Irving Fisher and the UIP Puzzle

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Abstract

In this paper we empirically verify much of the seminal work by Irving Fisher on uncovered interest parity, which he conducted over a century ago. Like Fisher, we find that the departures from UIP are connected to individual episodes in which errors surrounding exchange-rate expectations are persistent, but eventually transitory. We find considerable commonality in deviations from UIP and PPP, suggesting that both of these deviations are driven by a common factor. Using additional international parity conditions to model exchange rates, we find significant empirical evidence that deviations from UIP are almost entirely due to errors in forecasting exchange rates, rather than risk premia.
I. Introduction

Of the three major international parity relations, uncovered interest rate parity (UIP) has proven to be the most troublesome empirically. According to UIP, the difference between interest rates in two different currencies will equal the rate of change of the exchange rate between those currencies. However, most studies fail to find this positive one-to-one relation and, indeed, many find a negative relation.¹

To Irving Fisher, who was the first economist to investigate the UIP condition empirically, these anomalous results probably would not have come as much of a surprise.² Fisher viewed UIP as the cross-country dual of the within-country relation between interest rates and inflation that we now call the “Fisher Equation.” In his eyes, they were simply two sides of a coin – two facets of a more general relation linking interest rates in different standards, in his terminology, the relation between “appreciation and interest.”³

Fisher saw both as very often subject to violation in the real world. Concerning the Fisher Equation, he argued that people generally did not “adjust at all accurately and promptly” to changes in the behavior of prices but did so only with a long lag (1930). For UIP, he said much the same thing, presenting evidence of incomplete and delayed adjustment of nominal interest rate differentials to exchange-rate movements and also of

³ In the Fisher equation, the interest rates in question, of course, are the nominal and real rates of interest and the link between them, the expected rate of inflation – the rate at which money is expected to depreciate or appreciate in terms of goods. In the UIP relation, the interest rates are the nominal interest rates of the two countries in question and the link between them the expected rate of change of the exchange rate – the rate at which one currency is expected to depreciate or appreciate in terms of the other. Fisher discussed these relations first in his 1896 monograph Appreciation and Interest, and later in two books on interest-rate determination, The Rate of Interest (1907) and his more often cited The Theory of Interest (1930).
episodes which now fall under the heading of “peso problems” in which agents anticipate changes that have not yet occurred.

In this paper we re-examine the performance of UIP since the advent of floating exchange rates in the 1970s. Using data from the recent era of floating exchange rates, we find evidence that is entirely consistent with Fisher’s earlier conclusions. Like Fisher, we find that the failures of UIP are related to individual episodes in which exchange-rate forecasting errors have been persistent, but in the end transitory. The first piece of evidence that supports this inference is the improvement in performance of UIP as we average the data over progressively longer periods. Errors made in forecasting the exchange rate are much less important in the long run. The longer horizon analysis shows that the parity conditions become more stable. Our second piece of evidence comes from analysis of UIP in conjunction with the other two key international parity conditions, purchasing power parity (PPP) and real interest rate equality (RIE). The short-term deviations away from UIP and PPP are both substantial and highly correlated. This empirical evidence again points to exchange-rate forecast errors, as opposed to risk premia, as the major force driving the UIP deviations. The third piece of evidence to support the Fisherian view of exchange-rate forecasting errors as being the major source driving deviations from short-term UIP comes from estimating a dynamic latent factor model on deviations from both UIP and PPP. We find that errors in forecasting exchange rates are the major force driving short-term deviations from UIP. The estimation results from the dynamic latent factor model imply further that the long-term characteristics of the deviations from UIP die out. This is in line both Fisher’s results (1930) and the
descriptive statistics from the recent floating exchange-rate era that we presented earlier in the paper.

The paper is organized as follows. Because Fisher’s work on the subject is pertinent to the current debate surrounding UIP in a number of important ways, we begin with a review of his findings in Section I. In Section II, we provide empirical results, which substantiate the Fisherian view of exchange rates. In Section III, we analyze the sources of deviations away from UIP using a dynamic latent factor model. We show that forecast errors are the major source of departures from UIP in the short run, as posited by Irving Fisher over a century ago. Section V presents a brief summary and some conclusions derived from our findings.

II. The Fisherian View of Exchange Rates and Interest Rates

In his analyses of UIP and of the relation between appreciation and interest more generally, Fisher reached several important conclusions, all of which we are able to verify empirically in this paper. In the data that he examined, he found evidence in support of UIP, but that support was very far from perfect. Where UIP was violated, Fisher, moreover, was able to provide coherent explanations for the failure. The major culprit, he argued, was agents’ inability to accurately forecast the underlying monetary conditions affecting exchange rates. In some instances, agents appeared to have substantially underestimated the extent of the exchange-rate change; in others they grossly overestimated the changes along the lines of what now are termed “peso problems.” He went on to show, however, that in the long run the influence of
appreciation on interest rates was more certain, and departures from the theory much less prominent.

In line with Fisher’s premise, substantial evidence abounds that UIP has not in fact held in recent decades, or at least not in the short term. Engel (1996) and Chinn (2006) provide surveys of this literature. Indeed, one of the most puzzling features of exchange-rate behavior since the advent of floating exchange rates in the early 1970s is the tendency for countries with high interest rates to see their currencies appreciate rather than depreciate, as UIP would suggest. This UIP puzzle, also known as “the forward premium puzzle,” is now so well-documented that it has taken on the aura of a stylized fact. As a result, it has spawned an extensive second-generation body of literature that attempts to explain these departures from theory.

II. A. Fisher’s Empirical Evidence

A key feature of Fisher’s investigation of UIP was his research design. The data Fisher used were for yields of bonds of similar maturities issued by the same government, but denominated in different currencies.\footnote{As Fisher (1907, p. 259) put it, “A definite test may be made where two standards are simultaneously used.” As he pointed out a bit later (see the quote below), in such instances, factors other than agents’ expectations of currency appreciation or depreciation could be ruled out as influences on yield differentials. It is for this reason, perhaps, that Fisher made no comparisons of yield behavior across countries even though he clearly had the necessary data.} The result was something close to an ideal experiment, one in which differences in default risk were absent and in which errors in agents’ forecasts of exchange-rates were left as the force behind departures from UIP.

The first of the two bodies of data that Fisher analyzed were yields on long-term U.S. bonds over the period 1870 to 1896, one bond payable in gold and the other in paper, or "greenback," currency; the second was yields on long-term Indian bonds traded...
in London between 1865 and 1894, one bond payable in sterling and the other in silver rupees. Fisher discussed the results of this analysis first in his monograph *Appreciation and Interest* (1896), and then in his two books on the subject (1907, 1930).

