E} \right) - \frac{\omega}{2} \left( \frac{\gamma \mu + \eta}{\Psi + 2\gamma \mu + \eta} \frac{V_0 L}{C_E} \right)^{\mu + \eta}$$

$$\cdot \exp \left\{ \frac{1}{2} \left[ \{ 2\gamma \mu^2 \Omega + (\gamma \mu + \eta) + (\gamma \mu + \eta)^2 \} \Sigma_1 + (\eta + \eta^2) \Sigma_2 \right] \right\}$$

(A19)
Proof of Proposition 6

(i) Inspecting the optimisation problem (36), it is immediately obvious that for any given values of $\mu$ and $\eta$, the maximand increases if CRRA $(1 - \phi)$ falls, long-term risk $\Sigma_2$ falls, $\Sigma_1$ falls when $\gamma$ remains constant (i.e. $\Omega$ falls in proportion to $\Sigma_1$). It is also decreasing in $\Omega$ because a fall in $\Omega$ means an increase in $\gamma$; and an increase in $\gamma$, when $\mu$ varies in such a way as to keep $\mu \gamma$ constant, entails a decrease in $\mu$ which decreases the $\gamma \mu^2$ term in the exponent, increasing the maximand. Lastly, when $\Sigma_1$ falls holding $\Omega$ constant, $\gamma \equiv \frac{\Sigma_1}{\Sigma_1 + \Omega}$ falls. But suppose that you vary $\mu$ in such a way as to keep $\gamma \mu$ unchanged, then the first term in the exponent is $(\gamma \mu)^2 \Sigma_1 / \gamma = (\gamma \mu)^2 (\Sigma_1 + \Omega)$ and thus decreasing along with $\Sigma_1$, so that the entire maximand increases. All these arguments show that the maximand can be increased, even when the endogenous parameters $\mu$ and $\eta$ are not adjusted in an optimal manner; naturally it would increase even more if they were.

(ii) An optimal contract cannot have just short-term but not long-term incentives. Suppose not, that is, $\eta = 0$ and $\mu \neq 0$. The relevant FOCs are:

\[
\text{FOC w.r.t. } \mu : \quad \frac{V_0 L}{W C_E} = \left( \frac{\Psi}{\Psi + 2 \gamma \mu} \right)^{\Psi + 1} \cdot [2 \Psi + (1 - \phi) (\Psi + 2 \gamma \mu) \mu \Sigma_1] \\
\quad \cdot \exp \left\{ \frac{1 - \phi}{2} \gamma \mu^2 \Sigma_1 \right\}
\]

\[
\text{FOC w.r.t. } \eta : \quad \frac{V_0 L}{W C_E} \leq \frac{\Psi}{\Psi + \gamma \mu} \left( \frac{\Psi + 2 \gamma \mu}{\Psi} \right)^{\Psi + 1} \cdot [\Psi + (1 - \phi) (\Psi + 2 \gamma \mu) \gamma \mu \Sigma_1] \\
\quad \cdot \exp \left\{ \frac{1 - \phi}{2} \gamma \mu^2 \Sigma_1 \right\}
\]

Clearly these two conditions cannot be met simultaneously because the RHS of the first FOC is strictly greater than that of the second one (both due to a factor 2 on $\Psi$ in the FOC w.r.t. $\mu$, and the appearance of $\gamma \leq 1$ in two extra places reducing the RHS of the FOC w.r.t. $\eta$). Contradiction.

On the other hand it is conceivable that the solution has $\eta > 0$ but $\mu = 0$ simultaneously:
FOC w.r.t. $\mu$ : 

$$
\frac{V_0 L}{\overline{WC_E}} \leq \frac{\Psi}{\Psi - \eta} \left( \frac{\Psi + \eta}{\Psi} \right)^{\Psi+1} \cdot [2\Psi + (1 - \phi) (\Psi + \eta) \eta \Sigma_1] \\
\cdot \exp \left\{ \frac{1 - \phi}{2} \eta^2 [\Sigma_1 + \Sigma_2] \right\}
$$

FOC w.r.t. $\eta$ : 

$$
\frac{V_0 L}{\overline{WC_E}} = \left( \frac{\Psi + \eta}{\Psi} \right)^{\Psi+1} \cdot [\Psi + (1 - \phi) (\Psi + \eta) \eta (\Sigma_1 + \Sigma_2)] \\
\cdot \exp \left\{ \frac{1 - \phi}{2} \eta^2 [\Sigma_1 + \Sigma_2] \right\}
$$

In particular such a contract may be optimal if $\Sigma_2$ is small enough relative to $(\Sigma_1 + \Sigma_2)$ so that the extra risk attributable to long-term incentives is sufficiently small:

$$
\frac{\Psi}{\Psi - \eta} [2\Psi + (1 - \phi) (\Psi + \eta) \eta \Sigma_1] > [\Psi + (1 - \phi) (\Psi + \eta) \eta (\Sigma_1 + \Sigma_2)]
$$

i.e. 

$$
\Psi^2 > \Psi (1 - \phi) (\Psi + \eta) \eta \Sigma_2 - \eta [\Psi + (1 - \phi) (\Psi + \eta) \eta (\Sigma_1 + \Sigma_2)]
$$

$$
\Psi > (1 - \phi) \eta [\Psi \Sigma_2 - \eta (\Sigma_1 + \Sigma_2)]
$$

Fixing $\Sigma_1 + \Sigma_2$ and thus $\eta$, this condition is satisfied if $\Sigma_2$ is small enough.