In his analysis of the U.S. data, Fisher discussed two important episodes, the 1879 resumption of specie payments and the decades surrounding that episode, and the 1896 presidential election and three years preceding it. In both events, he found evidence of behavior consistent with theory. Prior to resumption, yields on currency bonds exceeded yields on gold bonds as they should have, given the expectations of an appreciation in the value of the paper currency relative to gold. At its peak in 1870, the spread between the two stood at 100 basis points. As time passed and the U.S. price level expressed in terms of the paper currency converged to the price level expressed in terms of gold, the spread narrowed, and by mid-1878 had reversed sign. Over the next 15 years the spread between the yields on currency and gold bonds averaged only -37 basis points, and in the earlier part of that period generally stood at -20 basis points or less.

Fisher went on to compare the expected rates of appreciation of the greenback implicit in the yield differentials prior to resumption. In his comparisons he used realized rates over progressively shorter periods, beginning in January 1870 and ending in each instance in January 1879, the actual date of resumption. The expected rate at the start of this sample was 0.8 percent per annum compared to a realized rate of 2.1 percent per annum, a ratio of a bit less than two fifths. Such underestimation was not at all unusual. Not until 1877 did the ratio finally break out of that general range. For a time in 1874 it actually went negative, implying expectations of depreciation rather than appreciation.
If adjustment was incomplete for most of the period prior to resumption, it was certainly not the case in the years leading up to the 1896 presidential election. During that episode, the first of the two peso-problems uncovered by Fisher, which we noted above, developed. Yields on currency bonds and gold bonds both increased, and the spread between the two progressively widened from 30 basis points in 1893 to a peak of 110 basis points in 1896. Fisher’s explanation, which subsequent research substantiates, attributed these developments to the free-silver agitation and the fears of impending inflation and dollar depreciation that it engendered.\textsuperscript{5} “Both the increases and the wedging apart of the two rates are explainable as effects of the free-silver proposal and its incorporation (July 1896) in the platform of the democratic [sic] party,” (Fisher, 1907, p. 261).

Fisher conducted a similar analysis using the yield data for India. In the period 1865-1874 when the exchange rate was stable, the yields on gold and silver rupee bonds were almost identical, differing on average by roughly 20 basis points. Then, in 1875, as the rupee began to depreciate, the spreads gradually widened, from an average of close to 40 basis points in the period between 1875 and 1878, to 64 basis points during the period 1879-1887, to over 100 basis points from 1888 through the first half of 1890. After further depreciation in the half decade that followed, the exchange rate stabilized at the par value of 16 pence/rupee.

Fisher pointed out that market reactions, both to the initial decline and to the eventual stabilization of the rupee, although basically in line with theory, came with substantial lags. His discussion of the first episode is revealing both with regard to his

\textsuperscript{5} Hallwood, et al. (2000) provide econometric evidence supporting this interpretation. For an historical discussion of this episode see Friedman and Schwartz (1982, Chapter 7).
choice of research design and the role he ascribed to expectations. He wrote (1930, pp. 405-406):

Inasmuch as the two bonds were issued by the same government, possessed the same degree of security, were quoted side by side in the same market, and were similar in all important respects except in the standard in which they are expressed, the results afford evidence that the fall of exchange (after it once began) was, to some extent, discounted in advance and affected the rates of interest in those standards. Of course investors did not form perfectly definite estimates of the future fall, but the fear of a fall predominated in varying degrees over the hope of a rise.

With regard to the latter episode, Fisher argued that market participants apparently anticipated a further depreciation in the exchange rate, but this depreciation never actually materialized. This incident is the second of the two peso problems highlighted by Fisher. In the Theory of Interest (1930, p. 407), Fisher wrote:

“[T]he legal par was reached in 1898 and was maintained thereafter, subject only to the slight variations of exchange due to the cost of shipping specie. But until the par was proved actually stable by two or three years’ experience, the public refused to have confidence that gold and the rupee were once more to run parallel. Their lack of confidence was shown in the difference in the rates of interest in gold and rupee securities during the transition period, 1893-1898, and the two or three succeeding years.” (Emphasis is ours)

The remainder of Fisher’s empirical investigation of the appreciation-versus-interest relation focused on the behavior of nominal interest rates and inflation rates within countries – seven countries in The Rate of Interest and six in The Theory of Interest. While he found evidence of various sorts in support of theory, the relationship, as in the case of UIP, was very far from perfect. The standard deviations of ex post real interest rates in all instances were many multiples of the standard deviations of the nominal interest rates. Increases in inflation went hand in glove with decreases in ex post real rates.
Fisher’s summation of this last bit of evidence is highly illuminating (1907, p. 278):

There are two possible explanations for [this inverse relation]. … One is that when prices are rising the cause may not be monetary but may lie in a progressive scarcity of commodities produced and exchanged ... The second reason is that these [price] movements are only imperfectly foreseen.

He opined further (1907, p. 279):

Doubtless both of these causes play a part in the explanation in particular cases. Nevertheless there is internal evidence to show that in general the latter factor – unforeseen monetary changes – is the more important. This evidence consists in the fact that commodity interest fluctuates so widely in some instances becoming negative. (Emphasis is ours)

He concluded on a more positive note, however, (1907, pp. 282-283) arguing that “When long periods of price movements are taken, the influence of appreciation on interest is more certain … because [i]n averages covering so many years we may be sure that accidental causes are almost wholly eliminated.” Using averages spanning a decade or more for Britain and the United States, he found evidence supporting this conclusion (1907, pp. 282-284).

Fisher’s reasoning here, though presented in a terse and somewhat offhand manner, is very much in line with the later emphasis of Friedman and Schwartz (1991) on the importance of accounting for errors in variables, defined as they put it, to include “all stochastic disturbances affecting the variables under study” and of filtering the data to capture fundamental long-run relations. As in much else, Fisher again was ahead of his time.

Using Irving Fisher’s original data for the United States and India we ran standard UIP-type regressions to examine how these relations hold, as in equation (1):
\[(1) \quad s_{t+1} - s_t = \alpha + \beta (i_t - i_*) + e_{t+1}, \]

where \( s_{t+1} - s_t \) is the one-period change in the log spot exchange rate and \( i_t - i_* \) is the corresponding yield differential. In the case of U.S. bonds the exchange rate is between gold and paper currency; in the case of the Indian bonds, it is the exchange rate between the British pound sterling and the Indian rupee. The yield differential in the U.S. case is between bonds payable in gold and in paper currency and in the Indian case the yield differential is between bonds payable in sterling and silver rupees. We collected these data from Tables 11 and 12 in Chapter 19 of The Theory of Interest (1930). The regression results are reported in Table 1. In the U.S. case, the estimate of the slope coefficient \( \beta \) is positive, and in the Indian case, negative. In both cases, however, these estimates are both insignificantly different from zero and insignificantly different from unity. Furthermore, the regressions explain relatively little of the variation in exchange-rate changes. So, while Fisher – quite legitimately we believe – was able to point to subperiods in which UIP had some degree of validity, the relation does not pass econometric muster over his two full sample periods.

Insert Table 1
**II. B. Evidence from the recent era of floating exchange rates**

We began by running UIP regressions in the form of equation (1) using monthly data\(^6\). We report the regression results in Appendix A. These results were very much in line with results reported in other studies. In 15 of the 20 countries, the estimates of \(\beta\) were significantly different from the theoretical value of unity at the five per cent level or below. In all instances the coefficients of determination in these regressions were extremely low, and in most instances close to zero.

Fisher’s explanation for the failures of UIP and the appreciation-interest relation more generally, as we have already discussed, centered on small-sample problems and Fisher’s other “accidental” factors affecting that relation. In order to investigate the possible effects of such transitory influences, we ran five-year rolling regressions for the G7 countries and regressions using pooled data for the full sample of countries averaged over progressively longer time periods.

We plot the coefficients for the rolling regressions in Figure 1. What stands out in the chart are the often sizable variations in the slope coefficients over time. We see periods in which the estimated coefficients are positive and UIP appears approximately to have held, but these are relatively brief and not always the same across countries. We see such behavior in the mid-1970s and then later in the late 1980s and early 1990s. In the late 1970s and early 1980s and then again from the mid 1990s on, however, we see the reverse – substantially negative coefficients typical of the UIP puzzle. Interestingly these

\(^6\) We collected monthly data for the period January 1976 to December 2005 for 20 countries relative to the United States: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, Netherlands, New-Zealand, Norway, Portugal, Spain, Sweden, Switzerland, and the United Kingdom. The data were mostly obtained from the CD version of the International Monetary Fund’s *International Financial Statistics*. Exchange rates are denominated in units of foreign currency per U.S. dollar; interest rates are short-term domestic Treasury bill or money market rates.
latter two episodes are associated with regime changes – the Reagan-Volcker move to disinflationary monetary policy in the United States and the adoption of the Euro.

**Insert Figure 1**

If the statistical issues related to UIP are in fact episodic phenomena that are due, as Fisher put it (1907, p. 282), to “accidental causes,” then his solution of averaging the data is a way to filter the data and thus mitigate the effects of temporary disturbances.\(^7\) We perform averaging of this sort in the data presented in Figure 2 and in the corresponding regressions reported in Table 2. In the three panels of Figure 2 we show the plots of the UIP relation based on non-overlapping samples of five-year, fifteen-year, and full-period averages of the data for our 20 countries. To provide a theoretical frame of reference we also draw a 45 degree line. In Table 2 we list the corresponding regression results.

**Insert Figure 2 & Table 2**

In the five-year averaged data, there is a positive, but nevertheless weak, relation between the exchange-rate change and the interest differential. However, the picture changes markedly as the period over which we average the data lengthens. We see this relation clearly improving in the bottom two panels of Figure 2. When we look at the

\(^7\) In this connection, see Lucas (1980) and Lothian (1985) for discussions of data filtering to isolate long-term relationships.
fifteen-year and full-period averages we find a strong positive relation between exchange-rate changes and interest-rate differentials.

The regression results in Table 2 confirm these observations. As the period over which we compute the averages lengthens, the slope coefficients in the regressions increase from less than 0.038 to 0.694, and the standard errors of those regressions decrease from close to 16.6 percentage points to 10.9 percentage points. Although we can always reject the hypothesis of a unit slope, it is clear from these results that as a long-run first approximation, UIP contains a substantial kernel of truth.8

III. A Model for the Fisherian View of Exchange Rates

We now turn our attention to developing a model that is capable of testing Fisher’s explanation for failures of the UIP relation. To do so, we decompose short-run deviations away from UIP into a risk premium and a forecast error component. The principal question at issue is whether the failure of UIP observed in the data for recent decades is, as Fisher had argued (based on his analyses of the nineteenth century U.S. and Indian data), (1) the result of systematic errors in agents’ forecasts of currency depreciation or (2) the consequence of time-varying risk premia. Are forecast errors sufficiently large and long-lived that they can account for the substantial deviations from UIP observed in Figure 1 over considerable periods? Do these errors largely cancel out in the end, as Figure 2 suggests and Fisher claimed, or are departures from UIP the rule over the long run too?

8 Flood and Taylor (1997) and Lothian and Simaan (1998) provide similar evidence for samples including many of these countries over time periods ending the mid-1990s.
III. A. A Three-parity framework to model exchange rates

Uncovered interest parity and the other two major international parity conditions – purchasing power parity in rate of change form (PPP) and real-interest equality (RIE) – are closely related. The deviation from any one of these parity conditions is equal to the algebraic sum of the deviations from the other two. We use this observation first to derive illustrative estimates of the effects of exchange-rate forecast errors and risk premia on UIP and then later in the construction of our model.⁹

Uncovered interest parity is an ex ante concept, positing an equality of expected nominal returns across countries, as in Equation (2):

\[ i_t = i_t^* + E_t[(s_{t+1} - s_t)] , \]

where \( E_t \) is the expectation conditional on all observable information up to and including time \( t \).

Empirical investigations of UIP, however, generally use actual, ex post changes in exchange rates as a proxy for their unobservable ex ante counterparts. Deviations from UIP, therefore, can arise from two sources: differences between actual and expected exchange-rate changes and differences in the riskiness of the two assets. This is denoted in equation (3) below:

\[ i_t - i_t^* - (s_{t+1} - s_t) = \rho + \varepsilon_{st} , \]

⁹ The following discussion draws on Marston (1997).
where $\varepsilon_{st}$ is the exchange-rate forecast error due to a difference between actual and expected exchange-rate changes and $\rho_t$ as the ex ante risk premium. The risk premium will be positive (or negative) if investors require an expected excess return on one of the currencies to compensate for the risk of holding assets denominated in that currency. Under the usual assumptions of rational expectations, exchange-rate forecast errors will be random, given that the true underlying distribution of the exchange rate is known. However, as Irving Fisher much earlier pointed out, there are conditions under which these errors might in fact be systematic over time. One situation in which there will be systematic errors is when investors continually anticipate changes in the underlying process generating the return distribution that have yet to occur – the “peso problem”. A second situation is when a monetary shock occurs, in the form of a sudden shift in the monetary regime. Before investors learn about the true process that generates the returns, there may be a period in which forecast errors again are systematic over time, rather than random. As alluded above, Fisher discussed the first of these two cases in the context of the 1896 U.S. presidential election and the second in the context of the stabilization of the rupee.