(iii) If we allow long term incentives, i.e. $\eta > 0$, then short term incentives are lower i.e. $\mu$ is reduced and a smaller proportion of time is devoted to manipulation. This is immediately obvious from at the FOC wrt $\mu$: the RHS is increasing in both $\eta$ and $\mu$ so holding all else constant an increase in the one must be offset by a decrease in the other. This also implies there will be less time spent on manipulation because of the ensuing fall in

$$
\frac{\gamma \mu}{\Psi + 2\gamma \mu + \eta}
$$

(iv) Effort is equal to:

$$
\frac{\overline{L}}{C_E} \left( \frac{\gamma \mu + \eta}{\Psi + 2\gamma \mu + \eta} \right)
$$

and first best effort is

$$
E^* = \frac{\overline{L}}{C_E} \left[ 1 - \left( \frac{C_E \overline{W}}{V_0 \overline{L}} \right)^{\frac{1}{\gamma \eta}} \right]
$$
so we need to show that

$$\frac{\gamma \mu + \eta}{\Psi + 2 \gamma \mu + \eta} \leq 1 - \left( \frac{C_E W}{V_0 L} \right)^{\Psi+1}$$

i.e.

$$\frac{V_0 L}{C_E W} \geq \Psi \left( \frac{\Psi + 2 \gamma \mu + \eta}{\Psi + \gamma \mu} \right)^{\Psi+1}$$

but the F.O.C. wrt $\eta$ is:

$$\frac{V_0 L}{C_E W} = \Psi \left( \frac{\Psi + 2 \gamma \mu + \eta}{\Psi + \gamma \mu} \right)^{\Psi+1} \left( \frac{\Psi + \gamma \mu}{\Psi} \right)^{\Psi \geq 1} \Psi + (1 - \phi) (\Psi + 2 \gamma \mu + \eta) \{(\gamma \mu + \eta) \Sigma_1 + \eta \Sigma_2\}_{\Psi \geq 1} \exp\{\ldots\} \geq 1$$

which proves the claim that effort is below first-best.

References


This figure illustrates how the optimal contract, effort-manipulation-leisure choice, and firm value vary with the uncertainty about manipulation, $\Omega$. $\mu$ and $\eta$ are the short term and long term elasticity of pay to gross firm value, respectively. $\gamma$ is the elasticity of the stock price to the manager’s report. The areas labeled $E$, $M$, and $L$ represent the proportion of time the manager spends on effort, manipulation and leisure. $E[V]$ is the ex-ante expected gross firm value and $E[V - W]$ is the ex-ante expected net firm value after subtracting expected managerial compensation. The parameter values used in the simulation are: $C_E = 1$, $\bar{L} = 1$, $\bar{W} = 1$, $V_0 = 100$, $\Sigma_1 = 1$, $\Sigma_2 = 5$, $\phi = -1$, and $\Psi = 1$. 

Figure 1. Optimal contract as a function of manipulation uncertainty.
Figure 2. Optimal contract as a function of manager’s relative risk aversion.

This figure describes how the optimal contract, effort-manipulation-leisure choice, and firm value vary with $1 - \phi$, the relative risk aversion coefficient of the manager. $\mu$ and $\eta$ are the short term and long term elasticity of pay to gross firm value, respectively. $\gamma$ is the elasticity of the stock price to the manager’s report. The left hand graph has two vertical axes, with the left one for $\eta$ and the right one for $\gamma\mu$. The areas labeled $E$, $M$, and $L$ represent the proportion of time the manager spends on effort, manipulation and leisure. $E[V]$ is the ex-ante expected gross firm value and $E[V - W]$ is the ex-ante expected net firm value after subtracting expected managerial compensation. The parameter values used in the simulation are: $C_E = 1$, $\underline{L} = 1$, $\overline{W} = 1$, $V_0 = 100$, $\phi = -1$, $\Sigma_2 = 5$, $\Omega = 1$, and $\Psi = 1$. 
Figure 3. Optimal contract as a function of short term uncertainty about firm fundamentals.

This figure illustrates how the optimal contract, effort-manipulation-leisure choice, and firm value vary with $\Sigma_1$, the short term uncertainty about firm fundamentals. $\mu$ and $\eta$ are the short term and long term elasticity of pay to gross firm value, respectively. $\gamma$ is the elasticity of the stock price to the manager’s report. The upper righthand graph has two vertical axes, with the left one for $\gamma \mu$ and the right one for $\eta$. The areas labeled $E$, $M$, and $L$ represent the proportion of time the manager spends on effort, manipulation and leisure. $E[V]$ is the ex-ante expected gross firm value and $E[V - W]$ is the ex-ante expected net firm value after subtracting expected managerial compensation. The parameter values used in the simulation are: $C_E = 1$, $L = 1$, $W = 1$, $V_0 = 100$, $\Sigma_1 = 1$, $\Sigma_2 = 5$, $\Omega = 1$, and $\Psi = 1$. 

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Figure 4. Optimal contract as a function of initial firm value.

This figure describes how the optimal contract, effort-manipulation-leisure choice, and firm value vary with $V_0$, the initial firm value. $\mu$ and $\eta$ are the short term and long term elasticity of pay to gross firm value, respectively. $\gamma$ is the elasticity of the stock price to the manager's report. The areas labeled $E$, $M$, and $L$ represent the proportion of time the manager spends on effort, manipulation and leisure. $E[V]$ is the ex-ante expected gross firm value and $E[V - W]$ is the ex-ante expected net firm value after subtracting expected managerial compensation. The parameter values used in the simulation are: $C_E = 1$, $\bar{L} = 1$, $\bar{W} = 1$, $\Sigma_1 = 1$, $\Sigma_2 = 5$, $\Omega = 1$, $\phi = -1$, and $\Psi = 1$. 