Now consider the ex ante form of PPP in equation (4), written here in terms of expected rates of change of the variables,

\[
E_t[\pi_{t+1} - \pi_{t+1}^*] = E_t[s_{t+1} - s_t],
\]

where $\pi_{t+1}$ and $\pi_{t+1}^*$ are the rates of inflation measured from time $t$ to $t+1$ in the home and foreign countries, respectively. Deviations from PPP arise either as a result of
exchange-rate forecast errors, $\varepsilon_{st}$, inflation forecast errors, $\varepsilon_{pt}$, or expected changes in the real exchange rate, $\theta_t$. Following Marston (1997), we assume that a modified form of relative PPP, which allows for expected changes in the real exchange rate, holds ex ante:\(^{10}\)

\[
(5) \quad \left(\pi_{t+1} - \pi_{t+1}^*\right) - \left(s_{t+1} - s_t\right) = \varepsilon_{st} + \varepsilon_{pt} + \theta_t.
\]

When we compare equations (5) and (3), we see that risk premia do not affect deviations from PPP, but exchange-rate forecast errors affect deviations from both UIP and PPP.

Given the interdependence of the three parity conditions, we obtain an analogous equation for RIE deviations simply by subtracting (5) from (3):

\[
(6) \quad r_t - r_t^* = \rho_t - \theta_t - \varepsilon_{pt},
\]

where $r - r^*$ is the difference between the domestic and foreign ex ante real interest rates. When we compare (6) with (3), we observe the risk premium as the only common component in the UIP and RIE equations. Exchange-rate forecast errors, which do matter for UIP and PPP, do not matter at all for RIE.

Comparing the behavior of deviations from UIP with deviations from PPP and RIE, therefore, allows us to make inferences with regard to the forces affecting UIP. We do this in Figure 3, where we plot the ex post deviations from the three parity conditions for the U.S. dollar-U.K. pound sterling (GBP) exchange rate and in Table 3 where we

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\(^{10}\) Dumas (1992) shows that imperfect goods arbitrage leads to a situation in which the ex ante real exchange rate can be written as a (linear) function of the ex ante real interest differential. A risk premium thus enters the PPP relation if we do not assume that PPP holds ex ante.
present the correlations between the deviations from the three parity conditions for the pound, the euro, and the yen against the dollar.\textsuperscript{11}

\textit{Insert Figure 3\& Table 3}

What immediately strikes the eye in Figure 3 are first the substantial magnitudes of the deviations from UIP and PPP and the close correspondence in their movements and second the much smaller magnitude of the deviations from RIE deviations and the near independence of those deviations from deviations from UIP and PPP.

Two related inferences follow from these results. The first is that risk premia appear not to matter very much. If they did deviations from RIE ought to be much more volatile and much more highly correlated with deviations from UIP.\textsuperscript{12} The second, which is consistent with Fisher’s conjectures, is that errors in exchange-rate forecasts appear to be the major driving force behind UIP deviations. This latter conclusion follows both from the high correlation that we observe between UIP and PPP deviations and the low correlations that we observe between both UIP and PPP deviations and RIE deviations, which, given the interdependence of the three, are as it were two sides of the same coin.

\textsuperscript{11} Prior to 1999 we proxy the euro by the deutschmark.
\textsuperscript{12} The only other way RIE deviations could appear so stable is if risk premia and errors in forecasts of relative inflation rates and/or expected changes in real-exchange rates were substantially negatively correlated. We can see no compelling reason why this would be so.
III. B. Identifying Fisherian forecast errors

To test Irving Fisher’s views more formally we apply the same three-parity framework and simultaneously test the set of joint parity conditions. We do this by estimating a dynamic latent factor model for UIP and PPP together.\(^\text{13}\) The common factor in these relations is the forecast error\(\varepsilon_s\). When tested empirically, instantaneous PPP is often rejected. A major reason cited in the literature is the sluggish reaction of international goods prices (e.g., Dornbusch, 1976; Mussa (1982)). To integrate price stickiness into our framework, we follow Kim (2005) and use an error correction model to describe the deviations from PPP. We write the resulting deviations from UIP and PPP as:

\[
\begin{pmatrix}
(i_t - i_t^*) - (s_{t+1} - s_t) \\
(\pi_t - \pi_t^*) - (s_{t+1} - s_t)
\end{pmatrix} = \begin{pmatrix} c \\
\phi (\mu + s_t + p_t^* - p_t) + \left(1 \begin{pmatrix} \varepsilon_{s,t} + \left(\frac{\rho_{s,t}}{r_{t}}\right) \end{pmatrix}ight)
\end{pmatrix},
\]

(7) with

\[
\begin{pmatrix} \rho_{s,t} \\
\theta_t 
\end{pmatrix} \sim N\left(0, \begin{pmatrix} \sigma_{\rho}^2 & 0 \\
0 & \sigma_{\theta}^2 \end{pmatrix}\right),
\]

where \(p_t\) and \(p_t^*\) are the logarithms of the foreign and U.S. price levels and \(\phi\) is an adjustment parameter. We consider the common factor for the forecast error \((\varepsilon_{s,t})\) to be a latent factor, governed by the AR(1) process:\(^\text{14}\)

\(^{13}\) Results for the combination of deviations from UIP and RIE are similar, but because instantaneous PPP is often rejected we choose to focus on UIP in conjunction with PPP, rather than in conjunction with RIE, allowing us to include a further long term component for PPP.

\(^{14}\) Alternative specifications for the risk premium do not substantially alter the results.

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We estimate the model parameters and the risk premium by maximum likelihood and compute the likelihood function recursively using the Kalman filter (Harvey, 1991). Once we have estimated the common factor we can identify the exchange-rate risk premium $\rho_t$ from Equation (3) and the joint component composed of the inflation forecast error and expected real exchange-rate change, $\tilde{\theta}_t$, from Equation (5). In Table 4, we present the estimation results for the dynamic factor model for the three main currencies in our sample, the euro (EUR), the British pound sterling (GBP), and the Japanese yen (JPY).\footnote{We note that our results are extremely robust to using the alternative combinations of UIP and PPP, and PPP and RIE to derive parameter estimates.}

\begin{equation}
\epsilon_{s,t+1} = \phi_t \epsilon_{s,t} + \eta_{t+1}, \quad \eta_{t+1} \sim N\left(0, \sigma^2_\eta\right).
\end{equation}

\textbf{Insert Table 4}

The low values of the autoregressive coefficients on the latent lagged risk premium $\phi_t$ show that the estimated forecast errors are not persistent. The standard deviations of the innovation errors are much larger for the UIP equations $\sigma_{rp}$, than for the PPP equations $\sigma_{\theta}$. Most important, and in line with our earlier results, we find that the innovation variances of the forecast errors $\sigma_{\eta}$ are very much larger than those for the risk premia, implying in turn that most of the variability in the deviations from UIP is caused by the forecast errors.
In Figure 4 we plot the time series of the estimated forecast errors and risk premia. The former is calculated as the smoothed estimate from the Kalman filter (see Harvey, 1991). In comparing the two, note that in all three cases, the scales for the estimates of the risk premia are much smaller than those for the exchange-rate forecast errors. In Table 5, we compare the moments from the estimated series from the dynamic latent factor model.

The monthly variances of the risk premia range from 0.02 and 0.05 for the three currencies, whereas strikingly, the monthly variances of the nominal exchange rates themselves and the associated forecast errors range from 9.35 to 11.95. Interestingly, Froot and Frankel (1989) find estimates similar in magnitude in their survey data. Importantly, these results show that the variability of the risk premium is much lower than the variability of expected exchange-rate returns. As a result, the rejection of UIP cannot be attributed completely, or even largely, to the existence of risk premia. Exchange-rate forecast errors appear to play a much more important role in terms of variability than risk premia. This, again, is line with the conclusions reached by Irving Fisher with regard to deviations from UIP over a century ago.
Using the procedure described in Engel (1996), we also have derived estimates of the relative impacts of risk premia and exchange-rate forecast error on the slope coefficients in the regressions based on equation (1). Engel shows that these OLS slope coefficients can be written as $\beta_{OLS} = 1 - \beta_{rp} - \beta_{ss}$, with $\beta_{rp}$ and $\beta_{ss}$ the components related to the risk premium and the forecast error, respectively. We refer the reader to Appendix B for details. The results from the decomposition can be found in Table 5. These again provide strong evidence that it is the exchange-rate forecast errors and not risk premia that are mainly responsible for the negative regression coefficients in the UIP regressions. For all currencies, the values of $\beta_{ss}$ are much larger than the values of $\beta_{rp}$.

Now we turn to the issue of long run versus short run. In Fisher’s view, departures from the theoretical relation between appreciation and interest in both its UIP and interest-versus-inflation forms were due to what he termed “accidental factors” that over the long run tended to cancel one another out. The results that we presented in Figure 2 and Table 2 earlier in this paper were consistent with this explanation. Here we analyze the same issue in the context of our dynamic factor model. We focus on two questions in particular: whether over the longer term the average deviations from UIP for the three currencies under investigation tend toward zero and whether the variances of those deviations progressively decline.

We define $y_{t, j} = \left( i_{t, j} - \hat{i}_{t, j} \right) - \left( s_{t, j+1} - \hat{s}_{t, j} \right)$ as the one-month deviation from UIP, measured on a monthly frequency, $j$ periods ahead.\(^{16}\) We measure the long-term

---

\(^{16}\) In accordance with the dynamic latent factor model we have that the interest rates are measured on money market instruments with a one-month time-to-maturity.
deviations from UIP over the time horizon \( k \) by the sum of the one-period deviations:

\[
\sum_{j=1}^{k} y_{t+j} .
\]

The average conditional expectation over \( k \) months is

\[
\frac{1}{k} E_t \left[ \sum_{j=1}^{k} y_{t+j} \right] = \frac{1}{k} E_t \left[ \sum_{j=1}^{k} \left( c + \epsilon_{s,t+j} + rp_{t+j-1} \right) \right] = c + \frac{1}{k} \epsilon_{s} \sum_{j=1}^{k} \rho \]

Where we have assumed, consistent with the formulation of the dynamic latent factor model (7), that the risk premium has a conditional mean of zero. Note that when \( k \) becomes larger the conditional expectation will converge to the constant term \( c \).

To measure the variability of the deviations over the horizon \( k \), we compute the conditional variance:

\[
\frac{1}{k} Var_t \left[ \sum_{j=1}^{k} y_{t+j} \right] = \frac{1}{k} Var_t \left[ \sum_{j=1}^{k} \left( \epsilon_{s,t+j} + rp_{t+j-1} \right) \right] =
\]

\[
\frac{1}{k} Var_t \left[ \sum_{j=1}^{k} \epsilon_{s,t+j} \right] + \frac{1}{k} Var_t \left[ \sum_{j=1}^{k} rp_{t+j-1} \right] =
\]

\[
\frac{1}{k} \sigma^2 \sum_{j=1}^{k} \rho^{2(j-1)} + \frac{1}{k} \sum_{j=1}^{k} \sigma^2 \rho^2 = \frac{1}{k} \sigma^2 \sum_{j=1}^{k} \rho^{2(j-1)} + \sigma^2 .
\]

This variance consists of a fixed component (the variance of the risk premium) plus a component related to the variance of the forecast error.

From our estimates of the monthly model, presented in Table 4, we can find the conditional means, given a zero forecast error at time \( t \) (\( \epsilon_{s,t} = 0 \)), as the estimated values for the constant \( c \). These means are 0.03%, 0.15% and 0.08%, for the euro, the pound and the yen exchange rates, respectively. In Figure 5, we plot the conditional variances for the
three exchange rates relative to elapsed time. It is clear from the chart that the variances of deviations from UIP do progressively diminish and nearly totally die out as the time horizon increases. After approximately 24 months (2 years) the variances are 0.450, 0.433, and 0.533, respectively – roughly one twentieth of their initial values. Most of the decline, moreover, takes place within about 12 months. The dynamic latent factor model, therefore, is able to capture both the behavior of short-term deviations from UIP, which are driven primarily by exchange-rate forecast errors, and the fact that in the longer term these deviations tend to disappear.

IV. Conclusion

Our results on the identification of the empirical failure of the uncovered interest parity are consistent with those reported a century and more ago by Irving Fisher in his studies of the relation between appreciation and interest, both in its UIP and interest-versus-inflation versions. Consistent with Fisher’s view, we find evidence of the important role played by episodic phenomena in disturbing that relation. Like Fisher, we too find that the influence of such phenomena dissipates over time.

We conclude that there are long-run deviations from parity conditions that appear to be caused by large, but infrequent, shocks to the monetary environment. These shocks systematically affect the error in forecasting the change in exchange rates. Over the long term, these errors are less important and we find empirical support for UIP.

Using a three-parity framework we investigate the possibility of a common factor driving short-run deviations from international parity conditions. We find extremely high correlation coefficients between UIP and PPP deviations that we identify with exchange-
rate forecasting errors. Our results are in line with the results from the studies using survey data (e.g. Froot and Frankel, 1989) on exchange-rate expectations that decompose expectations into a risk premium component and an exchange-rate forecast error defined as the difference between actual exchange rate changes and those forecast by survey participants. The common finding there is that the risk premium plays a limited role, and most variation is due to forecast errors defined in this way. Our model is also consistent with the fact that forecast errors dissipate over time, thereby rendering the UIP relation more empirical support.

Our empirical results for the major currencies confirm Fisher’s claim, made in 1907, that “unforeseen monetary changes” are the major cause of departures from UIP and the appreciation-interest relation more generally appear confirmed (1907, p. 279).
References


Lothian, J.R., Wu L., 2005, Uncovered Interest Rate Parity over the Past Two Centuries.” Unpublished working paper, Fordham University and Baruch College.


Appendix A: Summary of results of individual-country UIP regressions

We run the regressions summarized below by using monthly data from 1976:1-2005:12 obtained from *International Financial Statistics*. These regressions take the form

\[ s_{t+1} - s_t = \alpha + \beta (i_t - i^*_t) + e_{t+1}, \]  

(Eq. 1)

where \( s_{t+1} - s_t \) is the one-period change in the log of the spot foreign exchange rate measured as the foreign currency price of the U.S. dollar. The corresponding interest rate differential \( i_t - i^*_t \) is measured as the foreign minus the U.S. interest rate. We note that for some countries interest-rate data are only available at a later starting date.

<table>
<thead>
<tr>
<th>Full Sample</th>
<th>Standard Error</th>
<th>t-Stat</th>
<th>Standard Error</th>
<th>t-Stat</th>
<th>( R^2 )</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regression</td>
<td>( \alpha )</td>
<td>( \alpha = 0 )</td>
<td>( \beta )</td>
<td>( \beta = 1 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Austria</td>
<td>-2.700</td>
<td>-1.235</td>
<td>-1.016</td>
<td>0.739</td>
<td>-2.728</td>
<td>0.005</td>
</tr>
<tr>
<td>Australia</td>
<td>3.746</td>
<td>2.441</td>
<td>1.535</td>
<td>0.575</td>
<td>-2.993</td>
<td>0.004</td>
</tr>
<tr>
<td>Belgium</td>
<td>1.182</td>
<td>2.260</td>
<td>0.523</td>
<td>0.901</td>
<td>-2.823</td>
<td>0.008</td>
</tr>
<tr>
<td>Canada</td>
<td>1.920</td>
<td>1.389</td>
<td>0.891</td>
<td>0.585</td>
<td>-3.231</td>
<td>0.006</td>
</tr>
<tr>
<td>Denmark</td>
<td>1.007</td>
<td>2.475</td>
<td>0.407</td>
<td>0.537</td>
<td>-2.490</td>
<td>0.001</td>
</tr>
<tr>
<td>Finland</td>
<td>1.418</td>
<td>2.167</td>
<td>0.654</td>
<td>0.925</td>
<td>-1.746</td>
<td>0.001</td>
</tr>
<tr>
<td>France</td>
<td>1.732</td>
<td>2.461</td>
<td>0.704</td>
<td>0.831</td>
<td>-2.092</td>
<td>0.002</td>
</tr>
<tr>
<td>Germany</td>
<td>-0.233</td>
<td>0.188</td>
<td>-1.238</td>
<td>0.861</td>
<td>-2.545</td>
<td>0.005</td>
</tr>
<tr>
<td>Greece</td>
<td>-1.598</td>
<td>4.159</td>
<td>-0.384</td>
<td>0.386</td>
<td>-2.212</td>
<td>0.001</td>
</tr>
<tr>
<td>Ireland</td>
<td>0.660</td>
<td>2.397</td>
<td>0.275</td>
<td>0.460</td>
<td>-1.881</td>
<td>0.000</td>
</tr>
<tr>
<td>Italy</td>
<td>-0.237</td>
<td>3.257</td>
<td>-0.073</td>
<td>0.611</td>
<td>-0.426</td>
<td>0.005</td>
</tr>
<tr>
<td>Japan</td>
<td>-0.865</td>
<td>0.242</td>
<td>-3.574</td>
<td>0.861</td>
<td>-4.827</td>
<td>0.036</td>
</tr>
<tr>
<td>Netherlands</td>
<td>-0.557</td>
<td>2.199</td>
<td>-0.253</td>
<td>0.924</td>
<td>-3.263</td>
<td>0.015</td>
</tr>
<tr>
<td>New Zealand</td>
<td>2.040</td>
<td>2.005</td>
<td>1.017</td>
<td>0.245</td>
<td>-7.138</td>
<td>0.025</td>
</tr>
<tr>
<td>Norway</td>
<td>1.296</td>
<td>2.385</td>
<td>0.543</td>
<td>0.523</td>
<td>-2.341</td>
<td>0.001</td>
</tr>
<tr>
<td>Portugal</td>
<td>-0.318</td>
<td>2.686</td>
<td>-0.118</td>
<td>0.429</td>
<td>-1.798</td>
<td>0.001</td>
</tr>
<tr>
<td>Spain</td>
<td>4.462</td>
<td>0.487</td>
<td>9.171</td>
<td>0.102</td>
<td>-5.673</td>
<td>0.053</td>
</tr>
<tr>
<td>Sweden</td>
<td>1.847</td>
<td>2.340</td>
<td>0.789</td>
<td>0.608</td>
<td>-1.539</td>
<td>0.000</td>
</tr>
<tr>
<td>Switzerland</td>
<td>-6.173</td>
<td>2.966</td>
<td>-2.081</td>
<td>0.605</td>
<td>-3.616</td>
<td>0.011</td>
</tr>
<tr>
<td>UK</td>
<td>0.492</td>
<td>0.236</td>
<td>2.087</td>
<td>0.847</td>
<td>-3.757</td>
<td>0.018</td>
</tr>
</tbody>
</table>
Appendix B: Decomposition of the UIP regression coefficient

In this appendix we present the background of the decomposition of the OLS regression beta in the UIP regression (1) into a component related to the risk premium and a component related to the forecast error. To see the impact from errors made in forecasting exchange rates we first write the estimated slope coefficient for the UIP regression in terms of the standard OLS formula:

(B1) \[ \beta_{OLS} = \frac{\text{cov}(i_t - i_t^*, s_{t+1} - s_t)}{\text{var}(i_t - i_t^*)}. \]

From Equation (B1) we see that a negative slope coefficient can only occur if the covariance between the interest differential and the exchange-rate change is negative, i.e. the numerator in Equation (B1). To determine the specific effects of the risk premia and exchange-rate errors on the regression coefficient, we follow Engel (1996), who decomposes the beta into a beta related to the risk premium, \( \beta_{rp} \), and a beta for the forecast errors, \( \beta_{st} \). In our case we rewrite the numerator from Equation (B1) using the decomposition in Equation (3) from the text. More specifically, we find that

(B2) \[ \text{cov}(i_t - i_t^*, s_{t+1} - s_t) = \text{var}(i_t - i_t^*) - \text{cov}(\rho_{rs}, i_t - i_t^*) - \text{cov}(\epsilon_{st}, i_t - i_t^*). \]

This decomposition allows us to write the OLS beta as
\( \beta_{GLES} = 1 - \beta_{rp} - \beta_{ss}, \)

with the beta for the risk premium \( \hat{\beta}_{rp} \) defined by

\[(B4) \quad \hat{\beta}_{rp} = \frac{\text{cov}(\rho_{i^*,i}, i_t - i_t^*)}{\text{var}(i_t^* - i_t^*)}, \]

and the beta for the forecast errors as,

\[(B5) \quad \hat{\beta}_{ss} = \frac{\text{cov}(\varepsilon_{ss}, i_t - i_t^*)}{\text{var}(i_t^* - i_t^*)}. \]

The empirical analysis in the text is based on the replacement of the moments in (B4) and (B5) with their sample equivalents.
Table 1: Results of UIP regressions based on Irving Fisher’s (1930) data for U.S. gold and greenback bonds and Indian sterling and rupee bonds

In the regressions summarized below we use the data reported in Tables 11 and 12 of Fisher's *The Theory of Interest* (1930). These regressions take the form

\[ s_{t+1} - s_t = \alpha + \beta (i_t - i_t^*) + e_{t+1}, \]

where \( s_{t+1} - s_t \) is the one-period change in the log of the spot exchange rate measured as the implicit exchange rate between gold and paper money, and the Indian rupee price of the U.K. pound sterling, respectively. The corresponding differential \( i_t - i_t^* \) is measured as the difference in yields between bonds payable in gold and paper currency, and the difference in yields between sterling and rupee bonds, respectively.

<table>
<thead>
<tr>
<th></th>
<th>Standard Error</th>
<th>t-Stat</th>
<th>Standard Error</th>
<th>t-Stat</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>U.S. Bonds</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \alpha = 0 )</td>
<td>-1.037</td>
<td>0.724</td>
<td>-1.433</td>
<td>2.608</td>
<td></td>
</tr>
<tr>
<td>( \beta = 1 )</td>
<td>1.434</td>
<td>1.122</td>
<td>0.091</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Indian Bonds</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \alpha = 0 )</td>
<td>-0.020</td>
<td>1.369</td>
<td>-0.014</td>
<td>-2.012</td>
<td></td>
</tr>
<tr>
<td>( \beta = 1 )</td>
<td>2.435</td>
<td>-1.237</td>
<td>0.019</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 2: Results of UIP regressions for non-overlapping averages of the data

The regressions we summarize below are pooled regressions that we run using the averaged data. These regressions are \( s_{t+1} - s_t = \alpha + \beta (i_t - i_t^*) + e_{t+1} \), where \( s_{t+1} - s_t \) is the monthly change in the log of the spot foreign exchange rate measured as the foreign currency price of the U.S. dollar. The corresponding interest rate differential \( i_t - i_t^* \) is measured as the foreign minus the U.S. monthly interest rate.

Observations are missing for some countries, due to later starting dates of some series.

The countries we analyze are Austria, Australia, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, the Netherlands, New-Zealand, Norway, Portugal, Spain, Sweden, Switzerland, and the UK.

<table>
<thead>
<tr>
<th>Average</th>
<th>Intercept</th>
<th>Standar d Error</th>
<th>( \alpha=0 ) Beta</th>
<th>Standar d Error</th>
<th>( \beta=1 )</th>
<th>R(^2)</th>
<th>SEE</th>
<th>Nobs</th>
</tr>
</thead>
<tbody>
<tr>
<td>5-year</td>
<td>0.233</td>
<td>0.651</td>
<td>0.359</td>
<td>0.038</td>
<td>0.166</td>
<td>-5.783</td>
<td>0.001</td>
<td>6.044</td>
</tr>
<tr>
<td>15-year</td>
<td>-0.577</td>
<td>0.306</td>
<td>-1.886</td>
<td>0.694</td>
<td>0.109</td>
<td>-2.808</td>
<td>0.583</td>
<td>1.494</td>
</tr>
<tr>
<td>30-year</td>
<td>-0.481</td>
<td>0.433</td>
<td>-1.111</td>
<td>0.583</td>
<td>0.166</td>
<td>-2.516</td>
<td>0.421</td>
<td>1.583</td>
</tr>
</tbody>
</table>
Table 3: Correlations of deviations from ex-post UIP, PPP, and RIE

This table presents correlations between deviations from three international parities. The deviations are measured as

\[(i_t - i_t^*) - (s_{t+1} - s_t)\] uncovered interest rate parity (UIP),

\[(\pi_{t+1} - \pi_t^*) - (s_{t+1} - s_t)\] relative purchasing power parity (PPP),

\[(i_t - i_t^*) - (\pi_{t+1} - \pi_t^*)\] real interest rate parity (RIE).

The currencies are the euro (EUR), British pound sterling (GBP), and Japanese yen (JPY), all measured as the foreign currency price of the U.S. dollar. The estimation period is from January 1976 to December 2005. Corresponding interest \((i_t)\) and inflation rates \((\pi_{t+1})\) are measured on a monthly frequency. ‘*’ denote U.S. equivalents.

<table>
<thead>
<tr>
<th></th>
<th>Full sample</th>
<th>UIP &amp; PPP</th>
<th>UIP &amp; RIE</th>
<th>PPP &amp; RIE</th>
</tr>
</thead>
<tbody>
<tr>
<td>EUR</td>
<td>0.994</td>
<td>0.034</td>
<td>0.110</td>
<td></td>
</tr>
<tr>
<td>JPY</td>
<td>0.981</td>
<td>0.030</td>
<td>0.171</td>
<td></td>
</tr>
<tr>
<td>GBP</td>
<td>0.981</td>
<td>-0.020</td>
<td>0.153</td>
<td></td>
</tr>
</tbody>
</table>
Table 4: Estimation results for the dynamic factor model (7) and (8)

This table presents parameter estimates for the dynamic factor model consisting of equations:

\[
\begin{pmatrix}
(i_t - i^*_t) - (s_{t,1} - s_t) \\
(\pi_{t,1} - \pi^*_{t,1}) - (s_{t,1} - s_t)
\end{pmatrix} = \begin{pmatrix} c \\ \phi (\mu + s_t + p_t^r - p_t) \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \end{pmatrix} \varepsilon_{s,t} + \begin{pmatrix} \rho_s \\ \theta_s \end{pmatrix}
\]

with

\[
\begin{pmatrix} \rho_s \\ \theta_s \end{pmatrix} \sim N\left( \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_{\rho}^2 & 0 \\ 0 & \sigma_{\theta}^2 \end{pmatrix} \right)
\]

The latent factor \( \varepsilon_{s,t+1} \) is modeled as:

\[
\varepsilon_{s,t+1} = \phi \varepsilon_{s,t} + \eta_{t,1}, \text{ with } \eta_{t,1} \sim N(0, \sigma_{\eta}^2)
\]

The currencies we use are the euro (EUR), British pound sterling (GBP), and Japanese yen (JPY), all measured as the foreign currency price of the U.S. dollar. The estimation period is from January 1976 to December 2005. Standard errors are provided in the column “Std.err”.

<table>
<thead>
<tr>
<th></th>
<th>EUR Std.err</th>
<th>GBP Std.err</th>
<th>JPY Std.err</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c )</td>
<td>0.0305</td>
<td>0.1548</td>
<td>0.0757</td>
</tr>
<tr>
<td>( \mu )</td>
<td>0.0846</td>
<td>0.6006</td>
<td>-2.5514</td>
</tr>
<tr>
<td>( \phi )</td>
<td>0.4397</td>
<td>0.4188</td>
<td>0.0312</td>
</tr>
<tr>
<td>( \phi_s )</td>
<td>0.0251</td>
<td>0.0623</td>
<td>0.0417</td>
</tr>
<tr>
<td>( \sigma_{\eta} )</td>
<td>3.2215</td>
<td>3.0527</td>
<td>3.4541</td>
</tr>
<tr>
<td>( \sigma_{\rho} )</td>
<td>0.1321</td>
<td>0.2080</td>
<td>0.1874</td>
</tr>
<tr>
<td>( \sigma_{\theta} )</td>
<td>-0.0003</td>
<td>0.0082</td>
<td>-0.0001</td>
</tr>
</tbody>
</table>

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Table 5: Moments

The currencies we use are the euro (EUR), British pound sterling (GBP), and Japanese yen (JPY), all measured as the foreign currency price of the U.S. dollar. The estimation period is from January 1976 to December 2005. We calculate $\beta_{OLS}$, $\beta_{rp}$, and $\beta_{ss}$ as

\[
\frac{\text{cov}(i_t - i_t^*, s_{t+1} - s_t)}{\text{var}(i_t - i_t^*)}, \quad \frac{\text{cov}(\rho, i_t - i_t^*)}{\text{var}(i_t - i_t^*)}, \quad \text{and} \quad \frac{\text{cov}(\epsilon_{st}, i_t - i_t^*)}{\text{var}(i_t - i_t^*)},
\]

respectively. Note that $\beta_{OLS} = 1 - \beta_{rp} - \beta_{ss}$.

<table>
<thead>
<tr>
<th></th>
<th>EUR</th>
<th>GBP</th>
<th>JPY</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{var}(s_{t+1} - s_t)$</td>
<td>10.366</td>
<td>9.352</td>
<td>11.802</td>
</tr>
<tr>
<td>$\text{var}(i_t - i_t^*)$</td>
<td>0.039</td>
<td>0.036</td>
<td>0.043</td>
</tr>
<tr>
<td>$\text{cov}(i_t - i_t^*, s_{t+1} - s_t)$</td>
<td>-0.047</td>
<td>-0.078</td>
<td>-0.135</td>
</tr>
<tr>
<td>$\text{var}(\rho, i_t)$</td>
<td>0.017</td>
<td>0.047</td>
<td>0.035</td>
</tr>
<tr>
<td>$\text{var}(\epsilon_{st})$</td>
<td>10.388</td>
<td>9.361</td>
<td>11.952</td>
</tr>
<tr>
<td>$\text{cov}(\rho, i_t - i_t^*)$</td>
<td>0.015</td>
<td>0.017</td>
<td>0.025</td>
</tr>
<tr>
<td>$\text{cov}(\epsilon_{st}, i_t - i_t^*)$</td>
<td>0.072</td>
<td>0.096</td>
<td>0.153</td>
</tr>
<tr>
<td>$\beta_{OLS}$</td>
<td>-1.185</td>
<td>-2.176</td>
<td>-3.145</td>
</tr>
<tr>
<td>$\beta_{rp}$</td>
<td>0.369</td>
<td>0.481</td>
<td>0.582</td>
</tr>
<tr>
<td>$\beta_{ss}$</td>
<td>1.815</td>
<td>2.695</td>
<td>3.562</td>
</tr>
</tbody>
</table>
Figure 1: Averages of coefficients from five-year rolling regressions for the G7 countries and one-standard-deviation bounds

We report the beta estimates from the 5 year rolling regression summarized below by using monthly data from January 1976- December 2005 obtained from *International Financial Statistics*. These regressions take the form $s_{t+1} - s_t = \alpha + \beta (i_t - i_t^*) + e_{t+1}$, where $s_{t+1} - s_t$ is the one-period change in the log of the spot foreign exchange rate measured as the foreign currency price of the U.S. dollar. The corresponding interest rate differential $i_t - i_t^*$ is measured as the foreign minus the U.S. interest rate. We note that for some countries interest-rate data are only available at a later starting date. The results for the G7 countries are reported.
Figure 2: UIP relations based on five-year, 15-year and full-period averages

Fifteen-year Averages

Full-period Averages
Figure 3: Ex post deviations from UIP, PPP, and RIE for the British Pound

We plot ex post deviations from the following three international parities:

\[
(i_t - i_t^*) - (s_{t+1} - s_t) \quad \text{uncovered interest rate parity (UIP)},
\]

\[
(s_{t+1} - s_{t+1}^*) - (s_t - s_t^*) \quad \text{relative purchasing power parity (PPP)},
\]

\[
(i_t - i_t^*) - (\pi_{t+1} - \pi_{t+1}^*) \quad \text{real interest rate parity (RIE)},
\]

where \(s_{t+1} - s_t\) is the one-period change in the log of the spot foreign exchange rate measured as the GBP price of the U.S. dollar. Corresponding GBP interest \(i_t\) and inflation rates \(\pi_{t+1}\) are measured on a monthly frequency. ‘*’ denote U.S. equivalents. The estimation period is from January 1976 to December 2005.
Figure 4: Estimated factors from the dynamic factor model

The figure plots the parameter estimates from the dynamic factor model consisting of equations:

\[
\begin{pmatrix}
    (i_t - \hat{i} - (s_{r,1} - s_t) \\
    (\pi_{t+1} - \pi_{t+1} - (s_{t+1} - s_t)
\end{pmatrix}
= \begin{pmatrix}
    c \\
    \phi (\mu + s_t + p_t - p_t)
\end{pmatrix} + \begin{pmatrix}
    1 \\
    1
\end{pmatrix} \xi_t + \begin{pmatrix}
    \rho_i \\
    \theta_t
\end{pmatrix},
\]

with \(\begin{pmatrix}
    \rho_i \\
    \theta_t
\end{pmatrix} \sim N\left(\begin{pmatrix}
    0 \\
    0
\end{pmatrix}, \begin{pmatrix}
    \sigma_{\rho}^2 & 0 \\
    0 & \sigma_{\theta}^2
\end{pmatrix}\right)\)

The latent factor \(\xi_{x,t+1}\) is modeled as:

\[\xi_{x,t+1} = \phi \xi x_t + \eta_{t+1}, \text{ with } \eta_{t+1} \sim N(0, \sigma^2)\]

The currencies we use are the euro (EUR), British pound sterling (GBP), and Japanese yen (JPY), all measured as the foreign currency price of the U.S. dollar. The estimation period is from January 1976 to December 2005.

Euro
GBP:

JPY:
Figure 5: Implied conditional variances of the deviations from UIP

This figure shows the conditional variance

$$\frac{1}{k} \text{Var} \left[ \sum_{j=1}^{k} \eta_{t+j} \right] = \frac{1}{k} \sigma^2 \sum_{j=1}^{k} \rho^{2(j-1)} + \sigma_{\eta}^2.$$  

The parameters are from Table 4.